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Internal Assesment Test - III

Sub:	SIGNALS AND SYSTEMS							Code:	15EE54
Date:	20/11/2018	Duration:	90 mins	Max Marks:	50	Sem:	5th	Branch:	EEE

Answer Any FIVE FULL Questions

		Marks	OBE	
			CO	RBT
1	Determine x[n] if $X(z)=\frac{1-z^{-1}+z^{-2}}{(1-½z^{-1})(1-2z^{-1})(1-z^{-1})}$ for the following ROCs (i)$\lvert z\rvert < ½$ (ii) $\lvert z\rvert < 2$ (iii)$1< \lvert z\rvert < ½$	10	CO5	L2
2	Solve the following difference equation for y(n) using z-transform and the specified initial conditions. $y(n)- y(n-1)+\frac{1}{4} y(n-2) = x(n)$ where $x(n)=2(\frac{1}{8})^n u(n)$;y(-1)=2 and y(-2)=4	10	CO5	L2
3	Prove the time differentiation and Integration property of Fourier Transform	10	CO5	L1
4	Obtain the Fourier transform of the following a)$x(t)=e^{-a\lvert t\rvert}$;a>0 b)$x(t)=\sin(\pi t)\, e^{-2t}u(t)$ c)$x(t)=\frac{2}{t^2+1}$	10	CO5	L2
5	Find the frequency response and the impulse response of the system described by the difference equation y(n) +½ y(n-1) = x(n) - 2x(n-1)	10	CO5	L2
6	Compute DTFT for the signals a)$x(n) = 2^n u(-n)$ b) $a^{\lvert n\rvert}$	10	CO5	L2
7	Prove the convolution and Modulation property of DTFT	10	CO5	L1

1.

olution: We first factorize X(z).

$$X(z) = \frac{A_1}{1-\frac{1}{2}z^{-1}} + \frac{A_2}{1-2z^{-1}} + \frac{A_3}{1-z^{-1}}$$

$$A_1 = \frac{1-z^{-1}+z^{-2}}{\left(1-\frac{1}{2}z^{-1}\right)(1-2z^{-1})(1-z^{-1})} \times \left(1-\frac{1}{2}z^{-1}\right)\Bigg|_{z^{-1}=2}$$

$$= 1$$

$$A_2 = \frac{1 - z^{-1} + z^{-2}}{\left(1 - \frac{1}{2}z^{-1}\right)\cancel{(1 - 2z^{-1})}(1 - z^{-1})} \times \cancel{(1 - 2z^{-1})}\Bigg|_{z^{-1} = \frac{1}{2}}$$

$$= 2$$

$$A_3 = \frac{1 - z^{-1} + z^{-2}}{\left(1 - \frac{1}{2}z^{-1}\right)(1 - 2z^{-1})\cancel{(1 - z^{-1})}} \times \cancel{(1 - z^{-1})}\Bigg|_{z^{-1} = 1}$$

$$= -2$$

$$X(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} + \frac{2}{1 - 2z^{-1}} - \frac{2}{1 - z^{-1}}$$

The poles are at $z = \frac{1}{2}$, 2 and 1.

(i) ROC: $|z| < \frac{1}{2}$

The radius of ROC is less than all the poles. So all terms are left-sided gnals. Using (8.41)

$$x[n] = -\left(\frac{1}{2}\right)^n u[-n-1] - 2[2]^n u[-n-1] + 2u[-n-1]$$

(ii) ROC: $|z| > 2$

The radius of ROC is greater than all the poles. So all are right-sided nverse transforms. Using (8.40)

$$x[n] = \left(\frac{1}{2}\right)^n u[n] + 2(2)^n u[n] - 2u[n]$$

(iii) ROC: $1 < |z| < 2$

The terms corresponding to $d_k = \frac{1}{2}$ and 1 are right sided and $d_k = 2$ is left ided.

$$x[n] = \left(\frac{1}{2}\right)^n u[n] - 2u[n] - 2(2)^n u[-n-1]$$

2.

Given

$$y(n) - y(n-1) + \frac{1}{4}y(n-2) = x(n)$$

Taking z-transform on both sides

$$Y(z) - [z^{-1}Y(z) + y(-1)] + \frac{1}{4}[z^{-2}Y(z) + z^{-1}y(-1) + y(-2)] = X(z)$$

$$\frac{Y_2(z)}{z} = \frac{2z^2}{\left(z - \frac{1}{2}\right)^2\left(z - \frac{1}{8}\right)} = \frac{A}{z - \frac{1}{2}} + \frac{B}{\left(z - \frac{1}{2}\right)^2} + \frac{C}{z - \frac{1}{8}}$$

$$= \frac{16}{9}\frac{1}{z - \frac{1}{2}} + \frac{4}{3\left(z - \frac{1}{2}\right)^2} + \frac{2}{9}\cdot\frac{1}{z - \frac{1}{8}}$$

$$Y_2(z) = \frac{16}{9}\frac{z}{z - \frac{1}{2}} + \frac{4}{3}\cdot\frac{z}{\left(z - \frac{1}{2}\right)^2} + \frac{2}{9}\frac{z}{z - \frac{1}{8}}$$

$$y_2(n) = \frac{16}{9}\left(\frac{1}{2}\right)^n u(n) + \frac{8}{3}n\left(\frac{1}{2}\right)^n u(n) + \frac{2}{9}\left(\frac{1}{8}\right)^n u(n)$$

$$y(n) = y_1(n) + y_2(n) = \frac{25}{9}\left(\frac{1}{2}\right)^n u(n) + \frac{8}{3}n\left(\frac{1}{2}\right)^n u(n) + \frac{2}{9}\left(\frac{1}{8}\right)^n u(n)$$

$$A = \frac{1}{1!}\frac{d}{dz}\left(z - \frac{1}{2}\right)^2 \frac{2z^2}{\left(z - \frac{1}{2}\right)^2\left(z - \frac{1}{8}\right)}\Bigg|_{z = \frac{1}{2}} = \frac{\left(z - \frac{1}{8}\right)4z - 2z^2}{\left(z - \frac{1}{8}\right)^2}\Bigg|_{z = \frac{1}{2}}$$

$$= \frac{\left(\frac{1}{2} - \frac{1}{8}\right)2 - \frac{1}{2}}{\left(\frac{1}{2} - \frac{1}{8}\right)^2} = \frac{\frac{3}{4} - \frac{1}{2}}{(3/8)^2} = \frac{1}{4}\cdot\frac{8}{3}\cdot\frac{8}{3} = \frac{16}{9}$$

$$B = \left(z - \frac{1}{2}\right)^2 \frac{2z^2}{\left(z - \frac{1}{2}\right)^2\left(z - \frac{1}{8}\right)}\Bigg|_{z = \frac{1}{2}} = \frac{2(1/4)}{\frac{1}{2} - \frac{1}{8}} = (1/2)\cdot\frac{8}{3} = \frac{4}{3}$$

$$C = \left(z - \frac{1}{8}\right)\frac{2z^2}{\left(z - \frac{1}{2}\right)^2\left(z - \frac{1}{8}\right)}\Bigg|_{z = \frac{1}{8}} = \frac{2\left(\frac{1}{8}\right)^2}{\left(\frac{-3}{8}\right)^2} = 2\frac{1}{64}\cdot\frac{64}{9} = \frac{2}{9}$$

$$Y(z)\left[1 - z^{-1} + \frac{1}{4}z^{-2}\right] - 2 + \frac{1}{2}z^{-1} + 1 = X(z)$$

$$Y(z)\left[1 - z^{-1} + \frac{1}{4}z^{-2}\right] = 1 - 0.5z^{-1} + X(z) \qquad (10.117)$$

Given

$$x(n) = 2\left(\frac{1}{8}\right)^n u(n)$$

$$X(z) = 2\left(\frac{1}{1 - \frac{1}{8}z^{-1}}\right) \qquad (10.118)$$

Substituting Eq.(10.118) in Eq.(10.117) yields

$$Y(z)\left[1 - z^{-1} + \frac{1}{4}z^{-2}\right] = 1 - 0.5z^{-1} + \frac{2}{1 - \frac{1}{8}z^{-1}}$$

$$Y(z) = \frac{1 - 0.5z^{-1}}{1 - z^{-1} + \frac{1}{4}z^{-2}} + \frac{2}{\left(1 - z^{-1} + \frac{1}{4}z^{-2}\right)\left(1 - \frac{1}{8}z^{-1}\right)}$$

$$= \frac{z(z - 0.5)}{z^2 - z + \frac{1}{4}} + \frac{2z^3}{\left(z^2 - z + \frac{1}{4}\right)\left(z - \frac{1}{8}\right)}$$

$$= \frac{z(z - 0.5)}{\left(z - \frac{1}{2}\right)^2} + \frac{2z^3}{\left(z - \frac{1}{2}\right)^2\left(z - \frac{1}{8}\right)}$$

$$= \frac{z}{z - \frac{1}{2}} + \frac{2z^3}{\left(z - \frac{1}{2}\right)^2\left(z - \frac{1}{8}\right)}$$

$$= Y_1(z) + Y_2(z)$$

Then

$$y(n) = y_1(n) + y_2(n)$$

$$Y_1(z) = \frac{z}{z - \frac{1}{2}}$$

$$y_1(n) = \left(\frac{1}{2}\right)^n u(n)$$

$$Y_2(z) = \frac{2z^3}{\left(z - \frac{1}{2}\right)^2\left(z - \frac{1}{8}\right)}$$

3.(notes)

4.

$$X(j\omega) = \int_{-\infty}^{\infty} e^{-a|t|}\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{at}\, e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$X(j\omega) = \frac{2a}{a^2+\omega^2}$$

$$x(t) = \sin(\pi t)\, e^{-2t}\, u(t)$$

…tion : Given, $x(t) = \sin(\pi t)\, e^{-2t}\, u(t)$

$$= \left[\frac{e^{j\pi t} - e^{-j\pi t}}{2j}\right] e^{-2t}\, u(t)$$

$$= \frac{e^{-2t} \cdot e^{j\pi t}\, u(t)}{2j} - \frac{e^{-2t} \cdot e^{-j\pi t}\, u(t)}{2j}$$

We know that,

$$e^{-2t}\, u(t) \overset{FT}{\longleftrightarrow} \frac{1}{2+j\omega}$$

Using frequency shifting property, we get,

$$e^{j\pi t}\, e^{-2t}\, u(t) \overset{FT}{\longleftrightarrow} \frac{1}{2+j(\omega-\pi)}$$

Using Linearity property, we get,

$$\frac{1}{2j}\, e^{j\pi t}\, e^{-2t}\, u(t) \overset{FT}{\longleftrightarrow} \frac{1}{2j} \cdot \frac{1}{2+j(\omega-\pi)}$$

Similarly,

$$\frac{1}{2j}\, e^{-j\pi t}\, e^{-2t}\, u(t) \overset{FT}{\longleftrightarrow} \frac{1}{2j} \cdot \frac{1}{2+j(\omega+\pi)}$$

$$\therefore x(t) = \sin(\pi t) e^{-2t}\, u(t) \overset{FT}{\longleftrightarrow} \frac{1}{j2}\left[\frac{1}{2+j(\omega-\pi)} - \frac{1}{2+j(\omega+\pi)}\right]$$

$$\therefore X(j\omega) = \frac{1}{j2}\left[\frac{1}{2+j(\omega-\pi)} - \frac{1}{2+j(\omega+\pi)}\right]$$

Solution: (a) $x(t) = \frac{2}{t^2+1}$

From the duality property of FT, we have

if $x(t) \xleftrightarrow{FT} X(jw)$, the $X(jt) \xleftrightarrow{FT} 2\pi x(-w)$

we have the FT pair;

$$e^{-a|t|} \xleftrightarrow{FT} \frac{2a}{a^2+w^2}$$

$$\therefore \quad x(t) = e^{-|t|} \xleftrightarrow{FT} \frac{2}{1+w^2} = X(jw)$$

Using duality property, the above in modified as

$$X(jt) = \frac{2}{t^2+1} \xleftrightarrow{FT} 2\pi x(-w) = 2\pi e^{-|-w|} = 2\pi e^{-|w|}$$

Hence the FT of $\frac{2}{t^2+1}$ is $\boxed{2\pi e^{-|w|}}$

5.

Given ; $y(n) + ½\, y(n-1) = x(n) - 2x(n-1)$

Taking DTFT on both side, we get,

$$Y(e^{j\Omega}) + ½\, e^{-j\Omega}\, Y(e^{j\Omega}) = X(e^{j\Omega}) - 2e^{-j\Omega}\, X(e^{j\Omega})$$

∴ The frequency response

$$H(e^{j\Omega}) = \frac{Y(e^{j\Omega})}{X(e^{j\Omega})} = \frac{1-2e^{-j\Omega}}{1+½\, e^{-j\Omega}}$$

$$H(e^{j\Omega}) = \frac{1}{1+½\, e^{-j\Omega}} - \frac{2\, e^{-j\Omega}}{1+½\, e^{-j\Omega}}$$

Taking inverse DTFT, we get,

the impulse response $h(n) = \left(\frac{-1}{2}\right)^n u(n) - 2.\left(\frac{-1}{2}\right)^{n-1} u(n-1)$

6.

We have,

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

$$\therefore\ X(e^{j\Omega}) = \sum_{n=-\infty}^{0} 2^n\, e^{-j\Omega n}$$

Put m=-n, then

$$\therefore X(e^{j\Omega}) = \sum_{\infty}^{0} 2^{-m}\, e^{j\Omega m}$$

$$= \sum_{m=0}^{\infty} 2^{-m}\, e^{j\Omega m}$$

$$= \sum_{m=0}^{\infty} (2^{-1}\, e^{j\Omega})^m$$

$$= \frac{1}{1-2^{-1}\, e^{j\Omega}}$$

$$X(e^{j\Omega}) = \frac{2}{2-e^{j\Omega}}$$

$$X(e^{j\Omega}) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{\infty} a^{|n|}\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{-1} a^{-n}\, e^{-j\Omega n} + \sum_{n=0}^{\infty} a^n\, e^{-j\Omega n}$$

$$= \sum_{n=-\infty}^{-1} (ae^{j\Omega})^{-n} + \sum_{n=0}^{\infty} (ae^{-j\Omega})^n$$

$$= \sum_{n=1}^{\infty} (ae^{j\Omega})^n + \sum_{n=0}^{\infty} (ae^{-j\Omega})^n$$

$$= \frac{ae^{j\Omega}}{1-ae^{j\Omega}} + \frac{1}{1-ae^{-j\Omega}}$$

$$= \frac{1-a^2}{1-2a\cos\Omega + a^2}$$

7.(theory notes)