CMR
INSTITUTE OF
TECHNOLOGY

USN

Internal Assesment Test - II

Sub:	**SIGNALS AND SYSTEMS**							Code:	15EE54
Date:	16/10/2018	Duration:	90 mins	Max Marks:	50	Sem:	5th	Branch:	EEE

Answer Any FIVE FULL Questions

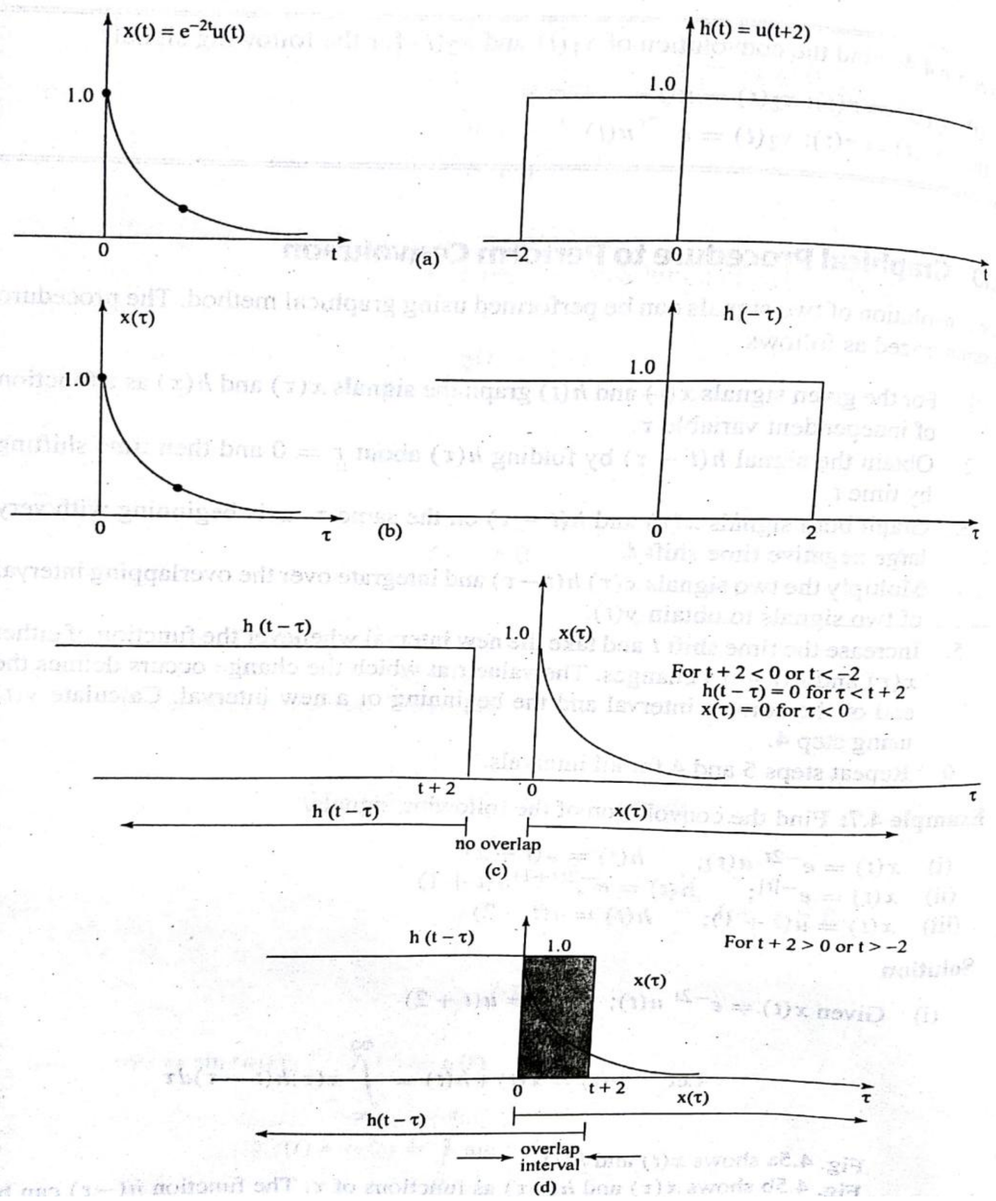
		Marks	OBE CO	OBE RBT
1	If $h(t)= e^{-2t}u(t)$ and $x(t)=u(t+2)$, Determine the output $y(t)$,given $h(t)$ is the impulse response and $x(t)$ is the input for the LTI system.	10	CO3	L2
2	The impulse response of a system 1 is $h(t) = e^{2t}u(t-1)$ and system 2 is $h(t)= e^{-2\|t\|}$.Check whether the systems are stable, causal and memory less system.	10	CO4	L2
3	Obtain direct form-1 and direct form 2 representations for the following a. $\frac{d^2y}{dt^2}+5\frac{dy}{dt}+4\ y(t)=\frac{dx}{dt}$ b. $y[n]+0.5y[n-1]-0.3y[n-3]=x[n]+2x[n-2]$	10	CO4	L2
4	Find the natural response, forced response and complete response of the given difference equation. $y[n]+1.5y[n-1]+0.5y[n-2]=x[n]$ given $y[-1]=1$, $y[-2]=0$, $x[n]=2^n u[n]$	10	CO4	L2
5	List and explain the properties of ROC with examples.	10	CO6	L2
6	Obtain Z-transform of the following functions a. $a^n\cos(\Omega n)u(n)$ b. $n(-1/2)^n u(n)*(3/4)^n u(-n)$	10	CO6	L2
7	Prove distributive and time shifting property of Convolution Integral	10	CO3	L2

Answers

1.
(i) Given $x(t) = e^{-2t}\,u(t)$; $h(t) = u(t+2)$

$$\text{Let}\qquad y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

Fig. 4.5a shows $x(t)$ and $h(t)$.
Fig. 4.5b shows $x(\tau)$ and $h(-\tau)$ as functions of τ. The function $h(-\tau)$ can be obtained by folding the signal $h(\tau)$ about $\tau = 0$. Fig. 4.5c shows the signals $h(t-\tau)$ and $x(\tau)$ on the same axis. Here the signal $h(t-\tau)$ is sketched for $t < -2$ by shifting $h(-\tau)$ to left. For $t < -2$, $x(\tau)$ and $h(t-\tau)$ does not overlap and the product $x(\tau)\,h(t-\tau) = 0$, that is



$$y(t) = 0 \quad \text{for} \quad t < -2$$

Now increase the time shift t until the signal $h(t-\tau)$ intersects $x(\tau)$. Fig 4.5d shows the situation for $t > -2$. Here $x(\tau)$ and $h(t-\tau)$ overlapped. But $x(\tau) = 0$ for $\tau < 0$ and $h(t-\tau) = 0$ for $\tau > t+2$. Therefore the integration interval is from $\tau = 0$ to $\tau = t+2$

$$\Rightarrow \quad y(t) = \int_{0}^{t+2} x(\tau)h(t-\tau)d\tau$$

$$= \int_{0}^{t+2} e^{-2\tau}d\tau$$

$$= \frac{-1}{2} e^{-2\tau}\Big|_{0}^{t+2}$$

$$= \frac{-1}{2}\left[e^{-2(t+2)} - 1\right]$$

$$= \frac{1 - e^{-2(t+2)}}{2}$$

$$\Rightarrow \quad y(t) = 0 \quad \text{for } t < -2$$

$$= \frac{1 - e^{-2(t+2)}}{2} \quad \text{for } t > -2$$

2

$h(t) = e^{-2|t|}$

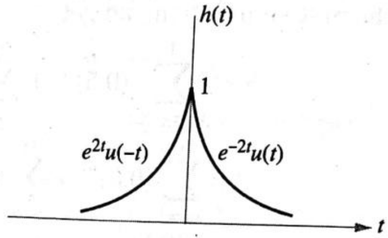


Fig. Ex.2.13

(i) System is not memoryless, because $h(t)$ is not of the form $h(t) = c\,\delta(t)$. evident in Fig. Ex.2.13.

(ii) Since, $h(t)$ is not zero for $t < 0$, the system under investigation is noncausa

(iii) Let
$$S = \int_{-\infty}^{\infty} |h(t)|\, dt$$

Then,
$$S = \int_{-\infty}^{0} e^{2t} dt + \int_{0}^{\infty} e^{-2t} dt = \frac{1}{2} + \frac{1}{2} = 1$$

Since S is finite, the system is BIBO stable.

3

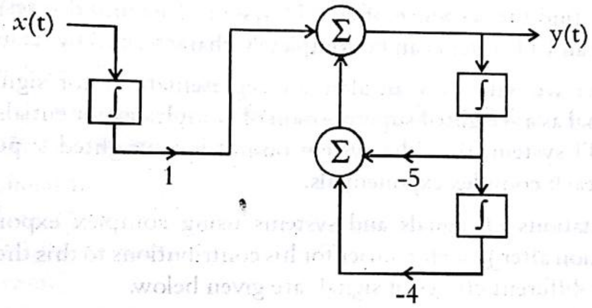


Fig. P2.77.1 Direct form I

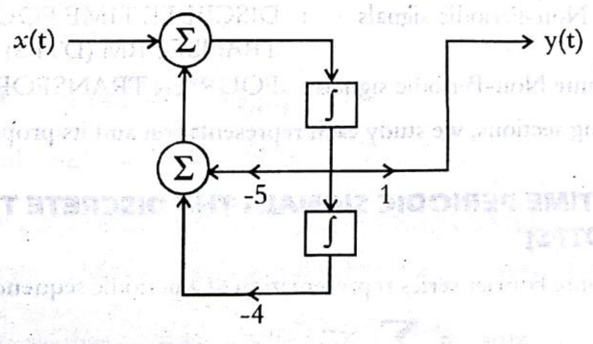
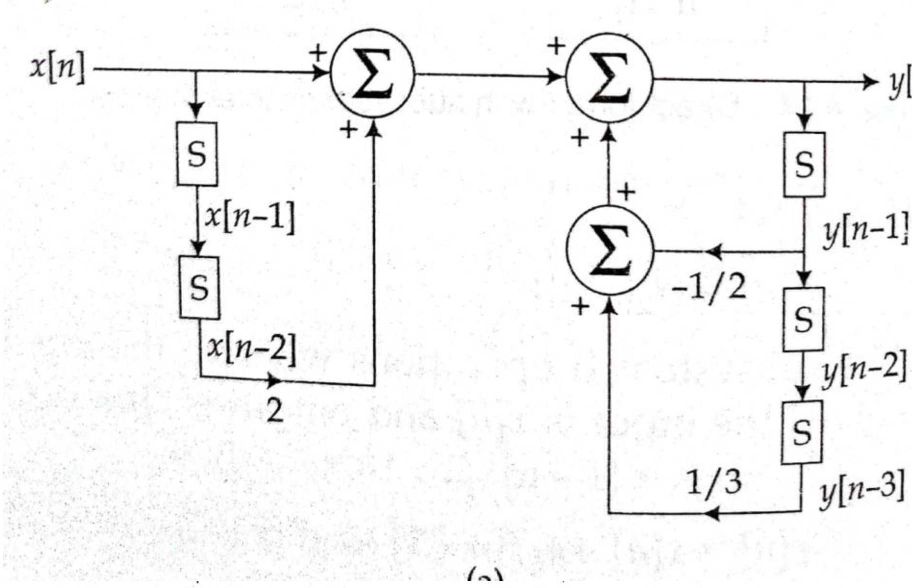


Fig. P2.77.2 Direct form II



(a)

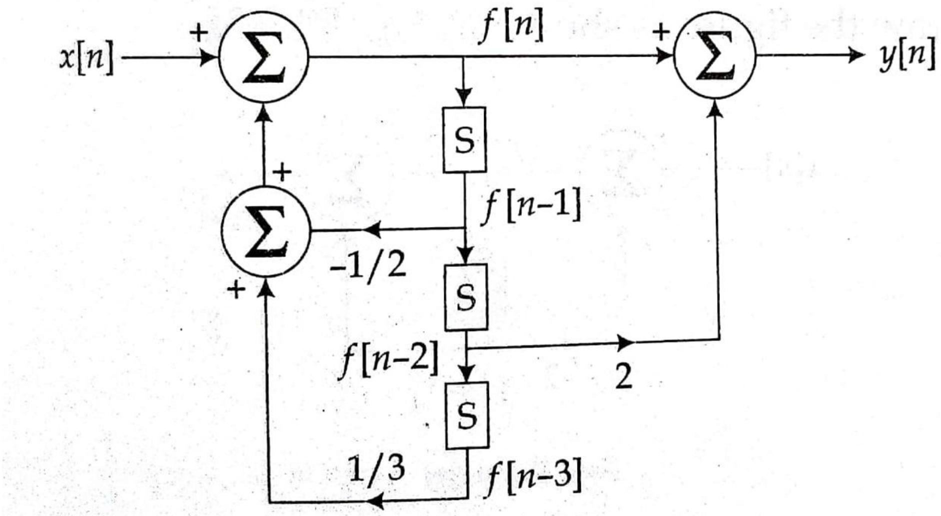


Fig. P4.9.1 (a) Direct form I (b) Direct form II

4.

1. Characteristic equation

$$\lambda^n - 1.5\lambda^{n-1} + 0.5\lambda^{n-2} = 0$$

$$\lambda^{n-2}[\lambda^2 - 1.5\lambda + 0.5] = 0$$

2. Root are $\lambda_1 = 1$ and $\lambda_2 = 0.5$
3. Zero-input response or natural response is given by

$$y^{(n)}[n] = c_1\lambda_1^n + c_2\lambda_2^n$$

$$= c_1[1]^n + c_2[0.5]^n$$

Substituting for $n = 0$ and $n = 1$, we get

$$y[0] = c_1 + c_2$$

$$y[1] = c_1 + c_2 \times 0.5$$

Coefficients c_1 and c_2 are solved considering zero input in the original difference equation.

$$y[n] = 1.5y[n-1] - 0.5y[n-2]; \quad y[-1] = 1; \quad y[-2] = 0$$

$$y[0] = 1.5y[-1] - 0.5y[-2]$$

$$= 1.5 \times 1 - 0.5 \times 0 = 1.5$$

$$y[1] = 1.5y[0] - 0.5y[-1]$$

$$= 1.5 \times 1.5 - 0.5 \times 1 = 1.75$$

Substituting for $y[-1]$ ...

$$c_1 + c_2 = 1.5$$
$$c_1 + 0.5c_2 = 1.75$$

Solving we get $c_1 = 2$ and $c_2 = -0.5$

The natural response is

$$y^{(n)}[n] = 2[1]^n - 0.5[0.5]^n, \quad n \geq 0$$
$$= 2u[n] - 0.5[0.5]^n u[n]$$

4. The particular solution is taken to be of the same form as input, $x[n] = 2^n u[n]$

$$y^{(p)}[n] = k2^n u[n]$$
$$y^{(p)}[n-1] = k2^{n-1}u[n-1]$$
$$y^{(p)}[n-2] = k2^{n-2}u[n-2]$$

5. $y^{(p)}[n]$ is a solution of the original difference equation. Therefore,

$$y^{(p)}[n] - 1.5y^{(p)}[n-1] + 0.5y^{(p)}[n-2] = x[n]$$

Substituting for $y^{(p)}[n]$,

$$k2^n u[n] - 1.5k2^{n-1}u[n-1] + 0.5k2^{n-2}u[n-2] = 2^n u[n]$$

k is calculated so that no term vanishes. Choose $n = 2$.

$$k2^2 - 1.5k2^1 + 0.5k2^0 = 2^2$$
$$k = \frac{8}{3}$$
$$y^{(p)}[n] = \frac{8}{3}[2]^n u[n].$$

6. Forced response is sum of particular solution and a component of the same form as natural response.

$$y^{(f)}[n] = \frac{8}{3}[2]^n + c_3[1]^n + c_4[0.5]^n; \quad n \geq 0 \qquad \text{...(4.53)}$$

$$y[0] = \frac{8}{3}[2]^0 + c_3[1]^0 + c_4[0.5]^0$$
$$= \frac{8}{3} + c_3 + c_4$$
$$y[1] = \frac{8}{3}[2]^1 + c_3[1]^1 + c_4[0.5]^1$$
$$= \frac{16}{3} + c_3 + 0.5c_4$$

We now compute $y[0]$ and $y[1]$ from the original difference equation with zero initial conditions.

Hence,

$$y[n] = 1.5y[n-1] - 0.5y[n-2] + x[n]; \quad y[-1] = 0,\ y[-2] = 0,\ x[n] = 2^n u[n]$$

$$y[0] = 1.5y[-1] - 0.5y[-2] + 2^0 u[0]$$
$$y[0] = 1$$
$$y[1] = 1.5y[0] - 0.5y[-1] + x[1]$$
$$= 1.5 \times 1 - 0.5 \times 0 + 2^1 = 3.5$$

Substituting for $y[0]$ and $y[1]$, we get

$$\frac{8}{3} + c_3 + c_4 = 1$$
$$\frac{16}{3} + c_3 + 0.5c_4 = 3.5$$

Solving, we get $c_3 = -2$ and $c_4 = \frac{1}{3}$

The forced response is $y^{(f)}[n] = -2[1]^n + \frac{1}{3}[0.5]^n + \frac{8}{3}2^n; \ n \geq 0$

Summarizing.

$$y^{(n)}[n] = \{2 - 0.5[0.5]^n\}u[n]$$
$$y^{(f)}[n] = \left\{-2 + \frac{1}{3}[0.5]^n + \frac{8}{3}2^n\right\}u[n]$$
$$y[n] = y^{(n)}[n] + y^{(f)}[n] = \left\{-\frac{1}{6}[0.5]^n + \frac{8}{3}2^n\right\}u[n].$$

5.

1. The ROC is a ring or disk in the z-plane centered at the origin.
2. The ROC cannot contain any poles.
3. If $x(n)$ is a causal sequence then the ROC is the entire z-plane except at $z = 0$.
4. If $x(n)$ is a anti-causal sequence then the ROC is the enitre z-plane except at $z = \infty$.

5. If $x(n)$ is a finite duration, two-sided sequence the ROC is entire z-plane except at $z = 0$ and $z = \infty$.
6. If $x(n)$ is an infinite duration, two-sided sequence the ROC will consist of a ring in the z-plane, bounded on the interior and exterior by a pole, not containing any poles.
7. The ROC of an LT1 stable system contains the unit circle (to be discussed in section (10.9).
8. The ROC must be a connected region.

6.

find $X(z)$ of

$$x[n] = a^n \cos(\Omega n)\, u[n].$$

Solution: Let $y[n] = a^n u[n]$.

From property of multiplication by an exponential $a^n y[n] \xleftrightarrow{z} Y(\alpha^{-1} z)$; ROC: $|d|\ Ry$

$$\cos(\Omega n) = \frac{e^{j\Omega n} + e^{-j\Omega n}}{2}$$

$$a^n u[n] \xleftrightarrow{z} \frac{z}{z-1} = \frac{1}{1 - az^{-1}};\ |z| > |a|$$

$$\frac{1}{2}\{e^{j\Omega n} + e^{-j\Omega n}\} a^n u[n] \xleftrightarrow{z} \frac{1}{2}\left\{\frac{1}{1 - az^{-1}e^{-j\Omega}} + \frac{1}{1 - az^{-1}e^{j\Omega}}\right\}$$

$$= \frac{1}{2}\left\{\frac{1 - az^{-1}e^{j\Omega} + 1 - az^{-1}e^{-j\Omega}}{(1 - az^{-1}e^{-j\Omega})(1 - az^{-1}e^{j\Omega})}\right\}$$

$$= \frac{1 - az^{-1}\left(\frac{e^{j\Omega} + e^{-j\Omega}}{2}\right)}{1 - az^{-1}(e^{-j\Omega} + e^{j\Omega}) + a^2 z^{-2}}$$

$$= \frac{1 - az^{-1}\cos(\Omega)}{1 - 2az^{-1}\cos(\Omega) + a^2 z^{-2}}$$

$$= \frac{z^2 - az\cos(\Omega)}{z^2 - 2az\cos(\Omega) + a^2};\ |z| > |a|$$

Specifically if $a = 1$ and $\Omega = \pi/2$, we get

$$\cos\left(\frac{\pi}{2} n\right) u[n] \xleftrightarrow{z} \frac{z^2}{z^2 + 1} \quad \text{...(8.36)}$$

Similarly, we can show that

$$a^n \sin(\Omega n)\, u[n] \xleftrightarrow{z} \frac{a \sin(\Omega) z^{-1}}{1 - 2a\cos(\Omega) z^{-1} + a^2 z^{-2}}$$

If $a = 1$ and $\Omega = \frac{\pi}{2}$

$$\sin\left(\frac{\pi}{2} n\right) u[n] \xleftrightarrow{z} \frac{z^{-1}}{1 + z^{-2}} = \frac{z}{z^2 + 1} \quad \text{...(8.37)}$$

Solution: Write $x[n] = y[n] * w[n]$

Compute $Y(z)$

$$u[n] \overset{z}{\longleftrightarrow} \frac{z}{z-1}\ ;\ |z| > 1$$

$$\left(-\frac{1}{2}\right)^n u[n] \overset{z}{\longleftrightarrow} \frac{z}{z+\frac{1}{2}};\quad \text{ROC: } |z| > \frac{1}{2}$$

$$n\left(-\frac{1}{2}\right)^n u[n] \overset{z}{\longleftrightarrow} -z\frac{d}{dz}\left(\frac{z}{z+\frac{1}{2}}\right) = \frac{-\frac{1}{2}z}{\left(z+\frac{1}{2}\right)^2};|z| > 1/2$$

Compute $W(z)$

$$u[n] \overset{z}{\longleftrightarrow} \frac{z}{z-1};\quad \text{ROC: } |z| > 1$$

$$u[-n] \overset{z}{\longleftrightarrow} \frac{1}{1-z};\quad \text{ROC: } |z^{-1}| > 1 \text{ or } |z| < 1$$

$$\left(\frac{3}{4}\right)^n u[-n] \overset{z}{\longleftrightarrow} \frac{1}{1-z/(3/4)} = \frac{1}{1-\frac{4}{3}z};\quad \text{ROC: } |z| < \frac{3}{4}$$

$X(z) = Y(z)\,W(z)$ (convolution property)

$$= -\frac{1}{2}\frac{z}{\left(z+\frac{1}{2}\right)^2} \times \frac{1}{1-\frac{4}{3}z};\quad \text{ROC: } \frac{1}{2} < |z| < \frac{3}{4}$$

$$= -\frac{1}{2}\left\{\frac{z}{\left(z+\frac{1}{2}\right)^2\left(1-\frac{4}{3}z\right)}\right\}; \frac{1}{2} < |z| < 3/4$$

7.

Distributive

Proof: Let the RHS be equal to a function $y(t)$.

$$y(t) = x_1(t) * x_2(t) + x_1(t) * x_3(t). \quad \text{...(3.14)}$$

We now substitute the integral representation of convolution in (3.14) to get,

$$y(t) = \int_{-\infty}^{\infty} x_1(\tau)x_2(t-\tau)d\tau + \int_{-\infty}^{\infty} x_1(\tau)x_3(t-\tau)d\tau$$

$$= \int_{-\infty}^{\infty} x_1(\tau)\{x_2(t-\tau) + x_3(t-\tau)\}d\tau$$

$$= x_1(t) * \{x_2(t) + x_3(t)\}$$

Time shifting

$$x_1(t) * x_2(t) = y(t)$$

$$x_1(t) * x_2(t-T) = y(t-T)$$

$$x_1(t) * x_2(t-T) = \int_{-\infty}^{\infty} x_1(\tau)x_2(t-T-\tau)d\tau$$

$$= y(t-T)$$

$$x_1(t-T) * x_2(t) = y(t-T)$$

$$x_1(t-T_1) * x_2(t-T_2) = y(t-T_1-T_2)$$