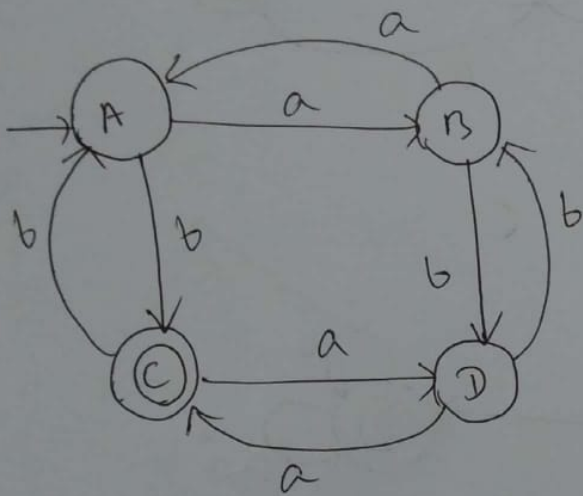


Sub: Automata Theory & Computability
 (15CS54)

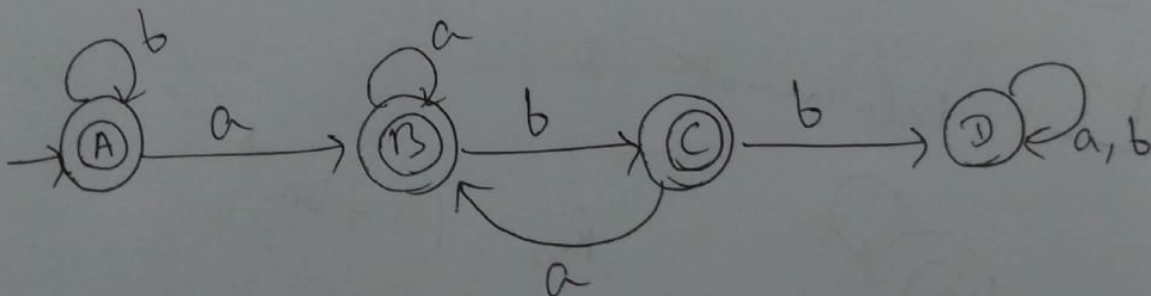
1. (i) $L = \{ w \in \{a,b\}^* : w \text{ contains an even no. of } a\text{'s and an odd no. of } b\text{'s} \}$ (5M)



Regular grammar

$A \rightarrow aB \mid bC$
 $B \rightarrow aA \mid bD$
 $C \rightarrow aD \mid bA \mid \epsilon$
 $D \rightarrow aC \mid bB$

(ii) $L = \{ w \in \{a,b\}^* : w \text{ doesn't contain the substring } abb \}$ (5M)

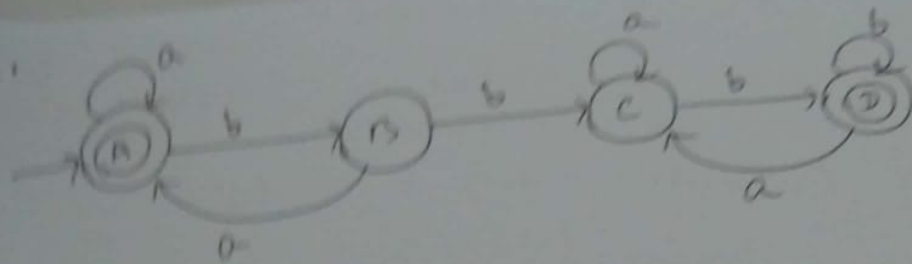


Regular grammar

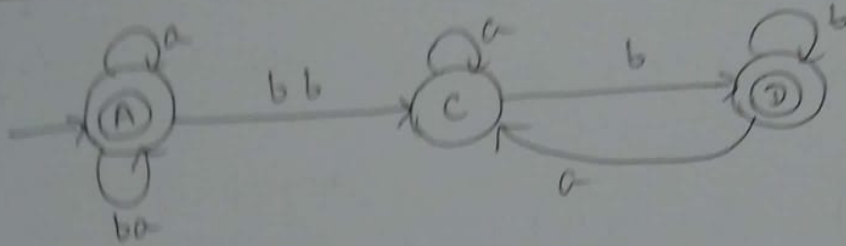
$A \rightarrow aB \mid bA \mid \epsilon$
 $B \rightarrow aB \mid bC \mid \epsilon$

$C \rightarrow aB \mid bD \mid \epsilon$
 $D \rightarrow aD \mid bD$

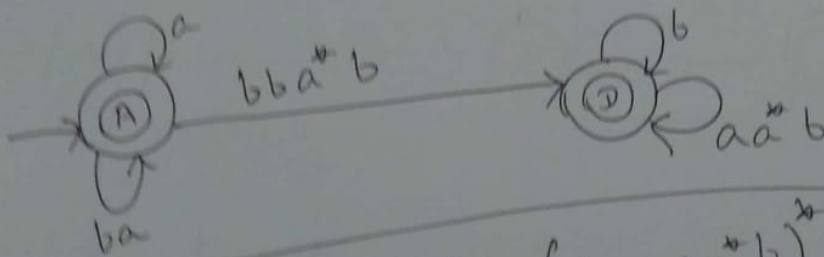
2.



Step 1 Eliminate B

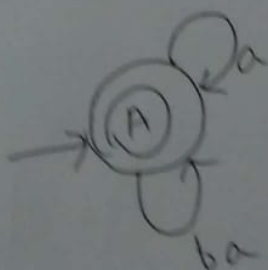


Step 2 Eliminate C



$$R_1 = (a+ba)^* bba^*b (b+aa^*b)^*$$

Step 3 Eliminate D



$$R_2 = (a+ba)^*$$

Final r.e is $R = R_1 + R_2 = (a+ba)^* [bba^*b (b+aa^*b)^* + E]$

Ambiguous Grammar

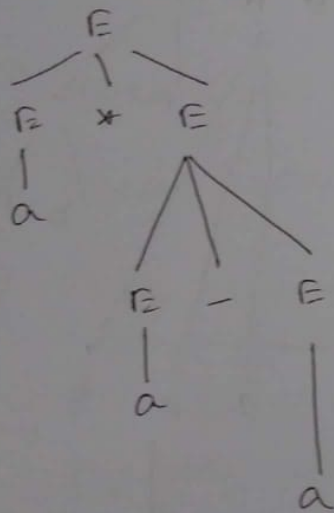
[6+4]

→ A grammar is ambiguous, if there exist more than one LMD or more than one RMD for the same string.

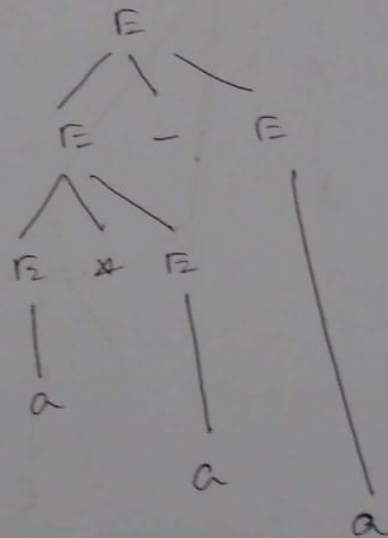
$$E \rightarrow E + E \mid E - E \mid E * E \mid E / E \mid (E) \mid E \wedge E \mid a$$

$$w = a * a - a$$

LMD1



LMD2



Yes, the grammar is ambiguous.

Equivalent Unambiguous grammar is

$$E \rightarrow E + T \mid E - T \mid T$$

$$T \rightarrow T * F \mid T / F \mid F$$

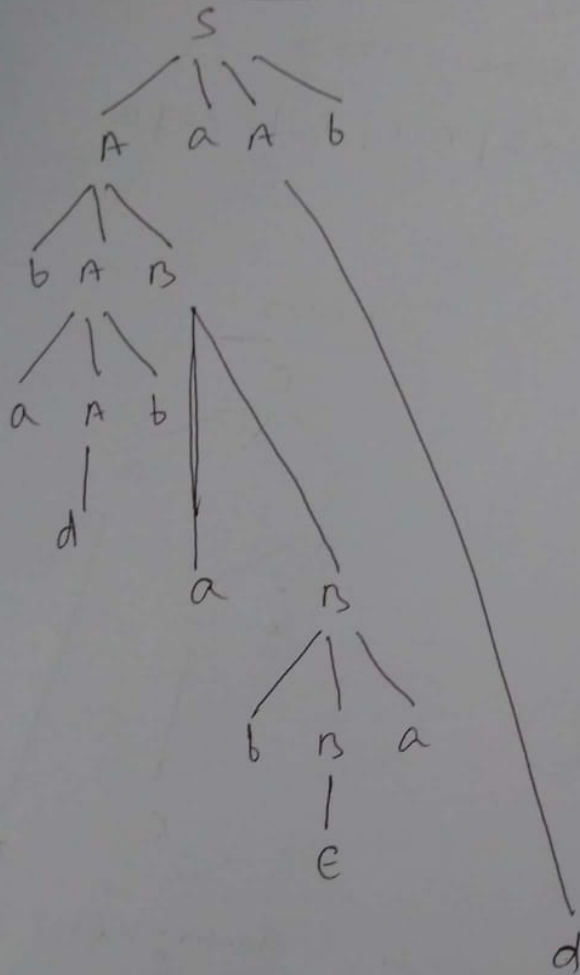
$$F \rightarrow G \wedge F \mid G$$

$$G \rightarrow (E) \mid a$$

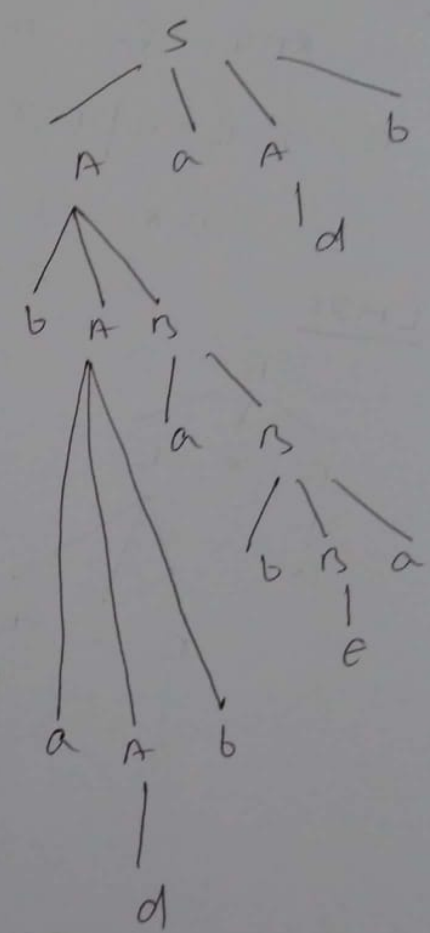
4. $W = badbabaadb$

(5)

LMD Parse Tree



RMD Parse Tree



$S \Rightarrow A a A b$

$\Rightarrow b A B a A b$ ($A \rightarrow b A B$)

$\Rightarrow b a A B B a A b$ ($A \rightarrow a A b$)

$\Rightarrow b a d B B a A b$ ($A \rightarrow d$)

$\Rightarrow b a d b A B a A b$ ($B \rightarrow a B$)

$\Rightarrow b a d b a B B a A b$ ($B \rightarrow b B a$)

$\Rightarrow b a d b a b B A a A b$ ($B \rightarrow E$) $\Rightarrow badbabaadb$ ($A \rightarrow d$)

MD

$S \Rightarrow AaAb$

$\Rightarrow Aadb \quad (A \rightarrow d)$

$\Rightarrow bAaadb \quad (A \rightarrow bAa)$

$\Rightarrow bAaadb \quad (a \rightarrow aa)$

$\Rightarrow bAabbaadb \quad (a \rightarrow bba)$

$\Rightarrow bAabEaadb \quad (a \rightarrow E)$

$\Rightarrow baAbabaaadb \quad (A \rightarrow aAb)$

$\Rightarrow badbabaaadb \quad (A \rightarrow d)$

5. (a) Push Down Automata (UP)

\rightarrow It is an automata to recognize the context free language.

It is defined by 7-tuples.

$P = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$

where Q is a finite set of states

Σ is finite set of input alphabets.

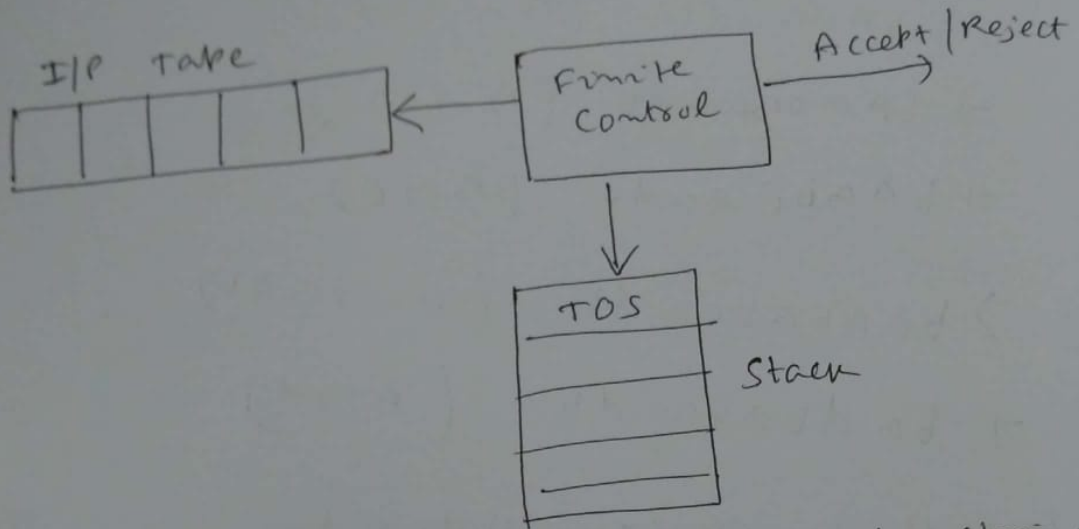
Γ is set of stack alphabets.

δ is transition function.

$\delta: Q \times \Sigma \cup \{E\} \times \Gamma \rightarrow Q \times \Gamma^+$

q_0 is start state.
 Z_0 is initial stack symbol
 F is final state.

PDA Block diagram



- The I/P tape contains input string.
- Finite Control checks the current input, top of stack (TOS), then decides which operation to do (Push or POP).
- When input becomes empty, stack is empty or it reached the final state, then it is accepted, otherwise it is rejected.

(6M)

$$(b) L = \{ w c w^R \mid w \in \{a, b\}^* \}$$

$$\delta(q_0, a, z_0) = (q_0, a z_0)$$

$$\delta(q_0, b, z_0) = (q_0, b z_0)$$

$$\delta(q_0, a, a) = (q_0, a a)$$

$$\delta(q_0, b, a) = (q_0, b a)$$

$$\delta(q_0, a, b) = (q_0, a b)$$

$$\delta(q_0, b, b) = (q_0, b b)$$

$$\delta(q_0, c, a) = (q_1, a)$$

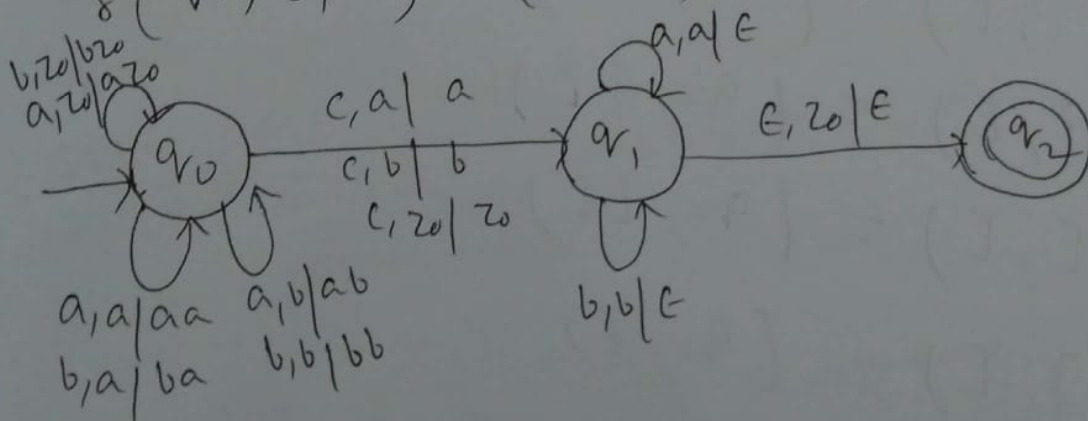
$$\delta(q_0, c, b) = (q_1, b)$$

$$\delta(q_0, c, z_0) = (q_1, z_0)$$

$$\delta(q_1, a, a) = (q_1, \epsilon)$$

$$\delta(q_1, b, b) = (q_1, \epsilon)$$

$$\delta(q_1, \epsilon, z_0) = (q_2, \epsilon)$$



ID for $w = abcba$

$$\begin{aligned}
 &(q_0, abcba, z_0) \vdash (q_0, bcba, a z_0) \vdash (q_0, cba, b a z_0) \\
 &\vdash (q_1, ba, b a z_0) \vdash (q_1, a, a z_0) \vdash (q_1, \epsilon, z_0) \\
 &\vdash (q_2, \epsilon, \epsilon) \quad \text{Accepted}
 \end{aligned}$$

$$6. \delta(q_0, \epsilon, z_0) = (q_0, \epsilon z_0)$$

$$\delta(q_0, \{, z_0) = (q_0, \{ z_0)$$

$$\delta(q_0, \lceil, z_0) = (q_0, \lceil z_0)$$

$$\delta(q_0, \lceil, \lceil) = (q_0, \lceil \lceil)$$

$$\delta(q_0, \{, \lceil) = (q_0, \{ \lceil)$$

$$\delta(q_0, \lceil, \lceil) = (q_0, \lceil \lceil)$$

$$\delta(q_0, \lceil, \lceil) = (q_0, \lceil \lceil)$$

$$\delta(q_0, \lceil, \{) = (q_0, \lceil \{)$$

$$\delta(q_0, \{, \{) = (q_0, \{ \{)$$

$$\delta(q_0, \lceil, \{) = (q_0, \lceil \{)$$

$$\delta(q_0, \{, \{) = (q_0, \epsilon)$$

$$\delta(q_0, \lceil, \lceil) = (q_0, \lceil \lceil)$$

$$\delta(q_0, \{, \lceil) = (q_0, \{ \lceil)$$

$$\delta(q_0, \lceil, \lceil) = (q_0, \lceil \lceil)$$

$$\delta(q_0, \lceil, \lceil) = (q_0, \epsilon)$$

$$\delta(q_0, \epsilon, z_0) = (q_1, \epsilon)$$

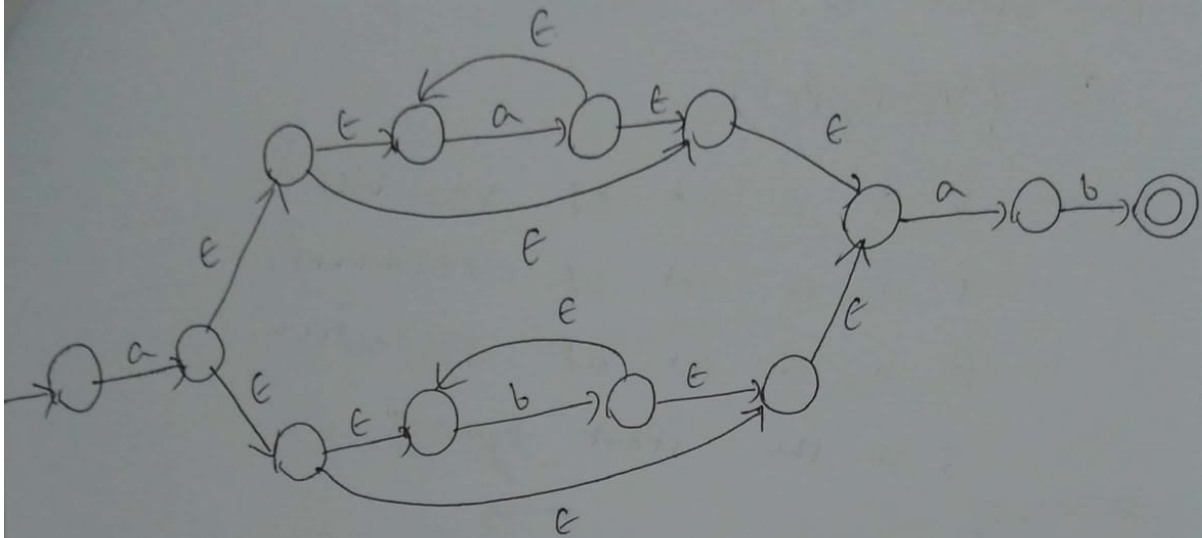
It is NPDA. Since there is a transition on $\delta(q_0, \lceil, z_0)$ & $\delta(q_0, \epsilon, z_0)$.

(2)
Defⁿ of PDA
& NPDA (2M)

Design of PDA (8M)

(a) $\gamma = a(a^* + b^*)ab$

(6M)



(b) Sentence

(4M)

→ A sentence is a string which contains only terminals.

Sentential form

When we derive a string from the start symbol S , the intermediate steps are called sentential form, which is a combination of terminals & non terminals.

Ex $E \Rightarrow E + E$ } Sentential form
 $\Rightarrow a + E$ }
 $\Rightarrow a + a \rightarrow$ sentence

8. CFG

A CFG G is defined by 4-tuples (10m)

$$G = (V, T, P, S)$$

where V is a set of variables
 T is a set of terminals
 P is a set of productions
 S is the start symbol.

$$(i) L = \{ a^m b^{m+3} \mid m \geq 0 \}$$

$$S \rightarrow a s b \mid b b b$$

$$(ii) L = \{ \text{Palindrome over } \{0,1\}^* \}$$

$$S \rightarrow 0 s 0 \mid 1 s 1 \mid 0 \mid 1 \mid \epsilon$$

$$(iii) L = \{ a^i b^j c^k \mid j = i + 2k, i, j, k \geq 0 \}$$

$$a^i b^j c^k$$

$$a^i b^{i+2k} c^k$$

$$a^i b^i b^{2k} c^k$$

$$S \rightarrow A B$$

$$A \rightarrow a A b \mid \epsilon$$

$$B \rightarrow b b B c \mid \epsilon$$