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Internal Assessment Test - III

Sub:	Power System Analysis II						Code:	15EE71	
Date:	22/10/2018	Duration:	90 mins	Max Marks:	50	Sem:	7	Branch:	EEE

Answer Any FIVE FULL Questions

		Marks	OBE	
			CO	RBT
1	Explain the Milne's predictor corrector method of solving transient stability equations.	[10]	CO6	L3
2	A 50 Hz synchronous generator having inertia constant $H=5.2$ MJ/MVA and $x_d'=0.3$ pu is connected to an infinite bus through a double circuit line as shown in fig. The reactance of the connecting HT transformer is 0.2 pu and reactance of each line is 0.4 pu. $ E_{gl} =1.2$ pu and $ V = 1.0$ pu and $P_e = 0.8$ pu. Plot the swing curve using point by point method if a 3 phase fault occurs at the middle of one of the transmission lines and is cleared by isolating the faulted line.	[10]	CO6	L2
3	Solve the question no 2 by Euler's method	[10]	CO6	L3
4	Illustrate clearly the steps involved in solving swing equation using Runge-Kutta method for transient analysis.	[10]	CO6	L3
5	Given that the incremental cost of 2 plant units are $dF_1/dP_1 = 0.008 P_1 + 8$ Rs/MWh $dF_2/dP_2 = 0.0096 P_2 + 6.4$ Rs/MWh Determine the economic operation schedule and corresponding cost of generation if the maximum and minimum loading on each unit is 625 MW and 100 MW respectively. The demand is 900 MW and losses are negligible. Also determine the saving in fuel cost in Rs /hr for economic distribution of the total load of 900 MW compared with equal distribution between the 2 units.	[10]	CO3	L3

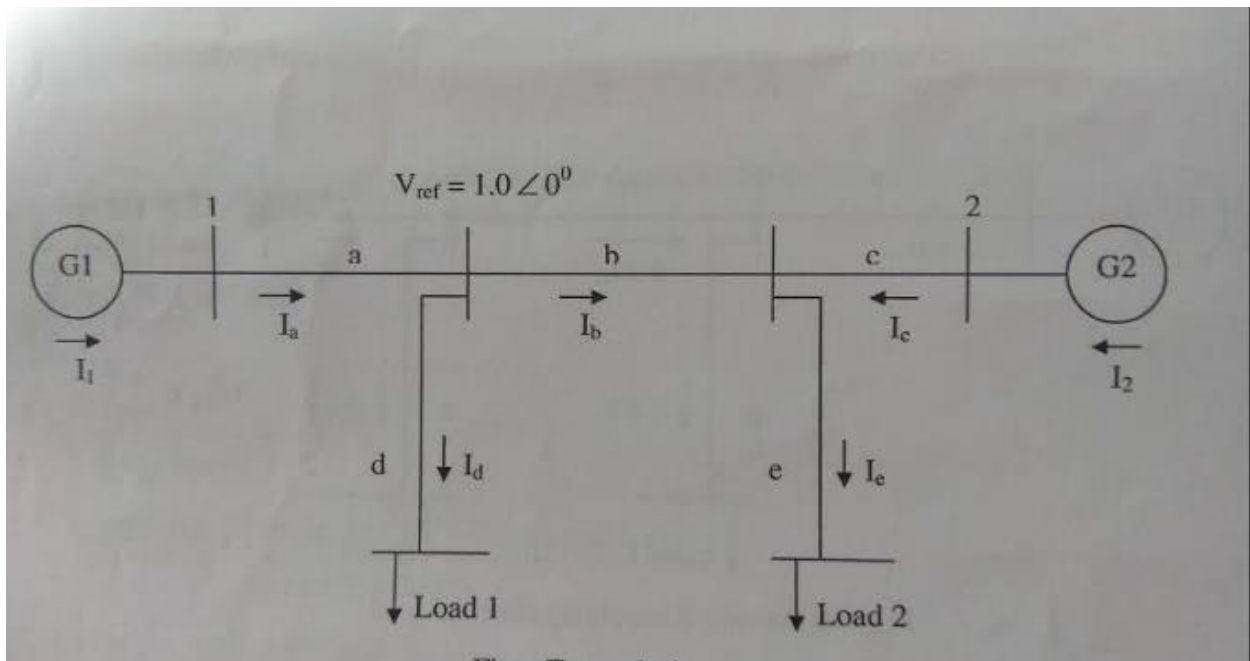
6 Derive the expressions for loss coefficients and transmission loss in terms of generation in an interconnected system. [10]

CO3	L2
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CO4	L3
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Calculate the loss coefficients in pu and MW⁻¹ on a base of 50 MVA for the network given in the fig. Corresponding data is given below

$$\begin{aligned} I_a &= 1.2 - j0.4 \text{ pu} & Z_a &= 0.02 + j0.08 \text{ pu} \\ I_b &= 0.4 - j0.2 \text{ pu} & Z_b &= 0.08 + j0.32 \text{ pu} \\ I_c &= 0.8 - j0.1 \text{ pu} & Z_c &= 0.02 + j0.08 \text{ pu} \\ I_d &= 0.8 - j0.2 \text{ pu} & Z_d &= 0.03 + j0.12 \text{ pu} \\ I_e &= 1.2 - j0.3 \text{ pu} & Z_e &= 0.03 + j0.12 \text{ pu} \end{aligned}$$



Solutions

1

Milne's Predictor-Corrector Method

$$\frac{dx}{dt} = f_x(x, y, t) \quad \Bigg| \quad \frac{dy}{dt} = f_y(x, y, t)$$

With the known four consecutive previous values

$$x_{n+1}^p = x_{n-3} + \frac{4h}{3} [2x'_{n-2} - x'_{n-1} + 2x'_n]$$

$$y_{n+1}^p = y_{n-3} + \frac{4h}{3} [2y'_{n-2} - y'_{n-1} + 2y'_n]$$

x' and y' are derivatives at the corresponding times.

Then the corrected values are

$$x_{n+1} = x_{n-1} + \frac{h}{3} [x'_{n-1} + 4x'_n + x'_{n+1}]$$

$$y_{n+1} = y_{n-1} + \frac{h}{3} [y'_{n-1} + 4y'_n + y'_{n+1}]$$

where $x'_{n+1} = f_x [x_{n+1}^p, y_{n+1}^p, t_{n+1}]$

$$y'_{n+1} = f_y [x_{n+1}^p, y_{n+1}^p, t_{n+1}]$$



M. Inés Predictor corrector method

$w_1' = 0.8 - 0.63 \sin$

~~$\delta_1' = 28.87$~~
 ~~$\delta_2' = 32.03$~~
 ~~$\delta_3' = 2.161$~~
 ~~$\delta_4' = 2.067$~~

From Runge Kutta method.

$t = 0.1 \text{ sec}$

$\delta_1 = 28.87$	$\omega_1 = 46.18$
$\delta_2 = 32.03$	$\omega_2 = 90.386$
$\delta_3 = 37.25$	$\omega_3 = 123.86$
$\delta_4 = 43.33$	$\omega_4 = 118.48$

$\delta_1' = 46.18$	$\omega_1 = \frac{0.8 - 0.63 \sin 28.87}{0.00594} = 911.43$ $\omega_2 = \frac{0.8 - 0.63 \sin 32.03}{0.00594} = 803.04$ $\omega_3 = \frac{0.8 - 0.63 \sin 37.25}{0.00594} = 725.588$ $\omega_4 = \frac{0.8 - 0.63 \sin 43.33}{0.00594} = 199.15$
$\delta_2' = 90.386$	
$\delta_3' = 123.86$	
$\delta_4' = 118.48$	

$$\delta_5^p = \delta_1 + \frac{4 \Delta t}{3} [2\delta_2' - \delta_3' + 2\delta_4'] = 28.87 + \frac{4 \times 0.05}{3} [2 \times 90.386 - 123.86 + 2 \times 118.48] = 78.37$$

$$\omega_5^p = \omega_1 + \frac{4 \Delta t}{3} [2\omega_2' - \omega_3' + 2\omega_4'] = 46.18 + \frac{4 \times 0.05}{3} [2 \times 803.04 - 725.588 + 2 \times 199.15] = 48.46$$

$$\delta_5' = 78.37$$

$$\omega_5' = \frac{0.8 - 0.63 \sin 78.37}{0.00594} = -342.7$$

$$p_5 = \delta_3 + \frac{\Delta t}{3} [\delta_4' + 4\delta_5' + \delta_5'] = 37.25 + \frac{0.05}{3} [123.86 + 4 \times 118.48 + 78.37] = 48.52$$

9



$$\begin{aligned}\omega_5 &= \omega_3 + \frac{Dt}{3} [\omega_3^1 + 4\omega_4^1 + \omega_5^1] \\ &= 123.86 + \frac{0.5}{3} [725.528 + 4 \times \overset{-199.15}{-487.9} + -342.7] = \underline{\underline{-92.282}} \\ &= \underline{\underline{116.98}}\end{aligned}$$

$$\delta_5^1 = \omega_5 = 116.98$$

$$\omega_5^1 = \frac{.8 - 1.333 \sin 48.52}{.000577} = \underline{\underline{-344.30}}$$

37

2 Pg. 393 Uma Rood

0.054 mVA
5 L

CMR

$E_g = 12$
 $X_s = 0.3$
 $R = 0.2$
 $X_L = 1.0$

$X_1 = 0.7$
 $P_1 = \frac{1.2 \times 1.0}{0.7} = 1.714$
 $P_c = 0.8 = P_m$
 $0.8 = 1.714 \sin \phi$
 $\therefore \phi = 27.82^\circ$

After fault.

$X_3 = 0.9$
 $P_3 = \frac{1.2 \times 1.0}{0.9} = 1.3334$

At $t = 0^-$

$P_a = P_g - P_c = 0.8 - 1.714 \sin 27.82 = 0$

At $t = 0^+$

$P_a = 0.8 - 1.3334 \sin 27.82 = 0.8 - 0.63 = 0.506$

$P_a = \frac{P_a(0^-) + P_a(0^+)}{2} = \frac{0 + 0.506}{2} = 0.253$

$\Delta \omega_1 = \frac{P_a \times \Delta t}{m} = \frac{0.253 \times 0.05}{0.0054} = 23.426$

$\omega_1 = \omega_0 + \Delta \omega_1 = 0 + 23.426 = 23.426$

$\Delta \delta_1 = \omega_1 \times \Delta t = 23.426 \times 0.05 = 1.1713$

$\delta_1 = \delta_0 + \Delta \delta_1 = 27.82 + 1.1713 = 28.9913 \Rightarrow t = 0.05 \Rightarrow \delta = 28.99$
 $\omega = 23.426$

Fault cleared in 2.5 cycles.

$$2.5 \text{ cycles} \Rightarrow 0.05 \text{ sec.}$$

$$P_a(0.05) \Rightarrow 0.8 - 0.63 \sin 28.99 = 0.8 - 0.3053 = 0.4947$$

$$P_a(0.05') \Rightarrow 0.8 - 1.33 \sin 28.99 = 0.8 - 0.6446 = 0.1554$$

$$P_a = 0.32505$$

$$\Delta \omega_1 = \frac{0.32505 \times 0.05}{m} = 28.16$$

$$\omega_1 = 23.42 + 28.16 = 51.5872$$

$$\Delta \delta_1 = 51.5872 \times 0.05 = 2.5794$$

$$\delta_1 = 28.99 + 2.5794 = 31.5694$$

t = 0.1

$$P_a = 0.8 - 1.33 \sin 31.5694 = 0.8 - 0.69787 = 0.10213$$

$$\Delta \omega_2 = \frac{0.10213 \times 0.05}{m} = 8.85008$$

$$\omega_2 = 51.5872 + 8.85008 = 60.4373$$

$$\Delta \delta_2 = 60.4373 \times 0.05 = 3.02186$$

$$\delta_2 = 31.5694 + 3.02186 = 34.5912$$

continue \rightarrow

Fault cleared in 6.25 cycles \Rightarrow 0.125 sec

t = 0.05 & t = 0.1 same as sustained fault.

t = 0.15

take angle
7.62 (same as
sustained fault)

$$P_a = 0.8 - 1.333 \sin 37.62 = 0.8 - 0.810 = -0.010$$

$$\Delta \omega = \frac{-0.010 \times 0.05}{m} = -8.8302$$

$$\omega = 106.4390 - 8.8302 = 105.5471$$

$$\Delta \delta = 105.5471 \times 0.05 = 5.277$$

$$\delta = 37.62 + 5.277 = 42.8978 // = \text{this is at } t = 0.2$$

modified Euler's method

considers two simultaneous diff equ

$$\frac{dx}{dt} = f_x(x, y, t)$$

$$\frac{dy}{dt} = f_y(x, y, t)$$

Starting with x_0, y_0, t_0 with a step size h

$$D_x = f_x(x_0, y_0, t_0) = \left. \frac{dx}{dt} \right|_0$$

$$D_y = f_y(x_0, y_0, t_0) = \left. \frac{dy}{dt} \right|_0$$

$$x^p = x_0 + D_x h \quad \left. \begin{array}{l} x^p \\ y^p \end{array} \right\} \text{ predicted values.}$$

$$y^p = y_0 + D_y h$$

$$D_{x,p} = \left. \frac{dx}{dt} \right|_p = f_x(x^p, y^p, t)$$

$$D_{y,p} = \left. \frac{dy}{dt} \right|_p = f_y(x^p, y^p, t)$$

$$x_1 = x_0 + \left(\frac{D_x + D_{x,p}}{2} \right) h$$

$$y_1 = y_0 + \left(\frac{D_y + D_{y,p}}{2} \right) h$$

From swing equ.

$$\frac{ds}{dt} = \omega \quad \text{if} \quad \frac{d\omega}{dt} = \frac{P_a}{m} = \frac{P_m - P_{max} \sin \delta}{m}$$

$$\frac{ds}{dt} = \omega_0 \quad \left. \frac{d\omega}{dt} \right|_0 = D_2 = \frac{P_m - P_{max} \sin \delta_0}{m}$$

$$s^p = s_0 + \omega_0 \Delta t \quad \text{if} \quad \omega^p = \omega_0 + D_2 \Delta t$$

$$\left. \frac{ds}{dt} \right|_p = D_{1,p} = \omega^p \quad \text{if} \quad \left. \frac{d\omega}{dt} \right|_p = D_{2,p} = \frac{P_m - P_{max} \sin \delta^p}{m}$$



$$d_1 = d_0 + \left(\frac{D_1 + D_1 P}{2}\right) \Delta t \quad \text{el } \omega_1 = \omega_0 + \left(\frac{D_2 + D_2 P}{2}\right) \Delta t$$

Same problem

$$t = 0^- \quad , \quad d_0 = 27.8 \quad \text{el } \omega_0 = 0 \quad P_{max} = 1.714$$

$$t = 0^+ \quad D_1 = \omega_0 = 0$$

$$D_2 = \frac{P_0 - P_{max} \sin d_0}{m} = \frac{0.8 - 0.63 \sin 27.8}{0.00574} = 930.4$$

$$d^P = d_0 + D_1 \Delta t = 27.8$$

$$\omega^P = \omega_0 + D_2 \Delta t = 46.52$$

$$D_1 P = \omega_P = 46.52$$

$$D_2 P = \frac{0.8 - 0.63 \sin 27.8}{m} = 930.47$$

$$d_1 = 27.8 + \left[\frac{0 + 46.52}{2} \right] \times 0.05 = 28.9$$

$$\omega_1 = 0 + \left[\frac{930.47 + 930.47}{2} \right] \times 0.05 = 46.52$$

$$t = 0.05 \Rightarrow \begin{cases} d = 28.9 \\ \omega = 46.52 \end{cases}$$

$$D_1 = 46.52$$

$$D_2 = \frac{0.8 - 0.63 \sin 28.9}{0.00544} = 910.90$$

$$d^P = 28.9 + 46.52 \times 0.05 = 31.226$$

$$\omega^P = 46.52 + 910.9 \times 0.05 = 92.065$$

$$D_1 P = 92.065, \quad D_2 P = \frac{0.8 - 0.63 \sin 31.226}{0.00544} = 870.2178$$

$$d_1 = 28.9 + \left[\frac{46.52 + 92.065}{2} \right] \times 0.05 = 32.36$$

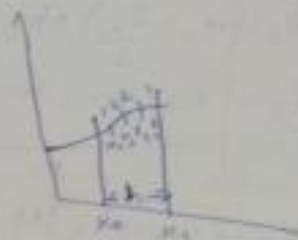
$$\omega_1 = 46.52 + \left[\frac{910.9 + 870.2178}{2} \right] \times 0.05 = 91.048$$

After 5 ms fault is cleared then $P_{max} = 1.333 //$

Runge kutta method

$$\frac{dx}{dt} = f_x(x, y, t)$$

$$\frac{dy}{dt} = f_y(x, y, t)$$



Starting with x_0, y_0, t_0 with step size h .

$$x_1 = x_0 + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = y_0 + \frac{1}{6} (l_1 + 2l_2 + 2l_3 + l_4)$$

$$k_1 = f_x(x_0, y_0, t_0)h$$

$$k_2 = f_x\left(x_0 + \frac{k_1}{2}, y_0 + \frac{l_1}{2}, t_0 + \frac{h}{2}\right)h$$

$$k_3 = f_x\left(x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + \frac{h}{2}\right)h$$

$$k_4 = f_x(x_0 + k_3, y_0 + l_3, t_0 + h)h$$

$$l_1 = f_y(x_0, y_0, t_0)h$$

$$l_2 = f_y\left(x_0 + \frac{k_1}{2}, y_0 + \frac{l_1}{2}, t_0 + \frac{h}{2}\right)h$$

$$l_3 = f_y\left(x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + \frac{h}{2}\right)h$$

$$l_4 = f_y(x_0 + k_3, y_0 + l_3, t_0 + h)h$$

Two differential equ.

$$\frac{dd}{dt} = \omega$$

$$\frac{d\omega}{dt} = \frac{P_a}{m} = \frac{P_m - P_{max} \sin \omega t}{m}$$

$$k_1 = \omega_0 \Delta t$$

$$f_1 = \left[\frac{P_m - P_{max} \sin(\omega_0 \Delta t)}{m} \right] \Delta t$$

$$k_2 = \left(\omega_0 + \frac{f_1}{2} \right) \Delta t$$

$$f_2 = \left[\frac{P_m - P_{max} \sin\left(\omega_0 + \frac{f_1}{2}\right) \Delta t}{m} \right] \Delta t$$



$$K_3 = \left(\omega_0 + \frac{l_2}{2} \right) \Delta t$$

$$l_3 = \left[\frac{P_m - P_{max} \sin\left(\delta_0 + \frac{K_2}{2}\right)}{m} \right] \Delta t$$

$$K_4 = (\omega_0 + l_3) \Delta t$$

$$l_4 = \left[\frac{P_m - P_{max} \sin(\delta_0 + K_3)}{m} \right] \Delta t$$

$$\delta_1 = \delta_0 + \frac{1}{6} [K_1 + 2K_2 + 2K_3 + K_4]$$

$$\omega_1 = \omega_0 + \frac{1}{6} [l_1 + 2l_2 + 2l_3 + l_4]$$

Same Problem

1st interval

$$\delta_0 = 27.8, \omega_0 = 0$$

$$K_1 = 0 \times 0.05 = 0, l_1 = \left[\frac{0.8 - 0.63 \sin 27.8}{0.00544} \right] \cdot 0.05 = 46.52$$

$$K_2 = \left(0 + \frac{46.52}{2} \right) \cdot 0.05 = 1.163, l_2 = \left[\frac{0.8 - 0.63 \sin(27.8)}{0.00544} \right] \cdot 0.05 = 46.52$$

$$K_3 = \left(0 + \frac{46.52}{2} \right) \cdot 0.05 = 1.163, l_3 = \left[\frac{0.8 - 0.63 \sin(27.8 + 1.163)}{0.00544} \right] \cdot 0.05 = 46.52$$

$$K_4 = (0 + 46) \cdot 0.05 = 2.3, l_4 = \left[\frac{0.8 - 0.63 \sin(27.8 + 1.163)}{0.00544} \right] \cdot 0.05 = 46.52$$

$$\delta_1 = 27.8 + \frac{1}{6} [0 + 2 \times 1.163 + 2 \times 46.52 + 2.3] = 28.9$$

$$\omega_1 = 0 + \frac{1}{6} [46.5 + 2 \times 46.52 + 2 \times 46 + 45.54] = 46.18$$

$$x_1 = 46.18 \times 0.05 = 2.309 \quad f_1 = \left(\frac{0.8 - 0.63 \sin 28.9}{0.00544} \right) \cdot 0.05 = \underline{\underline{45.54}}$$

$$x_2 = \left(46.18 + \frac{45.54}{2} \right) \cdot 0.05 = \underline{\underline{3.4475}} \quad f_2 = \left(\frac{0.8 - 0.63 \sin \left(28.9 + \frac{2.309}{2} \right)}{0.00544} \right) \cdot 0.05 = \underline{\underline{44.53}}$$

$$x_3 = 2.309 \left(46.18 + \frac{44.53}{2} \right) \cdot 0.05 = \underline{\underline{3.422}} \quad f_3 = \left(\frac{0.8 - 0.63 \sin \left(28.9 + \frac{3.4475}{2} \right)}{0.00544} \right) \cdot 0.05 = \underline{\underline{44.04}}$$

$$x_4 = \left(46.18 + 44.04 \right) \cdot 0.05 = \underline{\underline{4.5112}} \quad f_4 = \left(\frac{0.8 - 0.63 \sin \left(28.9 + 3.422 \right)}{0.00544} \right) \cdot 0.05 = \underline{\underline{42.56}}$$

$$S_2 = 28.9 + \frac{1}{6} \left[2 \cdot 3.09 + 2 \times 3 \cdot 4.475 + 2 \times 3 \cdot 4.22 + 4 \cdot 5.112 \right] = \underline{\underline{32.326}}$$

$$\omega_2 = 46.18 + \frac{1}{6} \left[45.54 + 2 \times 44.53 + 2 \times 44.04 + 42.56 \right] = \underline{\underline{90.386}}$$

DERIVATION OF TRANSMISSION LOSS FORMULA

An accurate method of obtaining general loss coefficients has been presented by Kron. The method is elaborate and a simpler approach is possible by making the following assumptions:

- (i) All load currents have same phase angle with respect to a common reference
- (ii) The ratio X / R is the same for all the network branches.

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Consider the simple case of two generating plants connected to an arbitrary number of loads through a transmission network as shown in Fig 8.9a

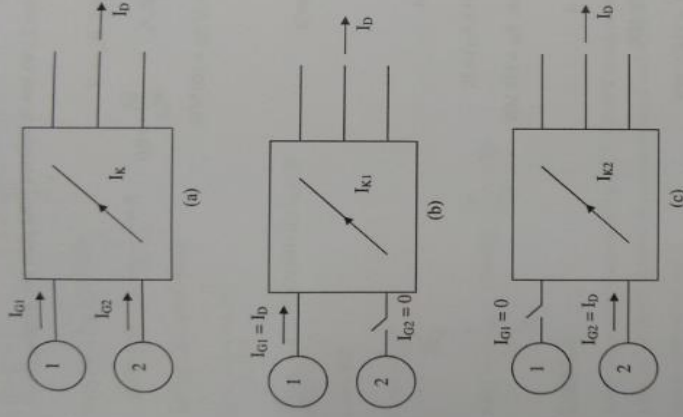


Fig. Two plants connected to a number of loads through a transmission network. Let's assume that the total load is supplied by only generator 1 as shown in Fig 8.9b. Let the current through a branch K in the network be I_{k1} . We define

$$N_{k1} = \frac{I_{k1}}{I_D}$$

$$N_{k2} = \frac{I_{k2}}{I_D}$$

It is to be noted that $I_{G1} = I_D$ in this case. Similarly with only plant 2 supplying the load current I_D , as shown in Fig 8.9c, we define

N_{k1} and N_{k2} are called current distribution factors and their values depend on the impedances of the lines and the network connection. They are independent of I_D . When both generators are supplying the load, then by principle of superposition

$$I_K = N_{k1} I_{G1} + N_{k2} I_{G2}$$

where I_{G1} , I_{G2} are the currents supplied by plants 1 and 2 respectively, to meet the demand I_D . Because of the assumptions made, I_{k1} and I_D have same phase angle, as do I_{k2} and I_D . Therefore, the current distribution factors are real rather than complex. Let

$$I_{G1} = |I_{G1}| \angle \sigma_1 \text{ and } I_{G2} = |I_{G2}| \angle \sigma_2,$$

where σ_1 and σ_2 are phase angles of I_{G1} and I_{G2} with respect to a common reference. We can write

$$\begin{aligned} |I_K|^2 &= (N_{k1}|I_{G1}| \cos \sigma_1 + N_{k2}|I_{G2}| \cos \sigma_2)^2 + (N_{k1}|I_{G1}| \sin \sigma_1 + N_{k2}|I_{G2}| \sin \sigma_2)^2 \\ &= N_{k1}^2 |I_{G1}|^2 [\cos^2 \sigma_1 + \sin^2 \sigma_1] + N_{k2}^2 |I_{G2}|^2 [\cos^2 \sigma_2 + \sin^2 \sigma_2] \\ &\quad + 2[N_{k1}|I_{G1}| \cos \sigma_1 N_{k2}|I_{G2}| \cos \sigma_2 + N_{k1}|I_{G1}| \sin \sigma_1 N_{k2}|I_{G2}| \sin \sigma_2] \\ &= N_{k1}^2 |I_{G1}|^2 + N_{k2}^2 |I_{G2}|^2 + 2N_{k1} N_{k2} |I_{G1}| |I_{G2}| \cos(\sigma_1 - \sigma_2) \end{aligned}$$

$$\text{Now } |I_{G1}| = \frac{P_{G1}}{\sqrt{3}|V_1| \cos \phi_1} \text{ and } |I_{G2}| = \frac{P_{G2}}{\sqrt{3}|V_2| \cos \phi_2}$$

where P_{G1} , P_{G2} are three phase real power outputs of plant 1 and plant 2; V_1 , V_2 are the line to line bus voltages of the plants and ϕ_1, ϕ_2 are the power factor angles.

The total transmission loss in the system is given by

$$P_L = \sum_K 3|I_K|^2 R_K$$

where the summation is taken over all branches of the network and R_K is the branch resistance. Substituting, we get

$$\begin{aligned} P_L &= \frac{P_{G1}^2}{|V_1|^2 (\cos \phi_1)^2} \sum_K N_{k1}^2 R_K + \frac{2P_{G1} P_{G2} \cos(\sigma_1 - \sigma_2)}{|V_1| |V_2| \cos \phi_1 \cos \phi_2} \sum_K N_{k1} N_{k2} R_K \\ &\quad + \frac{P_{G2}^2}{|V_2|^2 (\cos \phi_2)^2} \sum_K N_{k2}^2 R_K \end{aligned}$$

$$P_L = P_{G1}^2 B_{11} + 2P_{G1} P_{G2} B_{12} + P_{G2}^2 B_{22}$$

where

$$B_{11} = \frac{1}{|V_1|^2 (\cos \phi_1)^2} \sum_K N_{k1}^2 R_K$$

$$B_{12} = \frac{\cos(\sigma_1 - \sigma_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2} \sum_k N_{K1} N_{K2} R_K$$

$$B_{22} = \frac{1}{|V_2|^2(\cos\phi_2)^2} \sum_k N_{K2}^2 R_K$$

The loss – coefficients are called the B – coefficients and have unit MW⁻¹. For a general system with n plants the transmission loss is expressed as

$$P_L = \frac{P_{G1}^2}{|V_1|^2(\cos\phi_1)^2} \sum_k N_{K1}^2 + \dots + \frac{P_{Gn}^2}{|V_n|^2(\cos\phi_n)^2} \sum_k N_{Kn}^2 R_K$$

$$+ 2 \sum_{\substack{p,q=1 \\ p \neq q}}^n \frac{P_{Gp} P_{Gq} \cos(\sigma_p - \sigma_q)}{|V_p||V_q|\cos\phi_p\cos\phi_q} \sum_k N_{Kp} N_{Kq} R_K$$

In a compact form

$$P_L = \sum_{p=1}^n \sum_{q=1}^n P_{Gp} B_{pq} P_{Gq}$$

$$B_{pq} = \frac{\cos(\sigma_p - \sigma_q)}{|V_p||V_q|\cos\phi_p\cos\phi_q} \sum_k N_{Kp} N_{Kq} R_K$$

B – Coefficients can be treated as constants over the load cycle by computing average operating conditions, without significant loss of accuracy.

Example 8

Calculate the loss coefficients in pu and MW⁻¹ on a base of 50MVA for the net Fig below. Corresponding data is given below.



Fig : Example 8

Solution:

Total load current

$$I_L = I_1 + I_2 = 2.0 - j 0.5 = 2.061 \angle -14.03^\circ \text{ A}$$

$$I_{L1} = I_1 = 0.8 - j 0.2 = 0.8246 \angle -14.03^\circ \text{ A}$$

$$\frac{I_{L1}}{I_L} = 0.4; \quad \frac{I_{L2}}{I_L} = 1.0 - 0.4 = 0.6$$

If generator 1, supplies the load then $I_1 = I_L$. The current distribution is shown in Fig a.

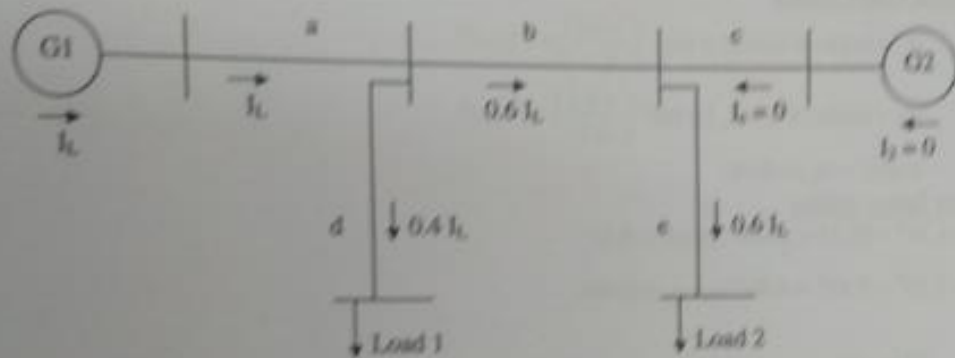


Fig a : Generator 1 supplying the total load

$$N_{a1} = \frac{I_L}{I_L} = 1.0; \quad N_{a2} = \frac{I_L}{I_L} = 0.6; \quad N_{c1} = 0; \quad N_{e1} = 0.4; \quad N_{e2} = 0.6$$

Similarly the current distribution when only generator 2 supplies the load is shown in Fig b.

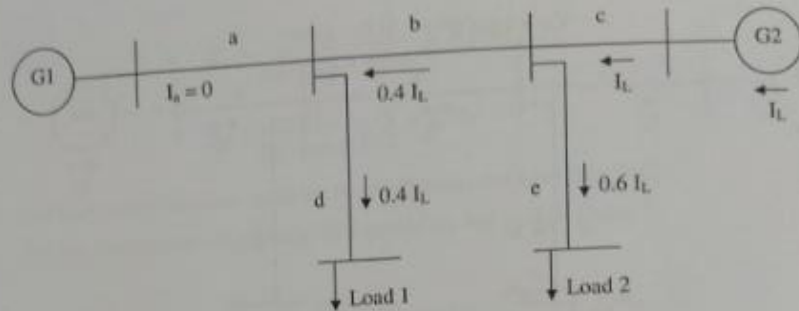


Fig b: Generator 2 supplying the total load

$$N_{a2} = 0; N_{b2} = -0.4; N_{c2} = 1.0; N_{d2} = 0.4; N_{e2} = 0.6$$

$$\begin{aligned} \text{From Fig 8.10, } V_1 &= V_{ref} + Z_a I_a \\ &= 1 \angle 0^\circ + (1.2 - j 0.4) (0.02 + j 0.08) \\ &= 1.06 \angle 4.78^\circ = 1.056 + j 0.088 \text{ pu.} \end{aligned}$$

$$\begin{aligned} V_2 &= V_{ref} - I_b Z_b + I_c Z_c \\ &= 1.0 \angle 0^\circ - (0.4 - j 0.2) (0.08 + j 0.32) + (0.8 - j 0.1) (0.02 + j 0.08) \\ &= 0.928 - j 0.05 = 0.93 \angle -3.10^\circ \text{ pu.} \end{aligned}$$

Current Phase angles

$$\sigma_1 = \text{angle of } I_1 (= I_a) = \tan^{-1} \left(\frac{-0.4}{1.2} \right) = -18.43^\circ$$

$$\sigma_2 = \text{angle of } I_2 (= I_c) = \tan^{-1} \left(\frac{-0.1}{0.8} \right) = -7.13^\circ$$

$$\cos(\sigma_1 - \sigma_2) = 0.98$$

Power factor angles

$$\phi_1 = 4.78^\circ + 18.43^\circ = 23.21^\circ; \cos \phi_1 = 0.92$$

$$\phi_2 = 7.13^\circ - 3.10^\circ = 4.03^\circ; \cos \phi_2 = 0.998$$

$$B_{11} = \frac{\sum_k N_{k1}^2 R_k}{|V_1|^2 (\cos \phi_1)^2} = \frac{1.0^2 \times 0.02 + 0.6^2 \times 0.08 + 0.4^2 \times 0.03 + 0.6^2 \times 0.03}{(1.06)^2 (0.920)^2}$$

$$= 0.0677 \text{ pu}$$

$$= 0.0677 \times \frac{1}{50} = 0.1354 \times 10^{-3} \text{ MW}^{-1}$$

$$B_{12} = \frac{\cos(\sigma_1 - \sigma_2)}{|V_1| |V_2| (\cos \phi_1) (\cos \phi_2)} \sum_k N_{k1} N_{k2} R_k$$

$$\begin{aligned}
&= \frac{0.98}{(1.06)(0.93)(0.998)(0.92)} [-0.4 \times 0.6 \times 0.08 + 0.4 \times 0.4 \times 0.03 + 0.6 \times 0.6 \times 0.03] \\
&= -0.00389 \text{ pu} \\
&= -0.0078 \times 10^2 \text{ MW}^{-1}
\end{aligned}$$

$$\begin{aligned}
B_{22} &= \frac{\sum_k N_{k2}^2 R_k}{|V_2|^2 (\cos \phi_2)^2} \\
&= \frac{(-0.4)^2 0.08 + 1.0^2 \times 0.02 + 0.4^2 \times 0.03 + 0.6^2 \times 0.03}{(0.93)^2 (0.998)^2} \\
&= 0.056 \text{ pu} = 0.112 \times 10^{-2} \text{ MW}^{-1}
\end{aligned}$$

5 Given that the incremental cost of 2 plant units are

[10] CO3 L3

$$dF_1/dP_1 = 0.008 P_1 + 8 \text{ Rs/MWh}$$

$$dF_2/dP_2 = 0.0096 P_2 + 6.4 \text{ Rs/MWh}$$

Determine the economic operation schedule and corresponding cost of generation if the maximum and minimum loading on each unit is 625 MW and 100 MW respectively. The demand is 900 MW and losses are negligible. Also determine the saving in fuel cost in Rs/hr for economic distribution of the total load of 900 MW compared with equal distribution between the 2 units.

$$dF_1/dP_1 = dF_2/dP_2 = \lambda$$

$$0.008 P_1 + 8 = 0.0096 P_2 + 6.4$$

$$P_1 + P_2 = 900 \quad \text{--- (2)}$$

$$P_1 = 400$$

$$P_2 = 500$$

$$F_1 = \int dF_1 = \frac{0.008 P_1^2}{2} + 8 P_1$$

$$= 3840$$

$$F_2 = \int dF_2/dP_2 = 0.0096 P_2^2 + 6.4 P_2$$

$$\text{Page 1 of 3} = 4400$$

$$\text{Total} = \underline{\underline{8240 \text{ Rs}}}$$

Equal distribution

$$P_1 = P_2 = 450$$

$$F_1 = 4410$$

$$F_2 = 4824$$

$$F_1 + F_2 = 9234$$

$$\text{Saving} = 9234 - 8240 = 994 \text{ Rs/hr}$$