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Internal Assesment Test - II

Sub:	Power System Analysis II							Code:	15EE71
Date:	16/10/2018	Duration:	90 mins	Max Marks:	50	Sem:	7	Branch:	EEE

Answer Any FIVE FULL Questions

		Marks	OBE CO	OBE RBT
1	Deduce the fast decoupled load flow model clearly stating all the assumptions made	[10]	CO2	L3
2	Derive the algorithm for the formation of bus impedance matrix Zbus for a single phase system when a link element is added to the partial network.	[10]	CO5	L2
3	For the positive sequence network data shown in table below, obtain Z bus by building procedure.	[10]	CO5	L3

#	P-Q(nodes)	Pos.Seq. reactance in pu
1	0-1	0.25
2	0-3	0.20
3	1-2	0.08
4	2-3	0.06

		Marks	CO	RBT
4	Obtain the voltages at all buses for the 3 bus system as shown in fig at the end of first iteration by NR method.	[10]	CO2	L3

From bus	To bus	R(pu)	X(pu)	$B_c/2$
1	2	0	0.1	0
1	3	0	0.2	0
2	3	0	0.2	0

Bus no	P_G	Q_G	P_L	Q_L	Vsp
1	-	-	-	-	1.0
2	5.3217	-	-	-	1.1
3	-	-	3.6392	0.5339	-

		Marks	CO	RBT
5	Solve Q.no 4 by fast decoupled method	[10]	CO2	L3
6a	Compare NR and GS method for load flow analysis	[5]	CO2	L2
	Form Zbus using step by step building algorithm of the system shown in fig.Take element connected between 1-2 (s) as LINK.	[5]	CO5	L3

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18.9 Fast-decoupled Load Flow

This is an extension of Newton-Raphson method formulated in polar coordinates with certain approximation which results into a fast algorithm for load flow solution. Before we discuss this method, we derive load flow equations in polar coordinates.

We know that

$$P_p - jQ_p = V_p^* I_p \text{ and } I_p = \sum_{q=1}^{n} Y_{pq} V_q$$

$$\therefore \quad P_p - jQ_p = V_p^* \sum_{q=1}^{n} Y_{pq} V_q \qquad (18.48)$$

The voltage and admittance in polar coordinates are expressed as

$$V_p = |V_p| \exp(j\delta_p) \text{ and } Y_{pq} = |Y_{pq}| \exp(-j\theta_{pq}).$$

Substituting these values in equation (18.48), we obtain

$$P_p - jQ_p = |V_p| \exp(-j\delta_p) \sum_{q=1}^{n} |Y_{pq}| \exp(-j\theta_{pq}) |V_q| \exp(j\delta_q)$$

$$= \sum_{q=1}^{n} |V_p|\,|V_q|\,|Y_{pq}| \exp\{-j(\theta_{pq} + \delta_p - \delta_q)\}$$

$$P_p = \sum_{q=1}^{n} |V_p V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q) \qquad (18.49)$$

and

$$Q_p = \sum_{q=1}^{n} |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \qquad (18.50,$$

$$p = 1, 2, \ldots n$$

Equations (18.49) and (18.50) are rewritten as

$$P_p = |V_p V_p Y_{pp}| \cos\theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} |V_p V_q Y_{pq}| \cos(\theta_{pq} + \delta_p - \delta_q) \quad (18.51)$$

$$Q_p = |V_p V_p Y_{pp}| \sin\theta_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q) \quad (18.52)$$

These equations after linearization can be rewritten in matrix form as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|E|/|E| \end{bmatrix} \qquad (18.53)$$

Here H, N, M and L are elements of Jacobian matrix.

The first assumption under decoupled load flow method is that real power changes (ΔP) are less sensitive to changes in voltage magnitude and are mainly sensitive to angle. Similarly, the reactive power changes are less sensitive to change in angle but mainly sensitive to change in voltage magnitude. With these assumptions, equation (18.53) reduce to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & O \\ O & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|E|/|E| \end{bmatrix} \qquad (18.54)$$

The equation (18.54) is decoupled equation which can be expanded as

$$[\Delta P] = [H][\Delta\delta] \tag{18.54a}$$

and

$$[\Delta Q] = [L][\Delta|E|/|E|] \tag{18.54b}$$

Using equations (18.51) and (18.52) the elements of the Jacobian matrices H and L are obtained as follows:

Off-diagonal element of H is

$$H_{pq} = \frac{\partial P_p}{\partial \delta_q} = |V_p V_q Y_{pq}| \sin[\theta_{pq} + \delta_p - \delta_q]$$

$$= |V_p V_q Y_{pq}|[\sin\theta_{pq}\cos(\delta_p - \delta_q) + \sin(\delta_p - \delta_q)\cos\theta_{pq}]$$

$$= |V_p V_q|[|Y_{pq}|\sin\theta_{pq}\cos(\delta_p - \delta_q) + |Y_{pq}|\cos\theta_{pq}\sin(\delta_q - \delta_q)]$$

$$= |V_p V_q|[-B_{pq}\cos(\delta_p - \delta_q) + G_{pq}\sin(\delta_p - \delta_q)] \tag{18.55}$$

Similarly off-diagonal element of L is

$$L_{pq} = \frac{\partial Q_p |V_q|}{\partial V_q} = |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q)$$

$$= |V_p V_q|[-B_{pq}\cos(\delta_p - \delta_q) + G_{pq}\sin(\delta_p - \delta_q)] \tag{18.56}$$

From equations (18.55) and (18.56), it is seen that

$$H_{pq} = L_{pq} = |V_p V_q|[G_{pq}\sin(\delta_p - \delta_q) - B_{pq}\cos(\delta_p - \delta_q)]$$

The diagonal elements of H are given as

$$H_{pp} = \frac{\partial P_p}{\partial \delta_p} = -\sum_{\substack{q=1\\ q\neq p}}^{n} |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q)$$

$$= -Q_p + |V_p V_p Y_{pp}| \sin\theta_{pp}$$

$$= -Q_p - V_p^2 B_{pp} \tag{18.57}$$

Similarly diagonal elements for the matrix are given by:

$$L_{pp} = \frac{\partial Q_p |V_p|}{\partial V_p} = |2V_p^2 Y_{pp}| \sin\theta_{pp} + \sum_{\substack{q=1\\ q\neq p}}^{n} |V_p V_q Y_{pq}| \sin(\theta_{pq} + \delta_p - \delta_q)$$

$$= |2V_p^2 Y_{pp}| \sin\theta_{pp} + Q_p - |V_p^2 Y_{pp}| \sin\theta_{pp}$$

$$= Q_p + |V_p^2 Y_{pp}| \sin\theta_{pp}$$

$$= Q_p - V_p^2 B_{pp} \tag{18.58}$$

In the case of fast decoupled load flow method following approximations are further made for evaluating Jacobian element.

$$\cos(\delta_p - \delta_q) \simeq 1$$

$$G_{pq}\sin(\delta_p - \delta_q) \leqslant B_{pq}$$

and

$$Q_p \ll B_{pp} V_p^2$$

∴ The Jacobian elements now become

$$L_{pq} = H_{pq} = -|V_p V_q| B_{pq} \text{ for } q \neq p$$

and

$$L_{pp} = H_{pp} = -B_{pp}|V_p|^2$$

With these Jacobian elements equations 18.54(a) and 18.54(b) become

$$[\Delta P_p] = [V_p][V_q][B'_{pq}][\Delta \delta_q] \quad (18.59)$$

and

$$[\Delta Q_p] = [V_p][V_q][B''_{pq}] \frac{\Delta|E_q|}{E_q} \quad (18.60)$$

where B'_{pq} and B''_{pq} are the elements of $[-B_{pq}]$ matrix.

Further decoupling is obtained as follows:

(i) Omit from B'' the angle shifting effects of phase shifters.

(ii) Omit from B' the representation of those network elements that affect MVAr flows i.e. shunt reactors and off-nominal in phase transformer taps.

(iii) Divide equations (18.59) and (18.60) by V_p and assuming $V_q = 1$ p.u. and also neglecting the series resistance in calculating the elements of B'.

With these assumptions, equations (18.59) and (18.60) for the load flow solution take the form

$$\left[\frac{\Delta P_p}{E_p}\right] = [B'][\Delta\delta] \quad (18.61)$$

and

$$\left[\frac{\Delta Q_p}{E_p}\right] = [B''][\Delta E] \quad (18.62)$$

It is to be noted that $[B']$ and $[B'']$ are real and sparse and have similar structures as those of H and L respectively. Since the two matrices are constant and do not change during successive iterations for solution of the load flow problem, they need be evaluated only once and inverted once during the first iteration and then used in all successive iterations. It is because of the nature of Jacobian matrices $[B']$ and $[B'']$ and the sparsity of these matrices that the method is fast.

Addition of a link

If the added element p–q is a link, the procedure for recalculating the elements of the bus impedance matrix is to connect in series with the added element a voltage source e_l as shown in Fig. 4.4. This creates a

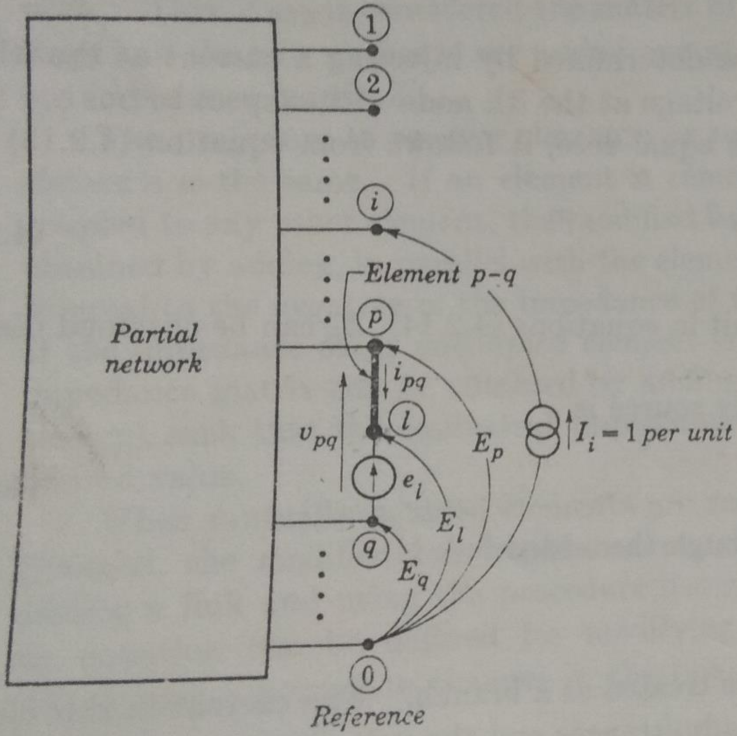

Fig. 4.4 Injected current voltage source in series with added link and bus voltages for calculation of Z_{li}.

fictitious node l which will be eliminated later. The voltage source e_l is selected such that the current through the added link is zero.

The performance equation for the partial network with the added element p–l and the series voltage source e_l is

$$\begin{bmatrix} E_1 \\ E_2 \\ \cdots \\ E_p \\ \cdots \\ E_m \\ e_l \end{bmatrix} = \begin{array}{c} 1 \\ \\ \\ p \\ \\ m \\ l \end{array} \overset{\begin{array}{ccccccc} 1 & & & p & & m & l \end{array}}{\begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1l} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2l} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pl} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{ml} \\ Z_{l1} & Z_{l2} & \cdots & Z_{lp} & \cdots & Z_{lm} & Z_{ll} \end{bmatrix}} \begin{bmatrix} I_1 \\ I_2 \\ \cdots \\ I_p \\ \cdots \\ I_m \\ I_l \end{bmatrix} \qquad (4.2.13)$$

$e_l = E_l - E_q$
$v_{pl} = E_p - E_l$
$e_l + v_{pl} = E_p - E_q$

Since

$e_l = E_l - E_q$

the element Z_{li} can be determined by injecting a current at the ith bus and calculating the voltage at the lth node with respect to bus q. Since all other bus currents equal zero, it follows from equation (4.2.13) that

$$E_k = Z_{ki} I_i \qquad k = 1, 2, \ldots, m \qquad (4.2.14)$$
$$e_l = Z_{li} I_i$$

Letting $I_i = 1$ per unit in equations (4.2.14), Z_{li} can be obtained directly by calculating e_l.

The series voltage source is

$$e_l = E_p - E_q - v_{pl} \qquad (4.2.15)$$

Since the current through the added link is

$i_{pq} = 0$

the element p–l can be treated as a branch. The current in this element in terms of primitive admittances and the voltages across the elements is

$$i_{pl} = y_{pl,pl} v_{pl} + \bar{y}_{pl,\rho\sigma} \bar{v}_{\rho\sigma}$$

where

$$i_{pl} = i_{pq} = 0$$

Therefore

$$v_{pl} = -\frac{\bar{y}_{pl,\rho\sigma}\bar{v}_{\rho\sigma}}{y_{pl,pl}}$$

Since

$$\bar{y}_{pl,\rho\sigma} = \bar{y}_{pq,\rho\sigma} \qquad \text{and} \qquad y_{pl,pl} = y_{pq,pq}$$

then

$$v_{pl} = -\frac{\bar{y}_{pq,\rho\sigma}\bar{v}_{\rho\sigma}}{y_{pq,pq}} \tag{4.2.16}$$

Substituting in order from equations (4.2.16), (4.2.6), and (4.2.14) with $I_i = 1$ into equation (4.2.15) yields

$$Z_{li} = Z_{pi} - Z_{qi} + \frac{\bar{y}_{pq,\rho\sigma}(\bar{Z}_{\rho i} - \bar{Z}_{\sigma i})}{y_{pq,pq}} \qquad \begin{matrix} i = 1, 2, \ldots, m \\ i \neq l \end{matrix} \tag{4.2.17}$$

The element Z_{ll} can be calculated by injecting a current at the lth bus with bus q as reference and calculating the voltage at the lth bus with respect to bus q. Since all other bus currents equal zero, it follows from equation (4.2.13) that

$$\begin{aligned} E_k &= Z_{kl}I_l \qquad k = 1, 2, \ldots, m \\ e_l &= Z_{ll}I_l \end{aligned} \tag{4.2.18}$$

Letting $I_l = 1$ per unit in equation (4.2.18), Z_{ll} can be obtained directly by calculating e_l.

The current in the element p–l is

$$i_{pl} = -I_l = -1$$

This current in terms of primitive admittances and the voltages across the elements is

$$i_{pl} = y_{pl,pl}v_{pl} + \bar{y}_{pl,\rho\sigma}\bar{v}_{\rho\sigma} = -1$$

Again, since

$$\bar{y}_{pl,\rho\sigma} = \bar{y}_{pq,\rho\sigma} \qquad \text{and} \qquad y_{pl,pl} = y_{pq,pq}$$

then

$$v_{pl} = -\frac{1 + \bar{y}_{pq,\rho\sigma}\bar{v}_{\rho\sigma}}{y_{pq,pq}} \tag{4.2.19}$$

Substituting in order from equations (4.2.19), (4.2.6), and (4.2.18) with $I_l = 1$ into (4.2.15) yields

$$Z_{ll} = Z_{pl} - Z_{ql} + \frac{1 + \bar{y}_{pq,\rho\sigma}(\bar{Z}_{\rho l} - \bar{Z}_{\sigma l})}{y_{pq,pq}} \tag{4.2.20}$$

If there is no mutual coupling between the added element and other elements of the partial network, the elements of $\bar{y}_{pq,\rho\sigma}$ are zero and

$$z_{pq,pq} = \frac{1}{y_{pq,pq}}$$

It follows from equation (4.2.17) that

$$Z_{li} = Z_{pi} - Z_{qi} \qquad i = 1, 2, \ldots, m \quad i \neq l$$

and from equation (4.2.20),

$$Z_{ll} = Z_{pl} - Z_{ql} + z_{pq,pq}$$

Furthermore, if there is no mutual coupling and p is the reference node,

$$Z_{pi} = 0 \qquad i = 1, 2, \ldots, m \quad i \neq l$$

and

$$Z_{li} = -Z_{qi} \qquad i = 1, 2, \ldots, m \quad i \neq l$$

Also

$$Z_{pl} = 0$$

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and therefore,

$$Z_{ll} = -Z_{ql} + z_{pq,pq}$$

The elements in the lth row and column of the bus impedance matrix for the augmented partial network are found from equations (4.2.17) and (4.2.20). It remains to calculate the required bus impedance matrix to include the effect of the added link. This can be accomplished by modifying the elements Z_{ij}, where $i, j = 1, 2, \ldots, m$, and eliminating the lth row and column corresponding to the fictitious node.

The fictitious node l is eliminated by short circuiting the series voltage source e_l. From equation (4.2.13),

$$\bar{E}_{BUS} = Z_{BUS}\bar{I}_{BUS} + \bar{Z}_{il}I_l \tag{4.2.21}$$

and

$$e_l = \bar{Z}_{lj}\bar{I}_{BUS} + Z_{ll}I_l = 0 \tag{4.2.22}$$

where $i, j = 1, 2, \ldots, m$. Solving for I_l from equation (4.2.22) and substituting into (4.2.21),

$$\bar{E}_{BUS} = \left(Z_{BUS} - \frac{\bar{Z}_{il}\bar{Z}_{lj}}{Z_{ll}}\right)\bar{I}_{BUS}$$

which is the performance equation of the partial network including the link p–q. It follows that the required bus impedance matrix is

$$Z_{BUS\,(\text{modified})} = Z_{BUS\,(\text{before elimination})} - \frac{\bar{Z}_{il}\bar{Z}_{lj}}{Z_{ll}}$$

where any element of $Z_{BUS\,(\text{modified})}$ is

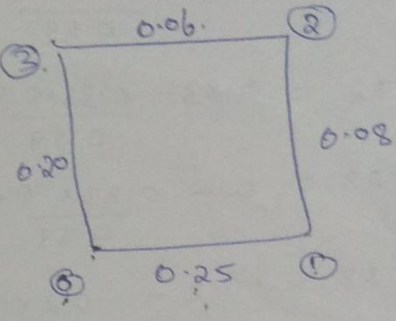
$$Z_{ij\,(\text{modified})} = Z_{ij\,(\text{before elimination})} - \frac{Z_{il}Z_{lj}}{Z_{ll}}$$

A summary of the equations for the formation of the bus impedance matrix is given in Table 4.1.

3

(a) Z_{bus} building Algorithm.

	pq	+ve reactance sequence in pu
1	0-1	0.25
2	0-3	0.20
3	1-2	0.08
4	2-3	0.06



① Add branch ⓪ to ① $Z_{bus} = [0.25]$

② Add branch ① to ② Z_{bus}

	1	2
1	0.25	0.25
2	0.25	0.33

③ Add branch ⓪ to ③ Z_{bus}

	1	2	3
1	0.25	0.25	0
2	0.25	0.33	0
3	0	0	0.2

$Z_{31} = Z_{pi} = 0$

$Z_{32} =$

Z_{33}

④ Add link ② & ③.

$Z_{bus} =$

	1	2	3	l
1	0.25	0.25	0	0.25
2	0.25	0.33	0	0.33
3	0	0	0.2	-0.2
l	0.25	0.33	-0.2	0.59

$Z_{li} = Z_{pi} - Z_{qi}$

$Z_{ll} = Z_{pl} - Z_{ql} + z_{pq\,pq}$

$Z_{l1} = Z_{21} - Z_{31} = 0.25 - 0 = 0.25$

$Z_{l2} = Z_{22} - Z_{32} = 0.33 - 0 = 0.33$

$Z_{l3} = Z_{23} - Z_{33} = 0 - 0.2 = -0.2$

$Z_{ll} = Z_{2l} - Z_{3l} + z_{pq\,pq} = 0.33 + 0.2 + 0.06 = 0.59$

$Z_{12} = 0.25 - \frac{0.25 \times 0.33}{0.59} = 0.110$

$Z_{11} = 0.25 - \frac{0.25 \times 0.25}{0.59} = 0.144$

$$Z_{13} = 0 - \frac{0.25 \times -0.2}{0.59} = 0.084$$

$$Z_{22} = 0.33 - \frac{0.33 \times 0.33}{0.59} = 0.145$$

$$Z_{23} = 0 - \frac{0.33 \times -0.2}{0.59} = 0.112$$

$$Z_{33} = -0.2 - \frac{-0.2 \times -0.2}{0.59} = \sout{0.268}\ .132$$

Z_{bus}

0.114	0.11	0.084
0.11	0.145	0.112
0.08	0.112	~~0.268~~ .132

4

$$y_{12} = \frac{1}{Z_{12}} = \frac{(0.08 - j0.24)}{(0.08 + j0.24)(0.08 - j0.24)}$$
$$= 1.25 - j3.75$$

Similarly $y_{13} = 5 - j15$ and $y_{23} = 1.6667 - j5.0$ and the nodal admittance matrix

$$Y = \begin{bmatrix} 6.25 - j18.75 & -1.25 + j3.75 & -5 + j15 \\ -1.25 + j3.75 & 2.916 - j8.75 & -1.666 + j5.0 \\ -5 + j15 & -1.666 + j5.0 & 6.666 - j20 \end{bmatrix}$$

Assuming a flat voltage profile for bus 2 and 3 and for bus 1,

$$V_1 = 1.06 + j0.0$$

From the nodal admittance matrix and the assumed voltage solution,

$G_{11} = 6.25$	$B_{11} = 18.75$	$e_1 = 1.06$	$f_1 = 0.0$
$G_{12} = -1.25$	$B_{12} = -3.75$	$e_2 = 1.0$	$f_2 = 0.0$
$G_{13} = -5.0$	$B_{13} = -15.0$	$e_3 = 1.0$	$f_3 = 0.0$
$G_{22} = 2.916$	$B_{22} = 8.75$		
$G_{23} = -1.666$	$B_{23} = -5.0$		
$G_{33} = 6.666$	$B_{33} = 20.0$		

$$P_2 = e_2(e_1G_{21} + f_1B_{21}) + f_2(f_1G_{21} - e_1B_{21}) + e_2(e_2G_{22} + f_2B_{22})$$
$$+ f_2(f_2G_{22} - e_2B_{22}) + e_2(e_3G_{23} + f_3B_{23}) + f_2(f_3G_{23} - e_3B_{23})$$
$$= 1.0\{1.06(-1.25) + 0.0(-3.75)\} + 0.0\{0.0(-1.25) - 1.06(-375)\}$$
$$+ 1.0\{1.0(2.916)\} + 0.0\{\quad\} + 1.0\{1.0(-1.666)\} + 0.0\{\quad\}$$
$$= -1.325 + 2.916 - 1.666$$
$$= -0.075$$

Similarly, $P_3 = -0.3$

$$Q_2 = f_2(\quad) - e_2(f_1G_{21} - e_1B_{21}) + f_2(\quad) - e_2(f_2G_{22} - e_2B_{22})$$
$$+ f_2(\quad) - e_2(f_3G_{23} - e_3B_{23})$$
$$= -0.225$$

$$Q_3 = -0.9$$

$$\therefore \quad \Delta P_2 = P_{2sp} - P_{2cal} = 0.2 - (-0.075) = 0.275$$
$$\Delta P_3 = -0.6 - (-0.3) = -0.3$$

Since the lower limit on Q_2 is 0.0 and the value of Q_2 as calculated above violates this limit, bus 2 is treated as a load bus where $Q_{2sp} = 0.0$

$$\therefore \quad \Delta Q_2 = 0.0 - (-0.225) = 0.225$$
$$\Delta Q_3 = -0.25 - (-0.9) = 0.65$$

Diagonal elements

$$\frac{\partial P_p}{\partial e_p} = 2e_pG_{pp} + \sum_{\substack{q=1\\q\neq p}}^{n} (e_qG_{pq} + f_qB_{pq})$$

$$\frac{\partial P_2}{\partial e_2} = 2e_2G_{22} + e_1G_{21} + f_1B_{21} + e_3G_{23} + f_3B_{23}$$

$$= 2 \times 1.0 \times 2.916 + 1.06(-1.25) + 0.0(-3.75) + 1.0(-1.666) + 0.0(-5.0)$$

$$= 2.848$$

$$\frac{\partial P_3}{\partial e_3} = 2e_3G_{33} + e_1G_{31} + f_1B_{31} + e_2G_{32} + f_2B_{32}$$

$$= 2(1.0)(6.666) + 1.06(-5.0) + 0.0(-15.0) + 1.0(-1.666) + 0.0(-5.0)$$

$$= 6.3666$$

$$\frac{\partial P_p}{\partial f_p} = 2f_pG_{pp} + \sum_{\substack{q=1\\q\neq p}}^{n} (f_qG_{pq} - e_qB_{pq})$$

$$\frac{\partial P_2}{\partial f_2} = 2f_2G_{22} + f_1G_{21} - e_1B_{21} + f_3G_{23} - e_3B_{23}$$

$$= 2(0.0)(2.916) + (0.0)(G_{21}) - 1.06(-3.75) - 1.0(-5.0)$$

$$= 8.975$$

$$\frac{\partial P_3}{\partial f_3} = 20.90$$

Off-diagonal elements:

$$\frac{\partial P_p}{\partial e_q} = e_pG_{pq} - f_pB_{pq}$$

$$\frac{\partial P_2}{\partial e_3} = e_2G_{23} - f_2B_{23} = -1.666$$

$$\frac{\partial P_3}{\partial e_2} = -1.666$$

$$\frac{\partial P_p}{\partial f_q} = + e_pB_{pq} + f_pG_{pq}$$

$$\frac{\partial P_2}{\partial f_3} = -5.0, \quad \frac{\partial P_3}{\partial f_2} = -5.0$$

Similarly we find out the partial derivatives of the reactive power.

Diagonal elements:

$$\frac{\partial Q_p}{\partial e_p} = 2e_pB_{pp} - \sum_{\substack{q=1\\q\neq p}}^{n} (f_qG_{pq} - e_qB_{pq})$$

$$\frac{\partial Q_2}{\partial e_2} = 2e_2B_{22} - f_1G_{21} + e_1B_{21} - f_3G_{23} + e_3B_{23}$$

$$= 2(1.0)(8.75) + 1.06(-3.75) + 1.0(-5.0)$$

$$= 8.525$$

$$\frac{\partial Q_3}{\partial e_3} = 19.1$$

$$\frac{\partial Q_p}{\partial f_p} = 2f_pB_{pp} + \sum_{\substack{q=1 \\ q \neq p}}^{n} (e_qG_{pq} + f_qB_{pq})$$

$$\frac{\partial Q_2}{\partial f_2} = -2.991$$

$$\frac{\partial Q_3}{\partial f_3} = -6.966$$

Similarly off-diagonal elements are calculated and the final set of linear equations at the end of iteration 1 are

$$\begin{bmatrix} 0.275 \\ -0.3 \\ 0.225 \\ 0.65 \end{bmatrix} = \begin{bmatrix} 2.846 & -1.666 & 8.975 & -5.0 \\ -1.666 & 6.366 & -5.0 & 20.90 \\ 8.525 & -5.0 & -2.991 & 1.666 \\ -5.0 & 19.1 & 1.666 & -6.966 \end{bmatrix} \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ \Delta f_2 \\ \Delta f_3 \end{bmatrix}$$

5

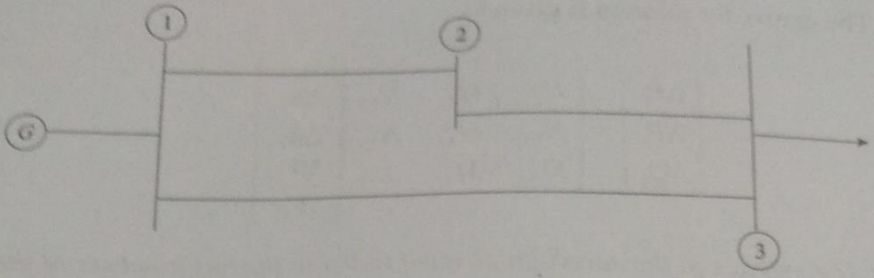


Fig. 7.8 Power system of example 7.7

Solution: Formulate Y_{bus}:

$$Y_{bus} = \begin{bmatrix} -j15.0 & j10.0 & j5.0 \\ j10.0 & -j15.0 & j5.0 \\ j5.0 & j5.0 & -j10.0 \end{bmatrix}$$

Initial Voltages:

$$V_1 = 1.0 + j0.0 = \angle 0°$$
$$V_2 = 1.1 + j0.0 = 1.1\ \angle 0°$$
$$V_3 = 1.0 + j0.0 = 1.0\ \angle 0°$$

(a) **NR Method:** $P_{2,sp} = 5.3217$; $P_{3,sp} = -3.6392$ (since it is a load P_{sp} is negative)

$$P_{2,cal} = P_2 = G_{22}|V_2|^2 + |V_2||V_1|(G_{21}\cos\delta_{21} + B_{21}\sin\delta_{21}) + |V_2||V_3|(G_{23}\cos\delta_{23} + B_{23}\sin\delta_{23})$$

$$\delta_{21} = \delta_2 - \delta_1 = 0°;\ \delta_{23} = \delta_2 - \delta_3 = 0°;\ G_{22} = 0.0$$

$\therefore$

$$P_{2,cal} = 0.0$$

$$\Delta P_2 = 5.3217 - 0.0 = 5.3217.$$

$$P_{3,cal} = P_3 = G_{33}|V_3|^2 + |V_3||V_1|(G_{31}\cos\delta_{31} + B_{31}\sin\delta_{31}) + |V_3||V_2|(G_{32}\cos\delta_{32} + B_{32}\sin\delta_{32})$$

$$= 0.0$$

$$\Delta P_3 = -3.6392 - 0.0 = -3.6392.$$

$$Q_{3,sp} = -0.5339.$$

$$Q_{3,cal} = Q_2 = -B_{33}|V_3|^2 + |V_3||V_1|(G_{31}\sin\delta_{31} - B_{31}\cos\delta_{31}) + |V_3||V_2|(G_{32}\sin\delta_{32} - B_{32}\cos\delta_{32})$$

$$= 10.0\,(1.0)^2 + (1.0 \times 1.0 \times -5.0)(1.0 \times 1.1 \times -5.0)$$

$$= 10.0 - 5.0 - 5.5 = -0.5 \text{ pu.}$$

$$\Delta Q_3 = -0.5339 - (-0.5) = -0.0339 \text{ pu.}$$

$$\begin{bmatrix} \dfrac{\Delta P_2}{|V_2|} \\ \dfrac{\Delta P_3}{|V_3|} \end{bmatrix} = \begin{bmatrix} -B_{22} & -B_{23} \\ -B_{32} & -B_{33} \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \end{bmatrix}$$

$$\frac{\Delta Q_3}{|V_3|} = [-B_{33}] \left[\frac{\Delta |V_3|}{|V_3|} \right]$$

$$\begin{bmatrix} \dfrac{5.3217}{|1.1|} \\ \dfrac{-3.63}{|1|} \end{bmatrix} = \begin{bmatrix} 15 & -5 \\ -5 & 10 \end{bmatrix} \begin{bmatrix} \Delta\theta_2 \\ \Delta\theta_3 \end{bmatrix}$$

$$\left[\frac{-0.0339}{|1|} \right] = [10] \left[\frac{\Delta E_3}{|E_3|} \right]$$

6a

18.6 Comparison of Solution Methods

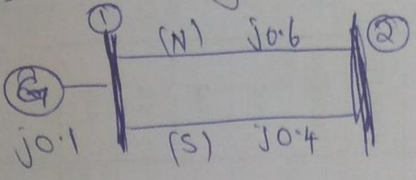
Since the Gauss-Seidel is undoubtedly superior to Gauss method, the comparison is restricted only between G-S method and Newton-Raphson method and that too when Y bus matrix is used for problem formulation. From the view point of computer memory requirements, polar coordinates are preferred for solùtion based on N-R method and rectangular coordinates for the G-S method.

The time taken to perform one iteration of the computation is relatively smaller in case of G-S method as compared to N-R method but the number of iterations required by G-S method for a particular system are greater as compared to N-R method and they increase with the increase in the size of the system. In case of N-R method the number of iterations is more or less independent of the size of the system and vary between 3 to 5 iterations. The convergence characteristics of N-R method are not affected by the selection of a slack bus whereas that of G-S method is sometimes very seriously affected and the selection of a particular bus may result in poor convergence.

The main advantage of G-S method as compared to N-R method is its ease in programming and most efficient use of core memory. Nevertheless, for large power systems N-R method is found to be more efficient and practical from the view point of computational time and convergence characteristics. Even though N-R method can solve most of the practical problems, it may fail in respect of some ill-conditioned problem where other advanced mathematical programming techniques like the non-linear programming techniques can be used. For the readers to have an approximate idea of the computation time taken by N-R method in solving the load flow problem is that, IBM 360/PS 44 system takes less than 10 seconds to obtain a load flow solution of a 14 bus system starting with a flat voltage solution of $(1 + j0.0)$. This includes the formulation of nodal admittance matrix and its storage time.

6b

(c) Z_{bus} building algorithm.



Step ① - Add branch b/w ① and ⓪

$Z_{bus} = [j0.1]$

Step ② - Add branch b/w ① and ②

$Z_{qi} = Z_{pi}$

$Z_{21} = Z_{11} = j0.1$

$Z_{12} = Z_{21} = j0.1$

$Z_{qq} = Z_{pq} + z_{pqpq}$

$Z_{22} = Z_{12} + z_{1212}$

$= j0.1 + j0.6 = j0.7$

$$Z_{bus} = \begin{bmatrix} j0.1 & j0.1 \\ j0.1 & j0.7 \end{bmatrix}$$

Step ③ - Add link b/w ① and ②

$Z_{li} = Z_{pi} - Z_{qi}$

$Z_{l1} = Z_{11} - Z_{21} = 0$

$Z_{l2} = Z_{12} - Z_{22} = j0.1 - j0.7$

$= -j0.6$

$Z_{ll} = Z_{1l} - Z_{2l} + z_{p1212}$

$= 0 + j0.6 + j0.4 = j1$

$$Z_{bus} \begin{bmatrix} j0.1 & j0.1 & 0 \\ j0.1 & j0.7 & -j0.6 \\ 0 & -j0.6 & j1 \end{bmatrix}$$

Z13 = Z23 ... = 0.33...

To eliminate the l^{th} node

$$Z_{11(new)} = Z_{11(old)} - \frac{Z_{1l} \cdot Z_{l1}}{Z_{ll}} = j0.1 - 0 = j0.1$$

$$Z_{12(new)} = Z_{12(old)} - \frac{Z_{1l} \cdot Z_{l2}}{Z_{ll}} = j0.1 - 0 = j0.1$$

$$Z_{21(new)} = Z_{12(new)}$$

$$Z_{22(new)} = Z_{22(old)} - \frac{Z_{2l} \cdot Z_{l2}}{Z_{ll}} = j0.7 - \frac{(-j0.6 \times -j0.6)}{j1}$$

$$= j0.7 - j0.36 = j0.34$$

$$Z_{22} = j0.34$$

$$\therefore Z_{bus} = \begin{bmatrix} j0.1 & j0.1 \\ j0.1 & j0.34 \end{bmatrix}$$