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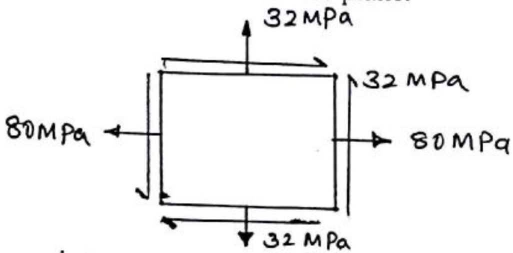
Internal Assesment Test – 2

Sub: Mechanics of Materials				Code: 15ME34
Date: 08/11/2017	Duration: 90 mins	Max Marks: 50	Sem: 3	Branch (sections): ME (A,B)

Answer any five questions. Good luck!

	PART A	Marks	OBE CO	OBE RBT
1	Derive torsion equation with usual notations.	[10]	CO5	L3
2	For the state of plane stress shown, determine (a) The principal planes and the principal stresses (b) Maximum and minimum shear stress and their planes and (c) Normal stress acting on maximum shear stress plane.	[10]	CO2	L3



3	The tensile stresses at a point across two mutually perpendicular planes are 120 N/mm^2 and 60 N/mm^2. Determine the normal, tangential and resultant stresses on a plane inclined at 30° to the axis of the minor stress by graphical method.	[10]	CO2	L4
4	Derive bending equation with usual notations.	[10]	CO3	L3

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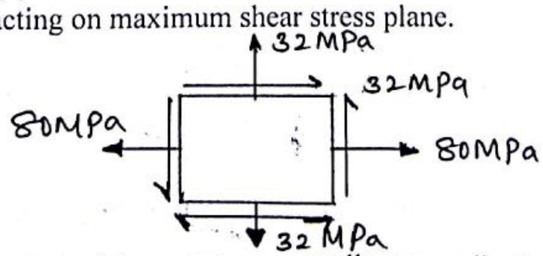

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5	A rod of circular section is to sustain a torsional moment of 300kNm and bending moment of 200kNm. Yield stress is 353MPa and assuming factor of safety = 3, determine the diameter of rod as per maximum shear stress theory.	[10]	CO6	L4
6	The load on bolt consists of an axial load of 10kN together with a transverse shear of 5kN. Find the size of bolt required according to Maximum principal stress theory and Maximum shear stress theory. Take permissible stress at elastic limit as 100MPa and Poisson's ratio = 0.3.	[10]	CO6	L4
7	A hollow shaft transmits 200kw of power at 150rpm. The total angle of twist in a length of 5m of shaft is 3°. Find the inner and outer diameters of the shaft, if the permissible shear stress is 60MPa. Take G=80GPa.	[10]	CO5	L4
8	A solid shaft rotating at 1000rpm transmits 50kw. Maximum torque is 20% more than the mean torque. Material of the shaft has the allowable shear stress of 50MPa and modulus of rigidity 80GPa. Angle of twist in the shaft should not exceed 1° in one meter length. Determine the diameter of the shaft.	[10]	CO5	L3

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8	A solid shaft rotating at 1000rpm transmits 50kw. Maximum torque is 20% more than the mean torque. Material of the shaft has the allowable shear stress of 50MPa and modulus of rigidity 80GPa. Angle of twist in the shaft should not exceed 1° in one meter length. Determine the diameter of the shaft.	[10]	CO5	L3

MECHANICS OF MATERIALS (15ME34)

IAT 2 - SOLUTION

10/11/2017

Sem: 3 A & B

Max Marks: 50

1. Torsion equation:

* Consider a shaft of length 'l', diameter 'd' fixed at one end and free at other end as shown in fig. a.
* Take any line PQ on the outer surface of the shaft. The line PQ is parallel to the longitudinal axis of the shaft.
* If torque 'T' is applied to the shaft at the free end, the line PQ is shifted to new position PQ'.
* The angle between PQ and PQ' i.e $Q\hat{P}Q'$ is ϕ, which is the shear strain.
* Angle between OQ and OQ' in the end view i.e $Q\hat{O}Q'$ is the angle of twist 'θ' in length 'l':

Let l = length of shaft

R = Radius of shaft

θ = Angle of twist

ϕ = shear strain

G = Modulus of rigidity

M_t or T = Torque applied at the end Q

τ = Shear stress induced at the outermost surface.

Shear strain at the outer surface = Distortion / unit length

$$= \frac{\text{Distortion at the outer surface}}{\text{Length of shaft}}$$

$$= \frac{QQ'}{l} = \frac{QQ'}{PQ} = \tan\phi = \phi \quad (\because \phi \text{ is small})$$

$\therefore$ Shear strain at the outer surface, $\phi = \frac{QQ'}{l}$

Also from the side view, Arc $QQ' = OQ \cdot Q' = R\theta$

i.e $\phi' = R\theta$

$$\therefore \ \phi = \frac{R\theta}{l} \quad \text{— (i)}$$

Shear strain at any radius r, $\phi_1 = \frac{r\theta}{l}$ — (ii)

From Hooke's law, Modulus of rigidity $= \frac{\text{Shear stress}}{\text{Shear strain}}$

$$\text{i.e} \quad G = \frac{\tau}{\phi} = \frac{\tau}{R\theta/l}$$

$$\therefore \ \frac{\tau}{R} = \frac{G\theta}{l} \quad \text{(where } \theta \text{ is in radians)} \quad \text{— (iii)}$$

Let 'q' is Shear stress at any radius r, then $\frac{\tau}{R} = \frac{G\theta}{l} = \frac{q}{r}$ — (iv)

Consider an elementary ring of thickness 'dr' at a radius 'r' from the centre as shown in fig. (b)

Area of the elementary ring $\delta a = 2\pi r \cdot dr$

From eqn (iv), $\frac{\tau}{R} = \frac{q}{r}$

$\therefore$ Shear stress at radius r, $\underline{q = \frac{\tau}{R} \cdot r}$

Shear force on the elemental ring, δF = Shear stress in the ring × Area of the ring

$$\delta F = \frac{\tau}{R} \cdot r \cdot 2\pi r dr = \frac{\tau}{R} 2\pi r^2 dr$$

Torque on the elemental ring $\delta T = \delta F \times$ Distance of the ring from the centre

$$= \left(\frac{\tau}{R} \times 2\pi r^2 dr\right) r$$

$$\therefore \ \underline{\delta T = \frac{\tau}{R} 2\pi r^3 dr}$$

$\therefore$ Total torque or Total resisting torsional moment

$$T = \int_0^R \frac{\tau}{R} 2\pi r^3 dr$$

$$= \frac{\tau}{R} 2\pi \int_0^R r^3 dr = \frac{\tau}{R} \cdot 2\pi \left(\frac{r^4}{4}\right)_0^R$$

$$= \frac{\tau}{R} 2\pi \left(\frac{R^4}{4}\right) = \left(\frac{\pi R^4}{2}\right) \frac{\tau}{R} \quad (\because R = d/2)$$

$$= \left(\frac{\pi d^4}{32}\right) \cdot \frac{\tau}{R}$$

i.e $T = J \cdot \frac{\tau}{R}$ where $J = \left(\frac{\pi d^4}{32}\right)$ = Polar moment of inertia

$$\therefore \frac{T}{J} = \frac{\tau}{R} \quad - (v)$$

From (ii) and (v)

$$\boxed{\frac{T}{J} = \frac{\tau}{R} = \frac{G\theta}{l}}$$

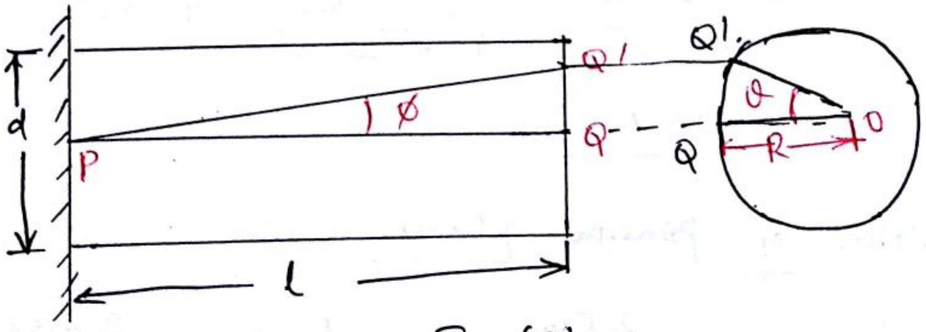


Fig.(a)

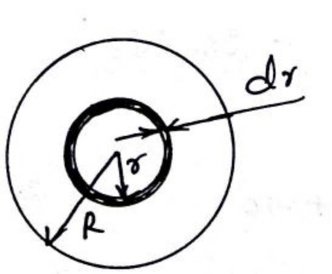
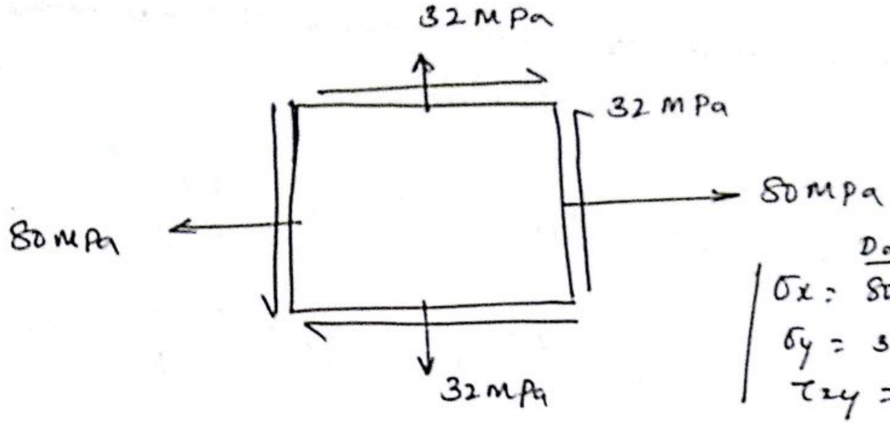


Fig (b)

2.



Data:
$\sigma_x = 80 MPa$
$\sigma_y = 32 MPa$
$\tau_{xy} = +32 MPa (ccw)$

(i) Principal stresses and their planes.

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{80+32}{2} + \sqrt{\left(\frac{80-32}{2}\right)^2 + 32^2}$$

$$= 96 \ N/mm^2$$

$$\sigma_2 = \frac{\sigma_x-\sigma_y}{2} - \sqrt{\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{80+32}{2} - \sqrt{\left(\frac{80-32}{2}\right)^2 + 32^2}$$

$$= 16 \ N/mm^2$$

Position of principal planes

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \ ; \ \tan 2\theta = \frac{2\times 32}{80-32}$$

$$\therefore \theta = -26.565^\circ = \theta_1$$

$$\therefore \theta_2 = \theta_1 + 90^\circ = -26.565^\circ + 90^\circ$$

$$= 63.435^\circ$$

(2) Maximum and minimum shear stress and their planes

$$\tau_{max} \text{ or } \tau_{min} = \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

or

$$\frac{\sigma_1 - \sigma_2}{2}$$

$$= \frac{96-16}{2} = 40 \text{ N/mm}^2$$

$$\therefore \tau_{max} \text{ or } \tau_{min} = \underline{\pm 40 \text{ N/mm}^2}$$

Position of their planes

$$\theta_1' = \theta_1 + 45^\circ = -26.565 + 45 = \underline{18.435^\circ}$$

$$\theta_2' = \theta_1 + 135^\circ = -26.565 + 135 = \underline{108.435^\circ}$$

(3) Normal stress on maximum shear stress plane

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \quad \text{or} \quad \frac{\sigma_1 + \sigma_2}{2}$$

$$= \frac{80+32}{2} \quad \text{or} \quad \frac{96+16}{2}$$

$$\therefore \sigma_{avg} = \underline{\underline{56 \text{ N/mm}^2}}$$

---

3. From Mohr's Circle $(\sigma_x = 120 \text{ N/mm}^2, \sigma_y = 60 \text{ N/mm}^2$

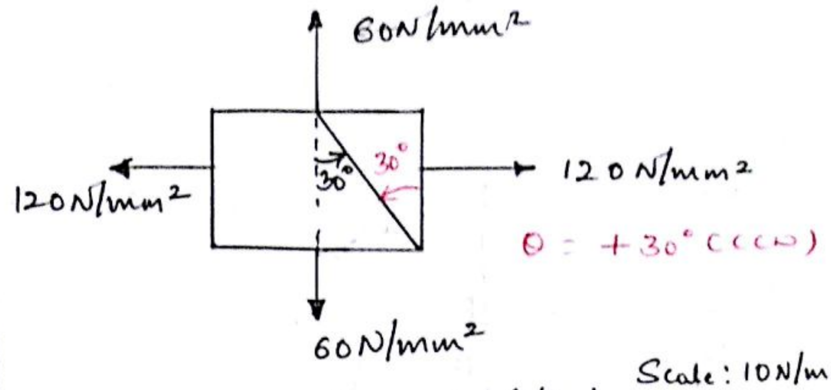
(From fig. 3

$\sigma_n = 160 \text{ N/mm}^2$

$\sigma_t = 26 \text{ N/mm}^2$

$\sigma_R = 190 \text{ N/mm}^2$

(3)



(Biaxial stress condition)

Scale: 10N/mm² = 1cm

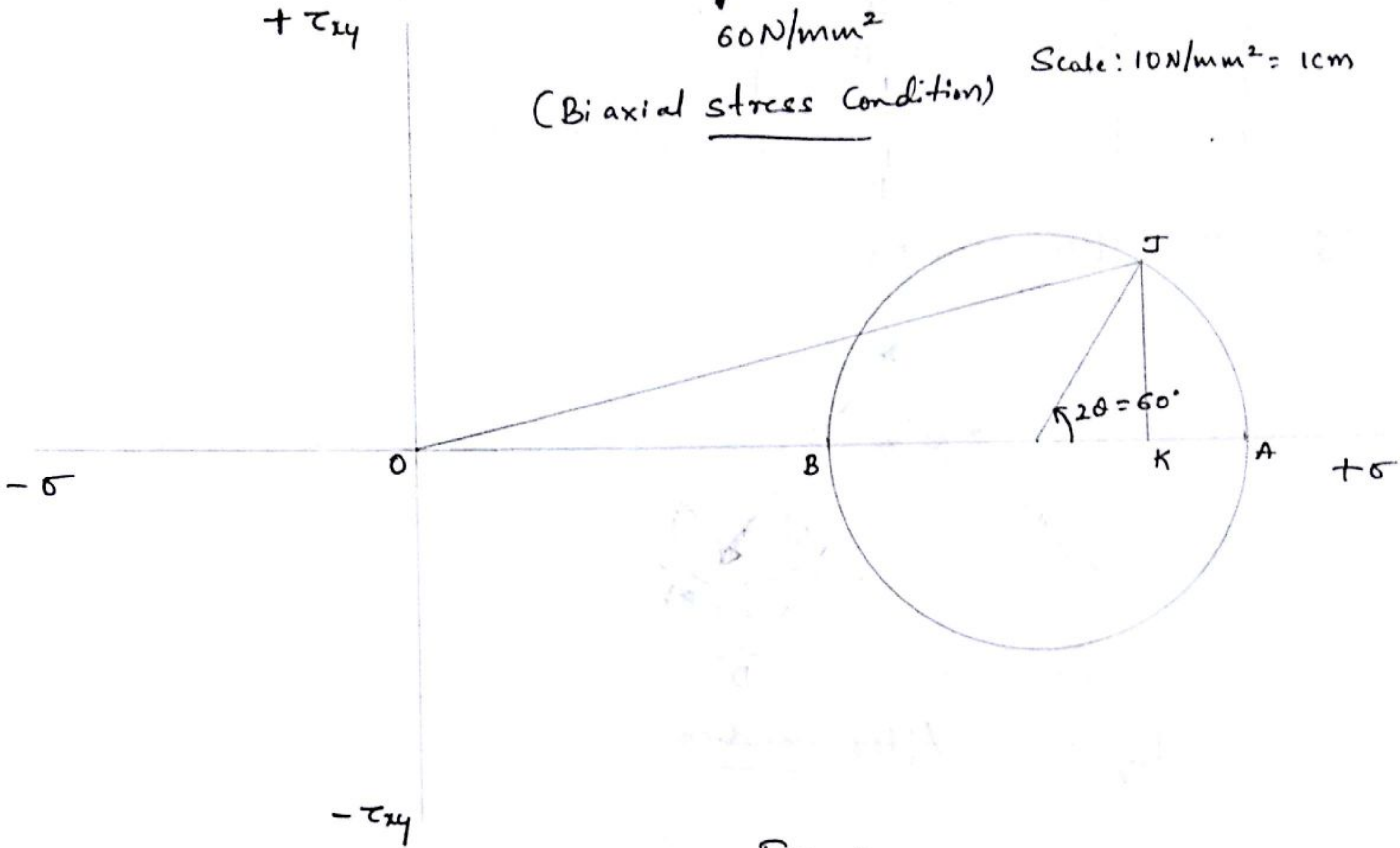


Fig. 3

$\sigma_R =$ OJ × scale = 19 × 10 = 190 N/mm²

$\sigma_n =$ OK × scale = 16 × 10 = 160 N/mm²

$\sigma_t =$ JK × scale = 2.6 × 10 = 26 N/mm²

4. Bending equation derivation

Consider a portion of beam between sections AC and BD as shown in fig. (a). Let 'EF' be the neutral axis and GH an element at a distance 'y' from the neutral axis

Fig (b) shows same portion after bending. Let 'R' be the radius of curvature and 'φ' be the angle subtended by C'A' and D'B' at the centre of radius of curvature

Since 'EF' is neutral axis, there is no change in its length

$$\therefore \; EF = E'F'$$
$$= R\phi$$

Now, strain in GH = $\dfrac{\text{Final length} - \text{Original length}}{\text{Original length}}$

$$= \frac{G'H' - GH}{GH}$$

But, $GH = EF = R\phi$ and

$$G'H' = (R+y)\phi$$

Hence Strain in layer $GH = \dfrac{(R+y)\phi - R\phi}{R\phi} = \dfrac{y}{R}$

If 'σ_b' is the bending stress and 'E' is the young's - modulus then strain is $\dfrac{\sigma_b}{E}$ and hence

$$\frac{\sigma_b}{E} = \frac{y}{R} \quad \text{or} \quad \frac{\sigma_b}{y} = \frac{E}{R} \quad -(1)$$

$$\text{or} \quad \sigma_b = \frac{E}{R} \times y$$

Thus bending stress varies linearly across the depth.

(4)

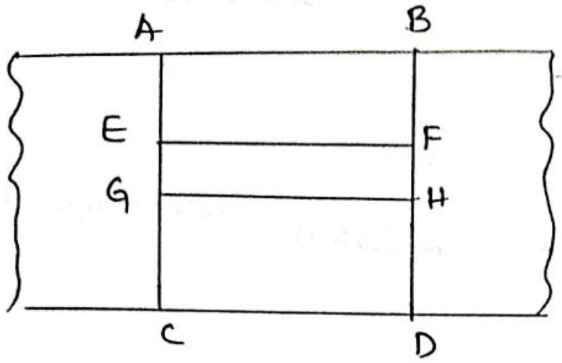


Fig. (a) Before bending

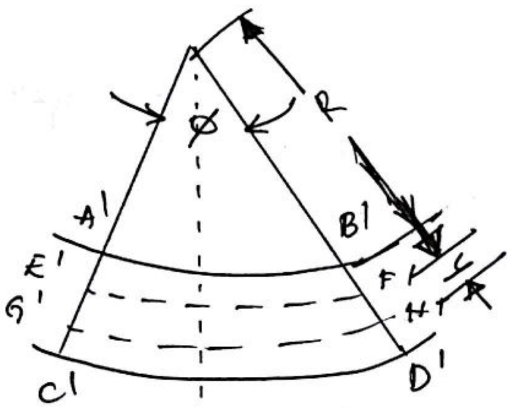


Fig (b): After bending.

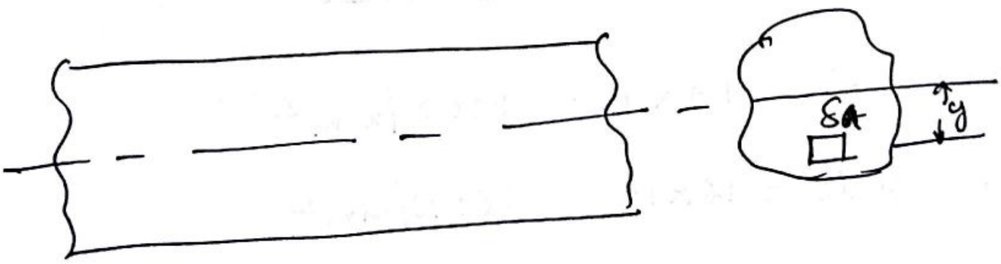


Fig (c):

Consider an elemental area 'δa' at a distance 'y' from neutral axis in the beam, the cross section of which is shown in fig(c).

Now stress 'σ_b' on the element is given by

$$\sigma_b = \frac{E}{R} \cdot y$$

$\therefore$ Force on this elemental area $= \sigma_b \cdot \delta a$

$$= \frac{E}{R} \cdot y \cdot \delta a$$

Moment of this resisting force about neutral axis

$$= \frac{E}{R} \cdot y \cdot \delta a \cdot y = \underline{\frac{E}{R} \cdot y^2 \delta a}$$

$\therefore$ Total moment of resistance (M') of the cross sectional area, $M' = \sum \frac{E}{R} y^2 \delta a$; $M' = \frac{E}{R} \sum y^2 \delta a$

From the definition of moment of inertia, which is Second moment of area about centroid, we can write

$$I = \sum y^2 \delta a.$$

where, 'I' is centroidal moment of inertia.

$$\therefore M' = \frac{E}{R} \cdot I$$

For equilibrium moment of resistance (M') should be equal to applied moment i.e $\underline{M' = M}$

Hence, we get $M = \frac{E}{R} \cdot I$

$$\text{(or)} \quad \frac{M}{I} = \frac{E}{R} \quad -(2)$$

From equations ① and ②

$$\boxed{\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}}$$

where,

M- Bending moment,

I - Moment of inertia about centroidal axis.

σ_b - Bending stress,

y - Distance of the fibre from neutral axis,

E - Young's modulus,

and R - Radius of curvature.

---

5. Data:

$T = 300 \text{ kNm} = M_t = 300 \times 10^6 \text{ Nmm}$

$M = 200 \text{ kNm} = 200 \times 10^6 \text{ Nmm}$

$\sigma_{yt} = 353 \text{ MPa}$

FOS = 3

Solution: We know that, $\tau = \frac{M_t}{J} \cdot R$

$$= \frac{3 \times 10^8}{\frac{\pi d^4}{32}} \times \frac{d}{2}$$

$$= \frac{48 \times 10^3}{\pi d^3} \text{ N/mm}^2$$

Bending stress $\sigma_b = \frac{M_b}{I} \cdot y = \frac{2\times10^8}{\frac{\pi d^4}{64}} \times \frac{d}{2}$

$$\therefore \sigma_b = \frac{64\times10^8}{\pi d^3} \text{ N/mm}^2 = \sigma_x$$

We Know that,

Maximum principal stress, $\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_x^2}$

$$= \frac{64\times10^8}{2\pi d^3} + \sqrt{\left(\frac{64\times10^8}{2\pi d^3}\right)^2 + \left(\frac{48\times10^8}{\pi d^3}\right)^2}$$

$$= \frac{2.855\times10^9}{d^3} \quad \text{- (i)}$$

Minimum principal stress, $\sigma_2 = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_x^2}$

$$= \frac{64\times10^8}{2\pi d^3} - \sqrt{\left(\frac{64\times10^8}{2\pi d^3}\right)^2 + \left(\frac{48\times10^8}{\pi d^3}\right)^2}$$

$$= -\frac{0.8177\times10^9}{d^3} \quad \text{- (ii)}$$

From maximum shear stress theory

W.K.T $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2}$

$$= \frac{2.855\times10^9 + 0.8177\times10^9}{2d^3}$$

$$= \frac{1.83635\times10^9}{d^3} \quad \text{- (iii)}$$

The design equation is

$$\tau_{max} = \frac{\sigma_{yt}}{2\times FOS}$$

$$\frac{1.83635\times10^9}{d^3} = \frac{353}{2\times3} \quad \therefore \text{dia. } d = 314.855 \text{ mm}$$

Diameter of rod, $d = 315$ mm

(11)

6. Data:

Axial tensile load = 10 KN = 10000 N

Shear load = 5 KN = 5000 N

Permissible stress at elastic limit is 100 N/mm^2

$\mu = 0.3$

Solution: For axial load, $\sigma = \frac{\text{Axial tensile load}}{\text{Core area of the bolt}} = \frac{10000}{A_c} = \sigma_x$

For transverse shear load, $\tau = \frac{\text{Shear load}}{\text{Core area of the bolt}}$

$= \frac{5000}{A_c} = \tau$; $(\sigma_y = 0)$

Max. Principal stress, $\sigma_1 = \frac{\sigma_x}{2} + \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}$

$= \frac{10000}{2A_c} + \sqrt{\left(\frac{10000}{2A_c}\right)^2 + \left(\frac{5000}{A_c}\right)^2}$

$= \frac{12071.07}{A_c}$ — (1)

Min. Principal stress, $\sigma_2 = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau^2}$

$= \frac{10000}{2A_c} - \sqrt{\left(\frac{10000}{2A_c}\right)^2 + \left(\frac{5000}{A_c}\right)^2}$

$= \frac{-2071.07}{A_c}$ — (2)

(i) Maximum principal stress theory

Since $\sigma_1 > \sigma_2$, the design equation is

$\sigma_1 = \frac{\sigma_{yt}}{n}$ $(n = FOS)$

(or)

$\sigma_1 = \sigma_e$

$\therefore \frac{12071.07}{A_c} = 100$ $(\because \sigma_e = 100)$

$\therefore$ $A_c = 120.7107\ mm^2 = \frac{\pi d_1^2}{4}$

$\therefore$ $d_1 = 12.4\ mm$

(ii) Maximum Shear stress theory

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{12071.07 + 2071.07}{2A_c} = \frac{7071.07}{A_c}$$

The design equation is $\tau_{max} = \frac{\sigma_e}{2}$

$$\therefore \frac{7071.07}{A_c} = \frac{100}{2}$$

$\therefore$ $A_c = 141.42\ mm^2 = \frac{\pi}{4} d_1^2$

$\therefore$ $d_1 = 13.42\ mm$ (Permissible diameter)

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7. Data: $P = 200\ kW$; $N = 150\ rpm$; $l = 5\ m$, $\theta = 3 \times \pi/180 = \frac{\pi}{60}$.

$\tau = 60\ MPa = 60\ N/mm^2$, $G = 80 \times 10^3\ N/mm^2$

Power transmitted by the shaft, $P = \frac{2\pi NT}{60000}$ kW

$$200 = \frac{2\pi \times 150 \times T}{60000}$$

$\therefore$ $T = 12732.4\ Nm$ (Torque)

$= 12732.4 \times 10^3\ Nmm$

Based on strength criterion

$$\frac{T}{J} = \frac{\tau}{R} \quad \text{— (i)}$$

Based on Rigidity criterion

$$\frac{T}{J} = \frac{G\theta}{l} \quad \text{(ii)}$$

From (i) and (ii)

$$\frac{\tau}{R} = \frac{G\theta}{l}$$

$$\frac{60}{d_o/2} = \frac{80 \times 10^3 \times \pi / 60}{5000}$$

$\therefore$ Outer diameter of shaft, $d_o = 143.24$ mm

From eqn (i), $\frac{T}{J} = \frac{\tau}{R}$; $\frac{T}{\frac{\pi}{32}(d_o^4 - d_i^4)} = \frac{\tau}{d_o/2}$

$$\frac{12732.4 \times 10^3}{\frac{\pi}{32}(143.24^4 - d_i^4)} = \frac{60}{\frac{143.24}{2}}$$

$\therefore$ Inner diameter of shaft, $d_i = 127.73$ mm

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8. Data: $P = 50$ kW ; $N = 1000$ rpm ; $T_{max} = 1.2 T_{mean}$; $\tau = 50$ MPa

$G = 80$ GPa $= 80 \times 10^3$ N/mm^2 ; $\theta = 1^\circ \times \frac{\pi}{180}$ rad, $l = 1$ m $= 1000$ mm

Power transmitted by the shaft, $P = \frac{2\pi N T_{mean}}{60000}$ kW

$$50 = \frac{2\pi \times 1000 \times T}{60000} \text{ kW}$$

$\therefore T_{mean} = 477.465$ Nm

$T_{max} = 1.2 \times T_{mean} = 1.2 \times 477.465 = 572.958$ Nm

(or) 572.958×10^3 Nmm

Based on torsional strength criterion

$$\frac{T}{J} = \frac{\tau}{R} \text{ ; } T = J \cdot \frac{\tau}{R} \text{ ; } T = \frac{\pi d^4}{32} \cdot \frac{\tau}{d/2}$$

$$\therefore T = \frac{\pi d^3}{16} \cdot \tau$$

$$572.958 \times 10^3 = \frac{\pi}{16} d^3 \times 50$$

$\therefore$ Diameter of shaft, $d = 38.8$ mm

Based on torsional rigidity criterion

$$\frac{T}{J} = \frac{G\theta}{l}$$

$$\frac{572.958 \times 10^3}{\frac{\pi d^4}{32}} = \frac{80 \times 10^3 \times 1 \times \pi/180}{1000}$$

$$\therefore d = \underline{\underline{45.2 \text{ mm}}}$$

Permissible dia. of the solid shaft is higher among the two values.

$\therefore$ Diameter of solid shaft, $d = \underline{\underline{45.2 \text{ mm}}}$