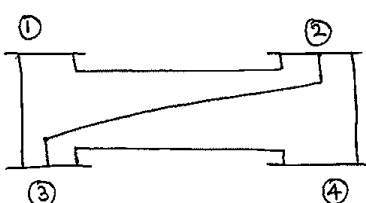


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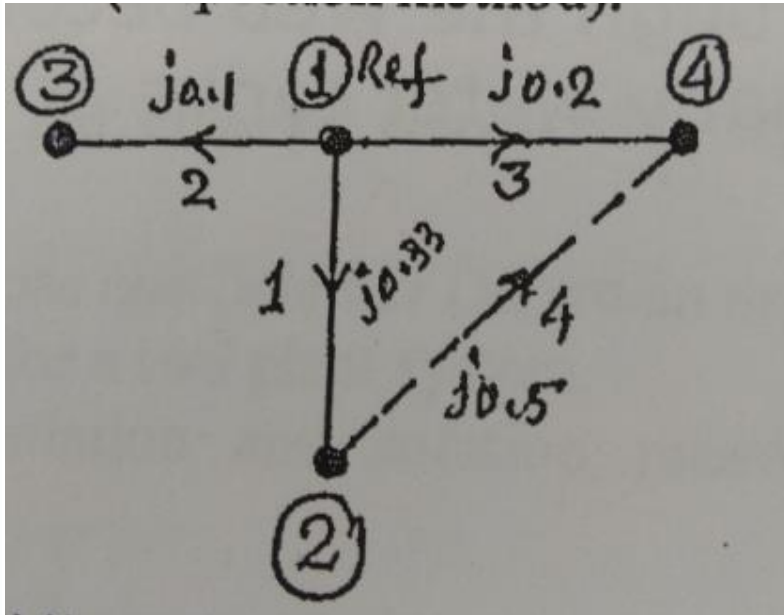
Internal Assessment Test - I

Sub:	Power System Analysis II					Code:	15EE71		
Date:	19/9/2018	Duration:	90 mins	Max Marks:	50	Sem:	7	Branch:	EEE
Answer Any FIVE FULL Questions									

	Marks	OBE																																														
		CO	RBT																																													
1a Derive an expression for obtaining the Ybus using singular transformation method	[5]	CO1	L2																																													
1b Explain in brief the various types of buses used in power system network and mention the significance of slack bus.	[5]	CO2	L2																																													
2 Obtain the Gauss –Seidal load flow solution at the end of first iteration for the power system shown in fig .Assume flat start for bus voltages V <sub>3</sub> and V <sub>4</sub> . Given :0.2<=Q2<=1.0	[10]	CO2	L3																																													
 <table border="1" style="display: inline-table; margin-right: 20px;"> <caption>Line Data</caption> <thead> <tr> <th>SB</th> <th>EB</th> <th>R(p.u)</th> <th>X(p.u)</th> </tr> </thead> <tbody> <tr><td>1</td><td>2</td><td>0.05</td><td>0.15</td></tr> <tr><td>1</td><td>3</td><td>0.10</td><td>0.30</td></tr> <tr><td>2</td><td>3</td><td>0.15</td><td>0.45</td></tr> <tr><td>2</td><td>4</td><td>0.10</td><td>0.30</td></tr> <tr><td>3</td><td>4</td><td>0.05</td><td>0.15</td></tr> </tbody> </table> <table border="1" style="display: inline-table;"> <caption>Bus Data</caption> <thead> <tr> <th>Bus No.</th> <th>P<sub>i</sub></th> <th>Q<sub>i</sub></th> <th>V<sub>i</sub></th> </tr> </thead> <tbody> <tr><td>1</td><td>-</td><td>-</td><td>1.04∠0°</td></tr> <tr><td>2</td><td>0.5</td><td>-</td><td>1.04</td></tr> <tr><td>3</td><td>-1.0</td><td>0.5</td><td>-</td></tr> <tr><td>4</td><td>-0.3</td><td>-0.1</td><td>-</td></tr> </tbody> </table>				SB	EB	R(p.u)	X(p.u)	1	2	0.05	0.15	1	3	0.10	0.30	2	3	0.15	0.45	2	4	0.10	0.30	3	4	0.05	0.15	Bus No.	P <sub>i</sub>	Q <sub>i</sub>	V <sub>i</sub>	1	-	-	1.04∠0°	2	0.5	-	1.04	3	-1.0	0.5	-	4	-0.3	-0.1	-	
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4 With the help of necessary equation explain how load flow analysis is conducted in NR method.	[10]	CO2	L2																																													
5a Explain briefly the primitive network used in the formation of bus admittance matrix by singular transformation	[5]	CO1	L2																																													
5b The bus incidence matrix A for a network of 8 elements and 5 nodes is as given below. Reconstruct the oriented graph. Hence obtain the one line diagram of the system indicating the generator positions.	[5]	CO1	L3																																													
<p>A=</p> <table border="1" style="margin-left: 20px;"> <thead> <tr> <th></th> <th>1</th> <th>2</th> <th>3</th> <th>4</th> <th>5</th> <th>6</th> <th>7</th> <th>8</th> </tr> </thead> <tbody> <tr> <th>1</th> <td>1</td> <td>0</td> <td>0</td> <td>0</td> <td>-1</td> <td>0</td> <td>-1</td> <td>0</td> </tr> <tr> <th>2</th> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> <td>-1</td> <td>0</td> <td>-1</td> </tr> <tr> <th>3</th> <td>0</td> <td>0</td> <td>1</td> <td>-1</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> </tr> <tr> <th>4</th> <td>0</td> <td>0</td> <td>0</td> <td>1</td> <td>0</td> <td>0</td> <td>1</td> <td>1</td> </tr> </tbody> </table>					1	2	3	4	5	6	7	8	1	1	0	0	0	-1	0	-1	0	2	0	1	0	0	1	-1	0	-1	3	0	0	1	-1	0	1	0	0	4	0	0	0	1	0	0	1	1
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4	0	0	0	1	0	0	1	1																																								

- 6 With the help of singular transformation method, determine the bus admittance matrix  $Y_{bus}$  for the power system whose oriented graph is shown in fig. Element no and self impedance of the elements in pu are marked on the diagram. Neglect mutual coupling. Verify the same using direct inspection method.

[10]	CO1	L2
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## Solution

1A

performance equation relating the bus voltages to bus current injections in bus frame of reference in admittance form is given by

$$I_{BUS} = Y_{BUS} E_{BUS} \quad (17)$$

Where  $E_{BUS}$  = vector of bus voltages measured with respect to reference bus

$I_{BUS}$  = Vector of currents injected into the bus

$Y_{BUS}$  = bus admittance matrix

The performance equation of the primitive network in admittance form is given by

$$i + j = [y] v$$

Pre-multiplying by  $A^t$  (transpose of  $A$ ), we obtain

$$A^t i + A^t j = A^t [y] v \quad (18)$$

However, as per equation (4),

$$A^t i = 0,$$

since it indicates a vector whose elements are the algebraic sum of element currents incident at a bus, which by Kirchhoff's law is zero. Similarly,  $A^t j$  gives the algebraic sum of all source currents incident at each bus and this is nothing but the total current injected at the bus. Hence,

$$A^t j = I_{BUS} \quad (19)$$

Thus from (18) we have,  $I_{BUS} = A^t [y] v$  (20)

However, from (5), we have

$$v = A E_{BUS}$$

And hence substituting in (20) we get,

$$I_{BUS} = A^t [y] A E_{BUS} \quad (21)$$

Comparing (21) with (17) we obtain,

$$Y_{BUS} = A^t [y] A \quad (22)$$

The bus incidence matrix is rectangular and hence singular. Hence, (22) gives a singular transformation of the primitive admittance matrix  $[y]$ . The bus impedance matrix is given by,

$$Z_{BUS} = Y_{BUS}^{-1} \quad (23)$$

Note: This transformation can be derived using the concept of power invariance, however, since the transformations are based purely on KCL and KVL, the transformation will obviously be power invariant.

## Load Flows

### Introduction

Load flow solution is a solution of the network under steady state condition subject to certain inequality constraints under which the system operates. These constraints can be in the form of load nodal voltages, reactive power generation of the generators, the tap settings of a tap changing under load transformer etc.

The load flow solution gives the nodal voltages and phase angles and hence the power injection at all the buses and power flows through inter-connecting power channels (transmission lines). Load flow solution is essential for designing a new power system and for planning extension of the existing one for increased load demand. These analyses require the calculation of numerous load flows under both normal and abnormal (outage of transmission lines, or outage of some generating source) operating conditions. Load flow solution also gives the initial conditions of the system when the transient behaviour of the system is to be studied.

Load flow solution for power network can be worked out both ways according as it is operating under (i) balanced, or (ii) unbalanced conditions. The following treatment will be for a system operating under balanced conditions only. For such a system a single phase representation is adequate. A load flow solution of the power system requires mainly the following steps:

- (i) Formulation of the network equations.
- (ii) Suitable mathematical technique for solution of the equations.

Since we are studying the system under steady state conditions the network equations will be in the form of simple algebraic equations. The load and hence generation are continually changing in a real power system. We will assume here that loads and hence generation are fixed at a particular value over a suitable period of time, e.g. half an hour or so.

### 18.1 Bus Classification

In a power system each bus or node is associated with four quantities, real and reactive powers, bus voltage magnitude and its phase angle. In a load flow solution two out of the four quantities are specified and the remaining two are required to be obtained through the solution of the equations. Depending upon which quantities have been specified, the buses are classified in the following three categories:

1. *Load bus:* At this bus the real and reactive components of power are specified. It is desired to find out the voltage magnitude and phase angle through the load flow solution. It is required to specify only  $P_D$  and  $Q_D$  at such a bus as at a load bus voltage can be allowed to vary within the permissible values e.g. 5%. Also phase angle of the voltage is not very important for the load.

2. *Generator bus or voltage controlled bus:* Here the voltage magnitude corresponding to the generation voltage and real power  $P_G$  corresponding to its ratings are specified. It is required to find out the reactive power generation  $Q_G$  and the phase angle of the bus voltage.

3. *Slack, swing or reference bus:* In a power system there are mainly two types of buses: load and generator buses. For these buses we have specified the real power  $P$  injections. Now  $\sum_{i=1}^n P_i = \text{real power loss } P_L$  where  $P_i$  is the power injection at the buses, which is taken as positive for generator buses and is negative for load buses. The losses remain unknown until the load flow solution is complete. It is for this reason that generally one of the generator buses is made to take the additional real and reactive power to supply transmission losses. That is why this type of bus is also known as the slack or swing bus. At this bus, the voltage magnitude  $V$  and phase angle  $\delta$  are specified whereas real and reactive powers  $P_G$  and  $Q_G$  are obtained through the load flow solution. The following table summarises the above discussion:

<i>bus type</i>	<i>Quantities specified</i>	<i>Quantities to be obtained</i>
Load bus	$P, Q$	$ V , \delta$
Generator bus	$P,  V $	$Q, \delta$
Slack bus	$ V , \delta$	$P, Q$

The phase angle of the voltage at the slack bus is usually taken as the reference. In the following analysis the real and reactive components of voltage at a bus are taken as the independent variables for the load flow equations i.e.

$$V_i \angle \delta_i = e_i + jf_i$$

where  $e_i$  and  $f_i$  are the real and reactive components of voltage at the  $i$ th bus. There are various other formulations wherein either voltage or current or both are taken as the independent variables. The load flow equations can be formulated using either the loop or bus frame of reference. However, from the viewpoint of computer time and memory, the nodal admittance formulation, using the nodal voltages as the independent variables is the most economic.

**Nodal Admittance Matrix**

Modified  $Y_{BUS}$  is written below

$$Y_{BUS} = \begin{bmatrix} 3 - j9 & -2 + j6 & -1 + j3 & 0 \\ -2 + j6 & 3.666 - j11 & -0.666 + j2 & -1 + j3 \\ -1 + j3 & -0.666 + j2 & 3.666 - j11 & -2 + j6 \\ 0 & -1 + j3 & -2 + j6 & 3 - j9 \end{bmatrix}$$

In Example 6.4, let bus 2 be a PV bus now with  $|V_2| = 1.04$  pu. Once again assuming a flat voltage start, find  $Q_2$ ,  $\delta_2$ ,  $V_3$ ,  $V_4$  at the end of the first iteration.

Given:  $0.2 \leq Q_2 \leq 1$ .

From Eq. (6.5), we get (Note  $\delta_2^0 = 0$ , i.e.  $V_2^0 = 1.04 + j0$ )

$$Q_2^1 = -\text{Im} \{ (V_2^0)^* Y_{21} V_1 + (V_2^0)^* [Y_{22} V_2^0 + Y_{23} V_3^0 + Y_{24} V_4^0] \}$$

$$= -\text{Im} \{ 1.04 (-2 + j6) 1.04 + 1.04 [(3.666 - j11) 1.04 + (-0.666 + j2) + (-1 + j3)] \}$$

$$= -\text{Im} \{ -0.0693 - j0.2079 \} = 0.2079 \text{ pu}$$

$\therefore Q_2^1 = 0.2079 \text{ pu}$

From Eq. (6.51)

$$\begin{aligned} \delta_2^1 &= \angle \left\{ \frac{1}{Y_{22}} \left[ \frac{P_2 - jQ_2}{(V_2^0)^*} - Y_{21} V_1 - Y_{23} V_3^0 - Y_{24} V_4^0 \right] \right\} \\ &= \angle \left\{ \frac{1}{3.666 - j11} \left[ \frac{0.5 - j0.2079}{1.04 - j0} - (-2 + j6)(1.04 + j0) \right. \right. \\ &\quad \left. \left. - (-0.666 + j2)(1 + j0) - (-1 + j3)(1 + j0) \right] \right\} \\ &= \angle \left( \frac{4.2267 - j11.439}{3.666 - j11} \right) = \angle (1.0512 + j0.0339) \end{aligned}$$

$$\text{or } \delta_2^1 = 1.84658^\circ = 0.032 \text{ rad}$$

$$\begin{aligned} \therefore V_2^1 &= 1.04 (\cos \delta_2^1 + j \sin \delta_2^1) \\ &= 1.04 (0.99948 + j0.0322) \\ &= 1.03946 + j0.03351 \end{aligned}$$

$$\begin{aligned} V_1^1 &= \frac{1}{Y_{11}} \left\{ \frac{P_1 - jQ_1}{(V_1^0)^*} - Y_{12} V_2^1 - Y_{13} V_3^0 - Y_{14} V_4^0 \right\} \\ &= \frac{1}{3.666 - j11} \left[ \frac{-1 - j0.5}{(1 - j0)} - (-1 + j3) 1.04 \right. \\ &\quad \left. - (-0.666 + j2)(1.03946 + j0.03351) - (-2 + j6) \right] \\ &= \frac{2.7992 - j11.6766}{3.666 - j11} = 1.0317 - j0.08937 \end{aligned}$$

$$\begin{aligned} V_4^1 &= \frac{1}{Y_{44}} \left\{ \frac{P_4 - jQ_4}{(V_4^0)^*} - Y_{41} V_1 - Y_{42} V_2^1 - Y_{43} V_3^0 \right\} \\ &= \frac{1}{3 - j9} \left[ \frac{0.3 + j0.1}{1 - j0} - (-1 + j3)(1.0394 + j0.0335) \right. \\ &\quad \left. - (-2 + j6)(1.0317 - j0.08937) \right] \\ &= \frac{2.9671 - j8.9962}{3 - j9} = 0.9985 - j0.0031 \end{aligned}$$

Now, suppose the permissible limits on  $Q_2$  (reactive power injection) are revised as follows:

Similarly  $V_4$  can be evaluated.

### 18.5 Newton-Raphson Method

*Development of load flow equations:* The load flow problem can also be solved by using Newton Raphson method. The equations for the method are derived as follows:

We know that at any bus  $p$ ,

$$P_p - jQ_p = V_p^* I_p = V_p^* \sum_{q=1}^n Y_{pq} V_q$$

Let

$$V_p = e_p + jf_p$$

and

$$Y_{pq} = G_{pq} - jB_{pq}$$

$$\begin{aligned} P_p - jQ_p &= (e_p + jf_p)^* \sum_{q=1}^n (G_{pq} - jB_{pq})(e_q + jf_q) \\ &= (e_p - jf_p) \sum_{q=1}^n (G_{pq} - jB_{pq})(e_q + jf_q) \end{aligned}$$

Separating the real and imaginary parts we have

$$P_p = \sum_{q=1}^n \{e_p(e_q G_{pq} + f_q B_{pq}) + f_p(f_q G_{pq} - e_q B_{pq})\} \quad (18.26)$$

and

$$Q_p = \sum_{q=1}^n \{f_p(e_q G_{pq} + f_q B_{pq}) - e_p(f_q G_{pq} - e_q B_{pq})\} \quad (18.25)$$

also

$$|V_p|^2 = e_p^2 + f_p^2 \quad (18.27)$$



These three sets of equations are the load flow equations and it can be seen that they are non-linear equations in terms of the real and imaginary components of nodal voltages. Here the left hand quantities i.e.  $P_p$ ,  $Q_p$  (for a load bus) and  $P_p$  and  $|V_p|$  for generator bus are specified and  $e_p$  and  $f_p$  are unknown quantities. For an  $n$ -bus system, the number of unknowns are  $2(n-1)$  because the voltage at the slack bus is known and is kept fixed both in magnitude and phase. Therefore, if bus 1 is taken as the slack, the unknown variables are  $(e_2, e_3, \dots, e_{n-1}, e_n, f_2, f_3, \dots, f_{n-1}, f_n)$ . Thus, to solve the problem for  $2(n-1)$  variables we need to solve  $2(n-1)$  set of equations.

Newton-Raphson method is an iterative method which approximates the set of non-linear simultaneous equations to a set of linear simultaneous equations using Taylor's series expansion and the terms are limited to first approximation.

The mathematical background of this method is explained as follows:

Let the unknown variables be  $(x_1, x_2, \dots, x_n)$  and the specified quantities  $y_1, y_2, \dots, y_n$ . These are related by the set of non-linear equations:

$$\begin{aligned} y_1 &= f_1(x_1, x_2, \dots, x_n) \\ y_2 &= f_2(x_1, x_2, \dots, x_n) \\ &\vdots \\ y_n &= f_n(x_1, x_2, \dots, x_n) \end{aligned} \quad (18.28)$$

To solve these equations we start with an approximate solution  $(x_1^0, x_2^0, \dots, x_n^0)$ . Here superscript zero means the zeroth iteration in the process of solving the above non-linear equations (18.28). It is to be noted that the initial solution for the equations should not be very far from the actual solution. Otherwise, there are chances of the solution diverging rather than converging and it may not be possible to achieve a solution whatever be the computer time utilized. At first glance it may appear to be a great drawback for the Newton-Raphson technique but the problem of initial guess for a power system is not at all difficult. A flat voltage profile i.e.  $V_p = 1.0 + j0.0$  for  $p = 1, 2, \dots, n$  except the slack bus has been found to be satisfactory for almost all practical systems.

The equations are linearized about the initial guess. We will expand first equation  $y_1 = f_1$  and the result for the other equations will follow.

Assume  $\Delta x_1^0, \Delta x_2^0, \dots, \Delta x_n^0$  are the corrections required for  $x_1^0, x_2^0, \dots, x_n^0$  respectively for the next better solution. The equation  $y_1 = f_1$  will be

$$\begin{aligned} y_1 &= f_1(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, \dots, x_n^0 + \Delta x_n^0) \\ &= f_1(x_1^0, x_2^0, \dots, x_n^0) + \Delta x_1^0 \left. \frac{\partial f_1}{\partial x_1} \right|_{x^0} + \Delta x_2^0 \left. \frac{\partial f_1}{\partial x_2} \right|_{x^0} + \dots + \Delta x_n^0 \left. \frac{\partial f_1}{\partial x_n} \right|_{x^0} + \phi_1 \end{aligned}$$

where  $\phi_1$  is function of higher order of  $\Delta x^0$  and higher derivatives which are neglected according to Newton-Raphson method. In fact it is this assumption which needs the initial solution to be close to the final solution. If all the

$$\begin{bmatrix} y_1 - f_1(x_1^0, x_2^0, \dots, x_n^0) \\ y_2 - f_2(x_1^0, x_2^0, \dots, x_n^0) \\ \vdots \\ y_n - f_n(x_1^0, x_2^0, \dots, x_n^0) \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \dots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \vdots \\ \Delta x_n^0 \end{bmatrix} \quad (18.29)$$

$$B = J \cdot C$$

Here  $J$  is the first derivative matrix known as the Jacobian matrix. The solution of the equations requires calculation of left hand vector  $B$  which is the difference of the specified quantities and calculated quantities at  $(x_1^0, \dots, x_n^0)$ . Similarly  $J$  is calculated at this guess. Solution of the matrix equation gives  $(\Delta x_1^0, \Delta x_2^0, \dots, \Delta x_n^0)$  and the next better solution is obtained as follows:

$$\begin{aligned} x_1^1 &= x_1^0 + \Delta x_1^0 \\ x_2^1 &= x_2^0 + \Delta x_2^0 \\ &\vdots \\ x_n^1 &= x_n^0 + \Delta x_n^0 \end{aligned}$$

The better solution is now available and is

$$(x_1^1, x_2^1, x_3^1, \dots, x_n^1)$$

With these values the process is repeated till (i) the largest (in magnitude) element in the left column of the equations is less than a prespecified value or (ii) the largest element in the column vector  $(\Delta x_1, \Delta x_2, \dots, \Delta x_n)$  is less than a prespecified value.

When referred to a power system problem (assuming there is only one generator bus which is taken as slack bus and all other buses are load buses), the above set of linearized equations become

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \vdots \\ \Delta P_n \\ \Delta Q_2 \\ \Delta Q_3 \\ \vdots \\ \Delta Q_n \end{bmatrix}_{2(n-1) \times 1} = \begin{bmatrix} \frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial e_3} & \dots & \frac{\partial P_2}{\partial e_n} & \frac{\partial P_2}{\partial f_2} & \frac{\partial P_2}{\partial f_3} & \dots & \frac{\partial P_2}{\partial f_n} \\ \frac{\partial P_3}{\partial e_2} & \frac{\partial P_3}{\partial e_3} & \dots & \frac{\partial P_3}{\partial e_n} & \frac{\partial P_3}{\partial f_2} & \frac{\partial P_3}{\partial f_3} & \dots & \frac{\partial P_3}{\partial f_n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial P_n}{\partial e_2} & \frac{\partial P_n}{\partial e_3} & \dots & \frac{\partial P_n}{\partial e_n} & \frac{\partial P_n}{\partial f_2} & \frac{\partial P_n}{\partial f_3} & \dots & \frac{\partial P_n}{\partial f_n} \\ \frac{\partial Q_2}{\partial e_2} & \frac{\partial Q_2}{\partial e_3} & \dots & \frac{\partial Q_2}{\partial e_n} & \frac{\partial Q_2}{\partial f_2} & \frac{\partial Q_2}{\partial f_3} & \dots & \frac{\partial Q_2}{\partial f_n} \\ \frac{\partial Q_3}{\partial e_2} & \frac{\partial Q_3}{\partial e_3} & \dots & \frac{\partial Q_3}{\partial e_n} & \frac{\partial Q_3}{\partial f_2} & \frac{\partial Q_3}{\partial f_3} & \dots & \frac{\partial Q_3}{\partial f_n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial Q_n}{\partial e_2} & \frac{\partial Q_n}{\partial e_3} & \dots & \frac{\partial Q_n}{\partial e_n} & \frac{\partial Q_n}{\partial f_2} & \frac{\partial Q_n}{\partial f_3} & \dots & \frac{\partial Q_n}{\partial f_n} \end{bmatrix}_{2(n-1) \times 2(n-1)} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \vdots \\ \Delta e_n \\ \Delta f_2 \\ \Delta f_3 \\ \vdots \\ \Delta f_n \end{bmatrix}_{2(n-1) \times 1} \quad (18.30)$$

In short form it can be written as

$$\begin{bmatrix} \Delta P \\ - \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ - & - \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta e \\ - \\ \Delta f \end{bmatrix}$$

In case the system contains all types of buses, the set of equations can be written as

$$\begin{bmatrix} \Delta P \\ - \\ \Delta Q \\ | \Delta V_p|^2 \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ - & - \\ J_3 & J_4 \\ - & - \\ J_5 & J_6 \end{bmatrix} \begin{bmatrix} \Delta e \\ - \\ \Delta f \end{bmatrix}$$

The elements of the Jacobian matrix can be derived from the three load flow equations (18.25)-(18.27).

The off-diagonal elements of  $J_1$  are

$$\frac{\partial P_p}{\partial e_q} = e_p G_{pq} - f_p B_{pq}, \quad q \neq p \quad (18.31)$$

and the diagonal elements of  $J_1$  are

$$\begin{aligned} \frac{\partial P_p}{\partial e_p} &= 2e_p G_{pp} + f_p B_{pp} - f_p B_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n (e_q G_{pq} + f_q B_{pq}) \\ &= 2e_p G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n (e_q G_{pq} + f_q B_{pq}) \end{aligned} \quad (18.32)$$

The off-diagonal elements of  $J_2$  are

$$\frac{\partial P_p}{\partial f_q} = e_p B_{pq} + f_p G_{pq}, \quad q \neq p \quad (18.33)$$

and the diagonal elements of  $J_2$  are

$$\frac{\partial P_p}{\partial f_p} = 2f_p G_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n (f_q G_{pq} - e_q B_{pq}) \quad (18.34)$$

The off-diagonal elements of  $J_3$  are

$$\frac{\partial Q_p}{\partial e_q} = e_p B_{pq} + f_p G_{pq}, \quad q \neq p \quad (18.35)$$

and the diagonal elements are

$$\frac{\partial Q_p}{\partial e_p} = 2e_p B_{pp} - \sum_{\substack{q=1 \\ q \neq p}}^n (f_q G_{pq} - e_q B_{pq}) \quad (18.36)$$

The off-diagonal and diagonal elements of  $J_4$ , respectively are

$$\frac{\partial Q_p}{\partial f_q} = -e_p G_{pq} + f_p B_{pq}, \quad q \neq p \quad (18.37)$$

$$\frac{\partial Q_p}{\partial f_p} = 2f_p B_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^n (e_q G_{pq} + f_q B_{pq}) \quad (18.38)$$

The off-diagonal and diagonal elements of  $J_5$  are

$$\frac{\partial |V_p|^2}{\partial e_q} = 0, \quad q \neq p \quad (18.39)$$

and

$$\frac{\partial |V_p|^2}{\partial e_p} = 2e_p \quad (18.40)$$

The off-diagonal and diagonal elements of  $J_6$  are

$$\frac{\partial |V_p|^2}{\partial f_q} = 0, \quad q \neq p \quad (18.41)$$

and

$$\frac{\partial |V_p|^2}{\partial f_p} = 2f_p \quad (18.42)$$

Next, we calculate the residual column vector consisting of  $\Delta P$ ,  $\Delta Q$  and  $|\Delta V|^2$ . Let  $P_{sp}$ ,  $Q_{sp}$ , and  $|V_{sp}|$  be the specified quantities at the bus  $p$ . Assuming a suitable value of the solution (flat voltage profile in our case) the value of  $P$ ,  $Q$  and  $|V|$  at the various buses are calculated. Then

$$\begin{aligned} \Delta P_p &= P_{sp} - P_p^0 \\ \Delta Q_p &= Q_{sp} - Q_p^0 \\ |\Delta V_p|^2 &= |V_{sp}|^2 - |V_p^0|^2 \end{aligned} \quad (18.43)$$

where the superscript zero means the value calculated corresponding to initial guess i.e. zeroth iteration.

Having calculated the Jacobian matrix and the residual column vector corresponding to the initial guess (initial solution) the desired increment voltage vector  $\begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix}$  can be calculated by using any standard technique (preferably Gauss elimination with sparsity techniques). The next better solution will be

$$e_p^1 = e_p^0 + \Delta e_p^0$$

$$f_p^1 = f_p^0 + \Delta f_p^0$$

These values of voltages will be used in the next iteration. The process will be repeated and in general the new better estimates for bus voltages will be

$$e_p^{k+1} = e_p^k + \Delta e_p^k$$

$$f_p^{k+1} = f_p^k + \Delta f_p^k$$

The process is repeated till the magnitude of the largest element in the residual column vector is less than the prespecified value. The sequence of steps for the solution of load flow problem using Newton-Raphson method is explained as follows (flow chart in Fig. 18.9):

1. Assume a suitable solution for all buses except the slack bus. Let  $V_p = 1 + j0.0$  for  $p = 1, 2, \dots, n, p \neq s, V_s = a + j0.0$ .
2. Set convergence criterion  $= \epsilon$  i.e. if the largest of absolute of the residues exceeds  $\epsilon$  the process is repeated, otherwise it is terminated.
3. Set iteration count  $K = 0$
4. Set bus count  $p = 1$ .
5. Check if  $p$  is a slack bus. If yes, go to step 10.
6. Calculate the real and reactive powers  $P_p$  and  $Q_p$  respectively using equations (18.26) and (18.25).
7. Evaluate  $\Delta P_p^k = P_{sp} - P_p^k$ .
8. Check if the bus in question is a generator bus. If yes, compare the  $Q_p^k$  with the limits. If it exceeds the limit, fix the reactive power generation to the corresponding limit and treat the bus as a load bus for that iteration and go to next step. If the lower limit is violated set  $Q_{psp} = Q_{p \min}$ . If the limit is not violated evaluate the voltage residue.

$$|\Delta V_p|^2 = |V_{p|spec}|^2 - |V_p^k|^2$$

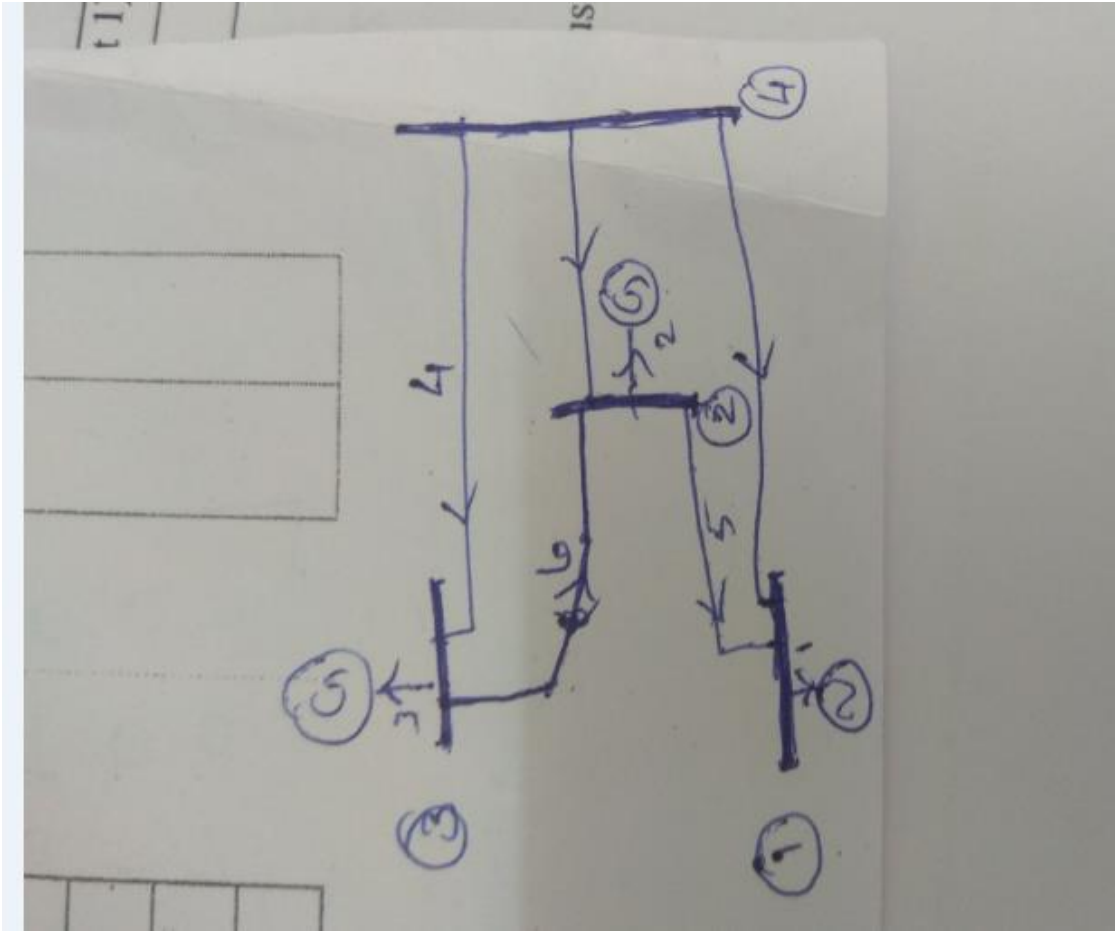
and go to step 10.

9. Evaluate  $\Delta Q_p^k = Q_{ps} - Q_p^k$ .
10. Advance the bus count by 1, i.e.  $p = p + 1$  and check if all the buses have been accounted. If not, go to step 5.
11. Determine the largest of the absolute value of the residue.
12. If the largest of the absolute value of the residue is less than  $\epsilon$ , go to step.17.
13. Evaluate elements for Jacobian matrix.
14. Calculate voltage increments  $\Delta e_p^k$  and  $\Delta f_p^k$ .
15. Calculate new bus voltages  $e_p^{k+1} = e_p^k + \Delta e_p^k$  and  $f_p^{k+1} = f_p^k + \Delta f_p^k$ .

Evaluate  $\cos \delta$  and  $\sin \delta$  of all voltages.

16. Advance iteration count  $K = K + 1$  and go to step 4.
17. Evaluate bus and line powers and print the results.

**EXAMPLE 18.4:** The load flow data for the sample power system are given below. The voltage magnitude at bus 2 is to be maintained at 1.04 p.u. The maximum and minimum reactive power limits of the generator at bus 2 are 0.35 and 0.0 p.u. respectively. Determine the set of load flow equations using Newton-Raphson method.



5B

## PRIMITIVE NETWORKS

So far, the matrices of the interconnected network have been defined. These matrices contain complete information about the network connectivity, the orientation of current, the loops and cutsets. However, these matrices contain no information on the nature of the elements which form the interconnected network. The complete behaviour of the network can be obtained from the knowledge of the behaviour of the individual elements which make the network, along with the incidence matrices. An element in an electrical network is completely characterized by the relationship between the current through the element and the voltage across it.

**General representation of a network element:** In general, a network element may contain active or passive components. Figure 2 represents the alternative impedance and admittance forms of representation of a general network component.

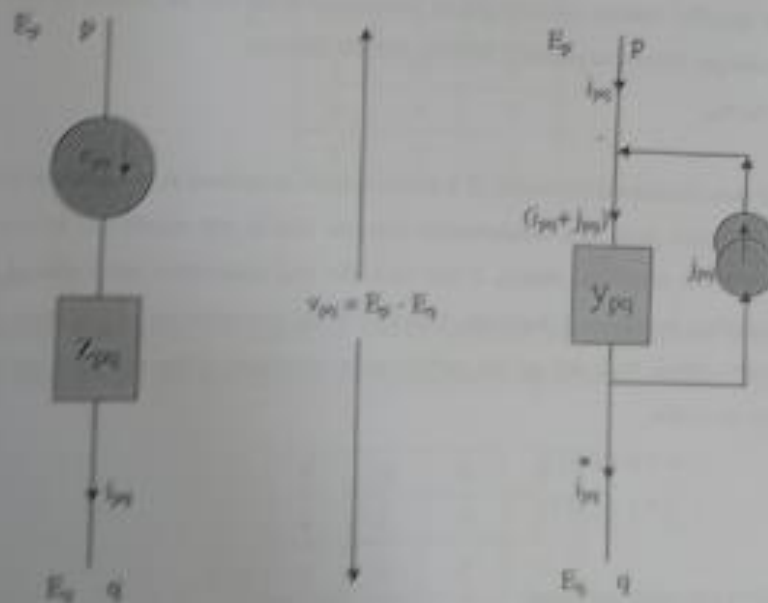


Fig.2 Representation of a primitive network element  
(a) Impedance form (b) Admittance form

The network performance can be represented by using either the impedance or the admittance form of representation. With respect to the element, p-q, let,

$v_{pq}$  = voltage across the element p-q,

$e_{pq}$  = source voltage in series with the element p-q,

$i_{pq}$  = current through the element p-q,

$j_{pq}$  = source current in shunt with the element p-q,

$z_{pq}$  = self impedance of the element p-q and

$y_{pq}$  = self admittance of the element p-q.

**Performance equation:** Each element p-q has two variables,  $v_{pq}$  and  $i_{pq}$ . The performance of the given element p-q can be expressed by the performance equations as under:

$$\begin{aligned} v_{pq} + e_{pq} &= z_{pq} i_{pq} && \text{(in its impedance form)} \\ i_{pq} + j_{pq} &= y_{pq} v_{pq} && \text{(in its admittance form)} \end{aligned} \quad (6)$$

Thus the parallel source current  $j_{pq}$  in admittance form can be related to the series source voltage,  $e_{pq}$  in impedance form as per the identity:

$$j_{pq} = - y_{pq} e_{pq} \quad (7)$$

A set of non-connected elements of a given system is defined as a *primitive Network* and an element in it is a fundamental element that is not connected to any other element. In the equations above, if the variables and parameters are replaced by the corresponding vectors and matrices, referring to the complete set of elements present in a given system, then, we get the performance equations of the primitive network in the form as under:

$$\begin{aligned} v + e &= [z] i \\ i + j &= [y] v \end{aligned} \quad (8)$$

**Primitive network matrices:**

A diagonal element in the matrices,  $[z]$  or  $[y]$  is the self impedance  $z_{pq,pq}$  or self admittance,  $y_{pq,pq}$ . An off-diagonal element is the mutual impedance,  $z_{pq,rs}$  or mutual admittance,  $y_{pq,rs}$ , the value present as a mutual coupling between the elements p-q and r-s. The primitive network admittance matrix,  $[y]$  can be obtained also by



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$$Y_{bus} = \begin{bmatrix} -j503 & 0 & j2 \\ 0 & -j10 & 0 \\ j2 & 0 & -j7 \end{bmatrix}$$

$$Y_{bus} = A^t \cdot y \cdot A$$

$$y = \begin{pmatrix} -303 & 0 & 0 & 0 \\ 0 & j0 & 0 & 0 \\ 0 & 0 & -j5 & 0 \\ 0 & 0 & 0 & j2 \end{pmatrix}$$

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$