CMR INSTITUTE OFTECHNOLOGY **USN** | | | | | | | | | | |

Third Internal Test -Answer Key

Sub:	Automata Theory and Computability						Code:	**15CS54**	
Date:	22 / 11 / 2018	Duration:	90 mins	Max Marks:	50	Sem:	V	Branch:	**ISE**

Answer **ANY 5 Full** Questions

		Marks	OBE CO	OBE RBT
1 (a)	Design a PDA for the language L=**{$a^n b^m$|n≥1 and n<m}.** Show the ID for the input string w=aabbbb.	[08] 3 marks for Transition Function, 3 marks for PDA diagram 2 marks for ID	CO2	L3

Ans

$\delta(q_0, a, Z_0) = (q_0, aZ_0)$

$\delta(q_0, a, a) = (q_0, aa)$

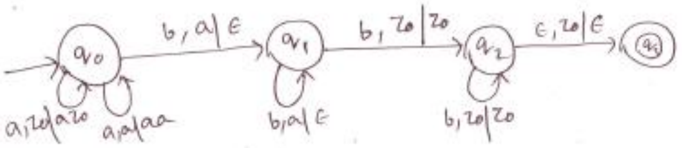
$\delta(q_0, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, a) = (q_1, \epsilon)$

$\delta(q_1, b, Z_0) = (q_2, Z_0)$

$\delta(q_2, b, Z_0) = (q_2, Z_0)$ // skips all the b's in input to reach at the end of string

$\delta(q_2, \epsilon, Z_0) = (q_3, \epsilon)$



$(q_0, aabbbb, Z_0) \vdash (q_0, abbbb, aZ_0) \vdash (q_0, bbbb, aaZ_0)$
$\vdash (q_1, bbb, aZ_0) \vdash (q_1, bb, Z_0) \vdash (q_2, b, Z_0)$
$\vdash (q_2, \epsilon, Z_0) \vdash (q_3, \epsilon, \epsilon)$

Accepted

		Marks	CO	RBT
(b)	Explain why CFG is not closed under Intersection.	[02] Example 2 marks	CO2	L3

The context-free languages are not closed under intersection:

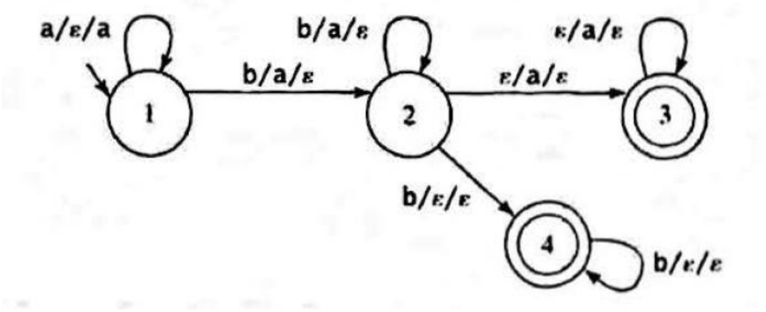
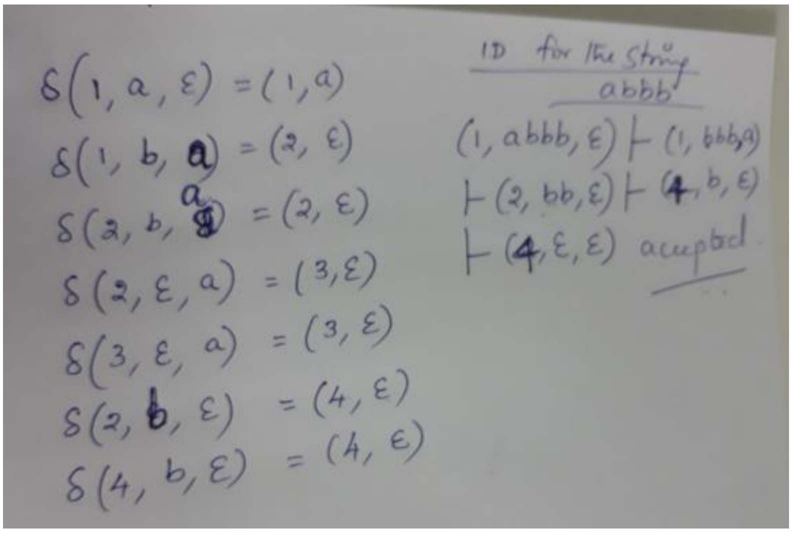
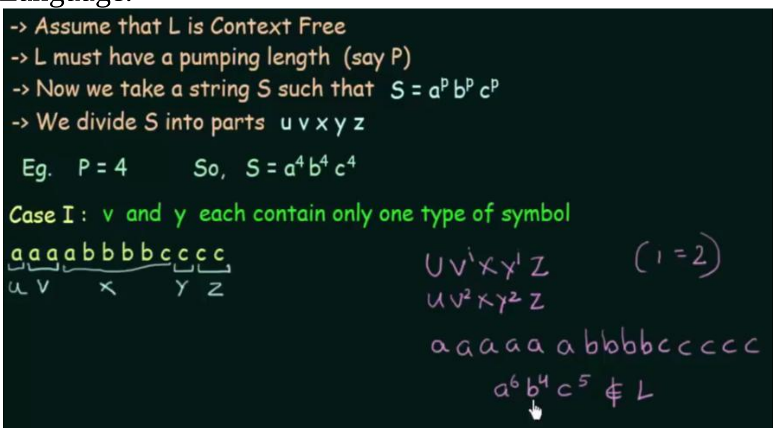
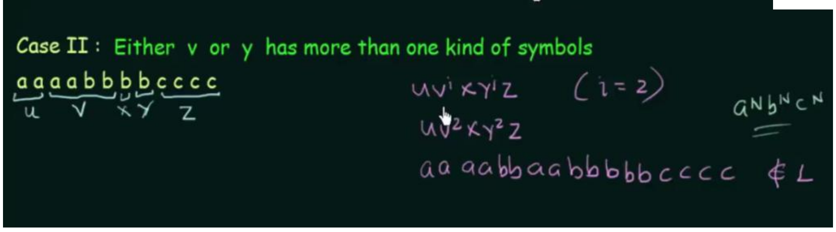
The proof is by counterexample. Let:

$L_1 = \{a^n b^n c^m : n, m \geq 0\}$ /* equal a's and b's.
$L_2 = \{a^m b^n c^n : n, m \geq 0\}$ /* equal b's and c's.

Both L_1 and L_2 are context-free, since there exist straightforward context-free grammars for them.

But now consider:
$L = L_1 \cap L_2$
$= \{a^n b^n c^n : n \geq 0\}$

		Marks	CO	RBT
2(a)	Design a PDA for the language L=**{$a^n b^m$|n,m≥1 and n≠m}.** Show the ID for the input string w=abbb.	[08] PDA 3 marks Transition Function 3 marks ID 2 marks	CO3	L3

	  			
(b)	Differentiate Decidable with Semi decidable languages Definition: for both 1 marks	[02]	CO5	L3
3.	Using pumping Lemma show that **L={$a^nb^nc^n$|n≥0 }** is not Context Free Language.   	[10] Initial Assumption 2 marks Each case: 3 marks Final Conclusion: 2marks	CO1	L2
4.	State and prove pumping lemma for Context Free Language(CFL). Show that **L= {ww | w Ɛ {a,b}*}** is not context free.	[10] Pumping Lemma: 2 marks Each case with example:	CO4	L1 &L3

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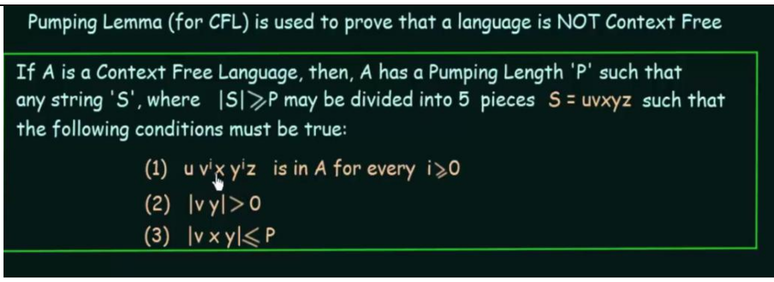

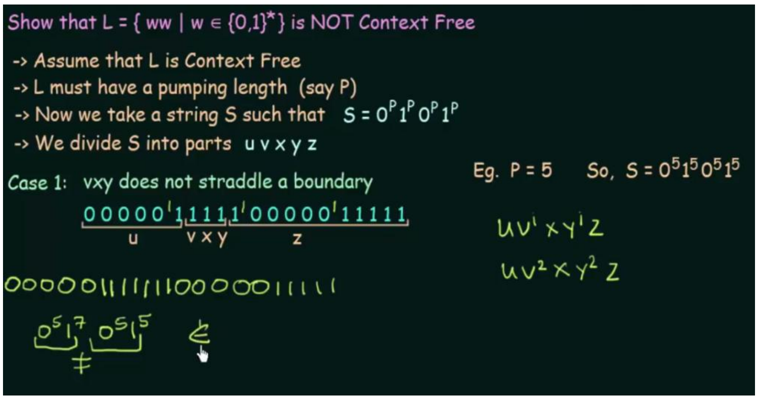

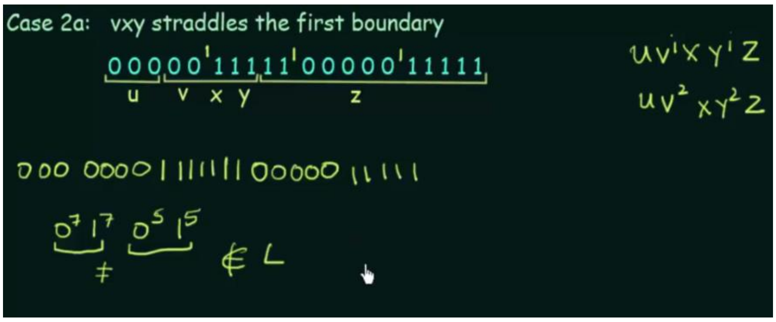

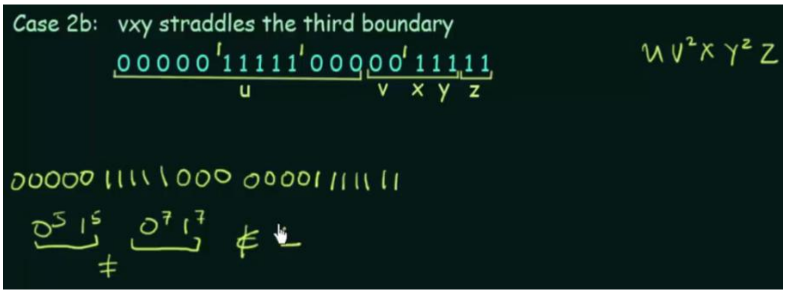

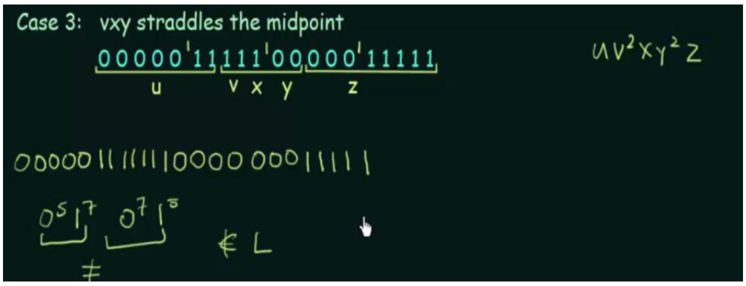

</td><td>2marks
Conclusion 2 marks</td><td></td><td></td></tr>
<tr><td>5.</td><td>Define a Turing machine. Explain the working of a basic TM with a neat diagram. Also define the language accepted by TM.</td><td>[10]

Model with Diagram: 4 marks,
Operations: 3 marks
Languages Definitions: 3 marks</td><td>CO4</td><td>L1</td></tr>
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Turing Machine Model

- TM has finite control connected to a R/W (read/write) head.
- It has one tape which is divided into a number of cells.

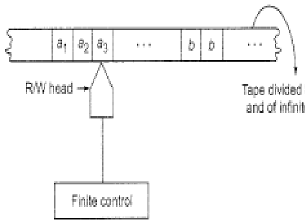


Fig. 9.1 Turing machine model.

TURING MACHINE MODEL

In one step TM does following opeartions

i. a new symbol to be written on the tape in the cell under the R/W head,

ii. a motion of the R/W head along the tape: either the head moves one cell left (L), or one cell right (R),

iii. the next state of the automaton, and

iv. whether to halt or not.

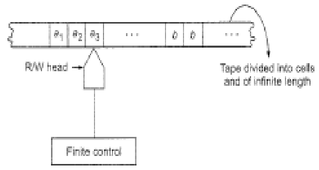


Fig. 9.1 Turing machine model.

Formal definition of TM

Definition 9.1 A Turing machine M is a 7-tuple, namely $(Q, \Sigma, \Gamma, \delta, q_0, b, F)$, where

1. Q is a finite nonempty set of states.
2. Γ is a finite nonempty set of tape symbols.
3. $b \in \Gamma$ is the blank.
4. Σ is a nonempty set of input symbols and is a subset of Γ and $b \notin \Sigma$.
5. δ is the transition function mapping (q, x) onto (q', y, D) where D denotes the direction of movement of R/W head: $D = L$ or R according as the movement is to the left or right.
6. $q_0 \in Q$ is the initial state, and
7. $F \subseteq Q$ is the set of final states.

(1) The acceptability of a string is decided by the reachability from the initial state to some final state. So the final states are also called the accepting states.

(2) δ may not be defined for some elements of $Q \times \Gamma$.

Recursive Language (Decidable) and Recursive Enumerable language (Semi Decidable) definition required.

6. Design TM that accepts **$\{1^n2^n3^n \mid n \geq 1\}$.** Write the ID's for 1223, 1123, 1233 and 112233.

[10] | CO5 | L3

Transition Table 3 marks: Diagram: 3 marks

ID for 4 strings 4 marks

Ex 4: design a TM to accept L= $\{1^n2^n3^n$:n>=1}

- Before designing the required Turing machine M, let us evolve a procedure for processing the input string 112233. After processing, we require the ID to be of the form bbbbbbq. The processing is done by using five steps:
- Step 1 : q1 is the initial state. The R/W head scans the leftmost 1, replaces 1 by b, and moves to the right. M enters q2.
- step 2: On scanning the leftmost 2, the R/W head replaces 2 by b and moves to the right. M enters q3.
- Step 3 :On scanning the leftmost 3. the R/W head replaces 3 by b, and moves to the right. M enters q4.
- Step 4: After scanning the rightmost 3, the R/W heads moves to the left until it finds the leftmost 1. As a result, the leftmost 1, 2 and 3 are replaced by b.
- Step 5 :Steps 1-4 are repeated until all 1's, 2's and 3's are replaced by blanks.

	TABLE 9.6 Transition Table for Example 9.7			

Present state	Input tape symbol 1	2	3	b
$\rightarrow q_1$	bRq_2			bRq_1
q_2	$1Rq_2$	bRq_3		bRq_2
q_3		$2Rq_3$	bRq_4	bRq_3
q_4			$3Lq_5$	bLq_7
q_5	$1Lq_5$	$2Lq_5$		bLq_5
q_6	$1Lq_6$			bRq_1
(q_7)				

- Write instantaneous descriptions for for 1223, 1233.

$$q_1 112233 \vdash bq_2 12233 \vdash b1q_2 2233 \vdash b1bq_3 233 \vdash b1b2q_3 33$$

$$\vdash b1b2bq_4 3 \vdash b1b_2 q_5 b3 \vdash b1bq_5 2b3 \vdash b1q_5 b2b3 \vdash bq_5 1b2b3$$

$$\vdash q_6 b1b2b3 \vdash bq_1 1b2b3 \vdash bbq_2 b2b3 \vdash bbbq_2 2b3$$

$$\vdash bbbbq_3 b3 \vdash bbbbbq_3 3 \vdash bbbbbbq_4 b \vdash bbbbbq_7 bb$$

7.	Write Short notes on: (a)Linear Bounded Automata (b) Non Deterministic TM Concept: 2 marks Diagram: 2 marks Difference b/w TM and LBA 1 marks	[2*5=10]	CO5	L1
8.	Design a Turing Machine for **L={0^n1^n|n≥1}.** Write the transition function for the same and also indicate the moves made by TM for input string **W=0011**.	[10]	CO4	L3

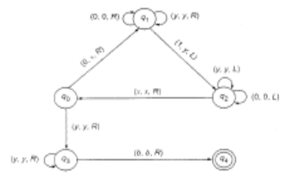
Ex 3: Design a TM to accept L= {0^n1^n: n>=1}

- ***Solution***

(a) If the leftmost symbol in the given input string w is 0, replace it by x and move right till we encounter a leftmost 1 in w. Change it to y *and* move backwards.

(b) Repeat (a) with the leftmost 0. If we move back and forth and no 0 or 1 remains, move to a final state.

(c) For strings not in the form 0^n1^n, the resulting state has to be nonfinal.



$$q_0 0011 \vdash xq_1 011 \vdash x0q_1 11 \vdash xq_2 0y1$$
$$\vdash q_2 x0y1 \vdash xq_0 0y1 \vdash xxq_1 y1 \vdash xxyq_1 1$$
$$\vdash xxq_2 yy \vdash xq_2 xyy \vdash xxq_0 yy \vdash xxyq_3 y$$
$$\vdash xxyyq_3 = xxyyq_3 b \vdash xxyybq_4 b$$

Hence 0011 is accepted by M.

$$q_0 010 \vdash xq_1 10 \vdash q_2 xy0 \vdash xq_0 y0 \vdash xyq_3 0$$

As $\delta(q_3, 0)$ is not defined, M halts. So 010 is not accepted by M.