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Internal Assessment Test III – Nov. 2018

Sub: Dynamics of Machinery **Code:** 15ME52

Date: 19/ 11 / 2018 **Duration:** 90 mins **Max Marks:** 50 **Sem:** V **Branch:** MECH

Note: Answer any **five** questions.

		Marks	OBE CO	OBE RBT
1	State the conditions for the equilibrium of following systems: i. Two force member ii) Three force member iii) Member with two force and torque	**10**	CO1	L2
2	Determine torque T_2 to keep the mechanism in equilibrium shown in fig (a). AC = 70 mm, AB = 150 mm, O_2A = 40 mm	**10**	CO1	L3

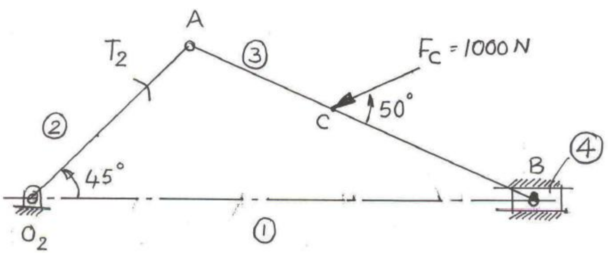


fig (a)

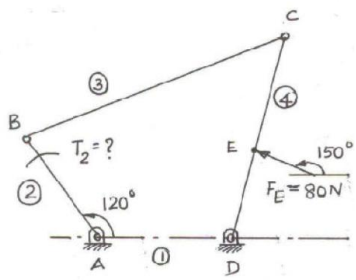


fig (b)

		Marks	CO	RBT
3.	A four link mechanism is acted upon by forces as shown in the fig (b). Determine the torque T_2 to be applied on link 2 to keep the mechanism in equilibrium. AD=50mm, AB=40mm, BC=100mm, DC=75mm, DE= 35mm.	**10**	CO1	L3
4	Define logarithmic decrement and show that it can be expressed as $\delta = \frac{1}{n}\log\left(\frac{x_0}{x_n}\right)$ where n = no of cycles, x_0 is initial amplitude and x_n is the amplitude after 'n' cycles.	**10**	CO5	L2
5	A TV set of 25 kg mass must be isolated from a machine vibrating with amplitude of 0.1 mm at 1000 rpm. The TV set is mounted on five isolators each having a stiffness of 30,000 N/m and a damping constant of 400 N-s/m. Determine i. Amplitude of vibration of the body (i.e TV Set) ii. Dynamic load on each isolator due to vibration.	**10**	CO5	L3
6	A vibrating system is defined by the following parameters m = 3 kg, K = 100 N/m, C = 3 N-Sec/m Determine i) damping factor (ii) natural frequency of damped vibration (iii) logarithmic decrement (iv) ratio of two consecutive amplitudes (v) No of cycles after which the original amplitude is reduced by 20%.	**10**	CO5	L2
7	A mass of 100 kg been mounted on a spring dashpot system having spring stiffness of 19,600 N/m and damping coefficient of 100 N-s/m. The mass is acted upon by a harmonic force of 39 N at the undamped natural frequency of the system. Determine i. Amplitude of vibration of the mass. ii. Phase difference between force & displacement iii. Force transmissibility ratio	**10**	CO5	L3

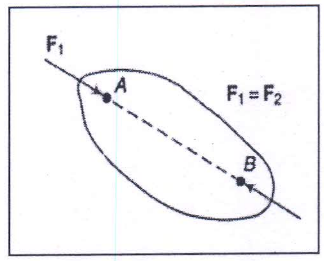
SUB : 15ME52 (DOM)

SOLUTION FOR IAT-3 2018 NOV.

1. **Equilibrium of Two Force Members**

A member under the action of two forces will be in equilibrium if

- The forces are of the same magnitude,
- The forces act along the same line, and the forces are in opposite directions



Equilibrium of Three Force Members

A member under the action of three forces will be in equilibrium if

- The resultant of the forces is zero, and
- The lines of action of the forces intersect at a point (known as *point of concurrency).*

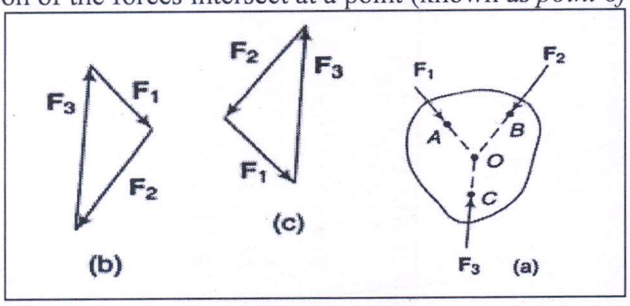


Figure (a) indicates an example for the three force member and (b) and (c) indicates the force polygon to check for the static equilibrium.

Member with two forces and a torque

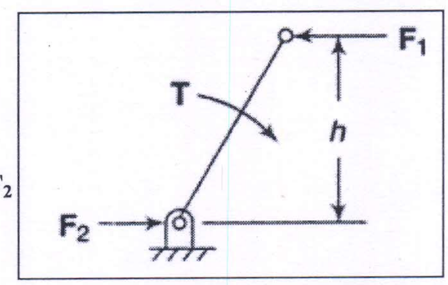
A member under the action of two forces and an applied torque will be in equilibrium if

- The forces are equal in magnitude, parallel in direction and opposite in sense and
- The forces form a couple which is equal and opposite to the applied torque.

Figure shows a member acted upon by two equal forces $\mathbf{F_1}$, and $\mathbf{F_2}$ and an applied torque **T** for equilibrium,

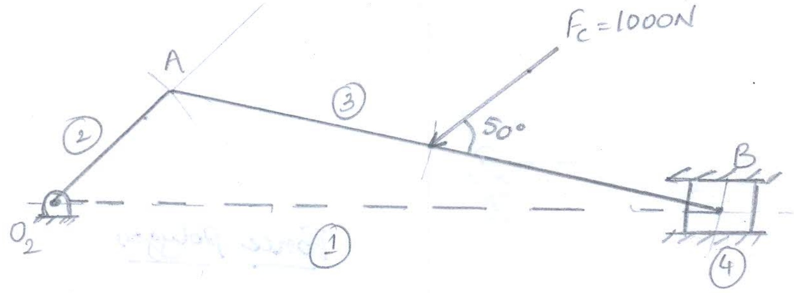
$$T = F_1 h = F_2 h$$

Where T, F_1 *and* F_2 *are the magnitudes of* **T**, $\mathbf{F_1}$ *and* $\mathbf{F_2}$ respectively.
T is clockwise whereas the couple formed by $\mathbf{F_1}$, and $\mathbf{F_2}$ is counter-clockwise.

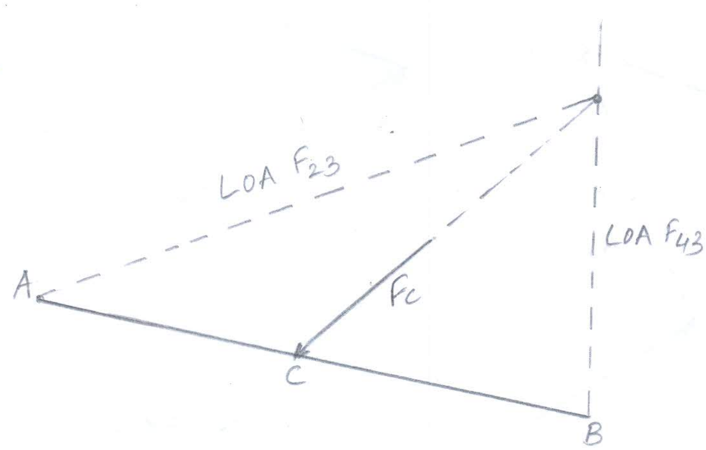


(2) <u>Given</u> : $AC = 70mm$; $AB = 150mm$; $O_2A = 40mm$

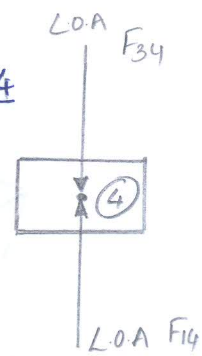
<u>Scale</u> 1cm = 20mm



<u>Link 3</u>

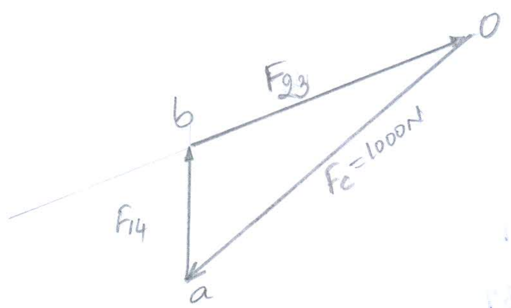


<u>Link 4</u>



<u>Force Polygon</u>

<u>Scale</u> 1cm = 200N



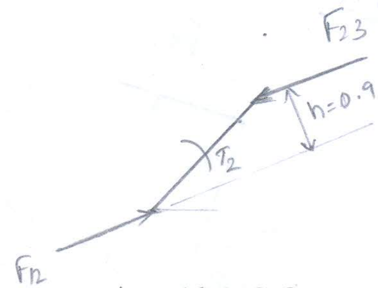
$F_{23} = 06 \times Scale$

$= 3.9 \times 200$

$F_{23} = 780N$ //

<u>Scale</u> 1cm = 20mm

<u>Link 2</u>



h = 180mm

$F_{14} = ab \times Scale$

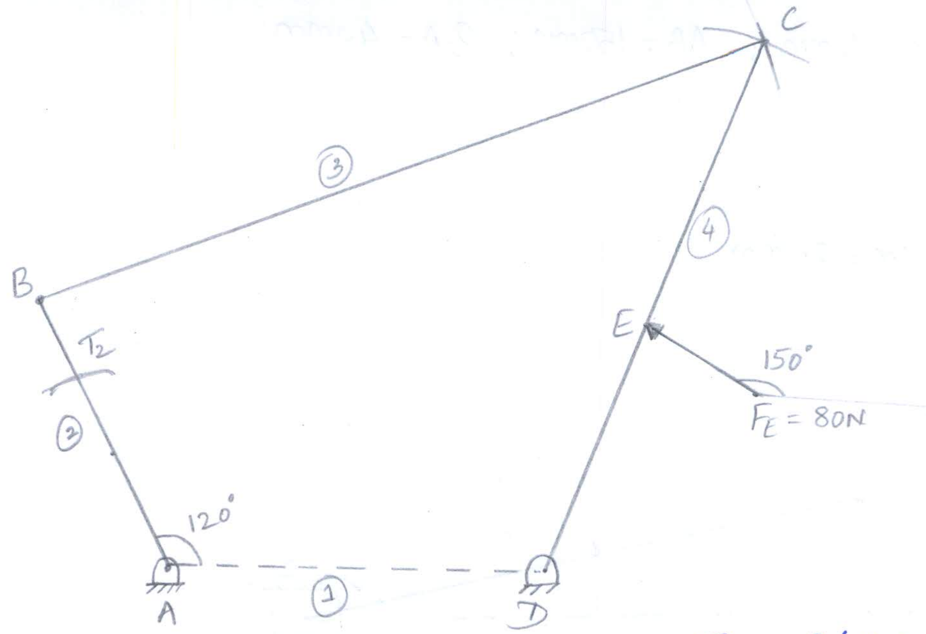
$= 1.3 \times 200$

$F_{14} = 260N$ //

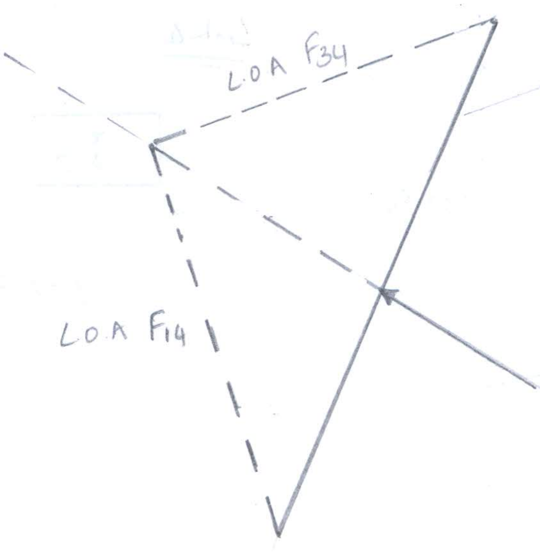
$T_2 = F_{23} \times h = 780 \times 180$

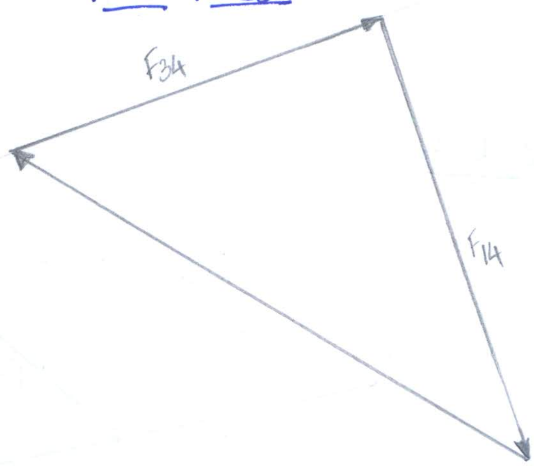
$T_2 = 140.4 N\text{-}m$ //

(3)



Force Polygon





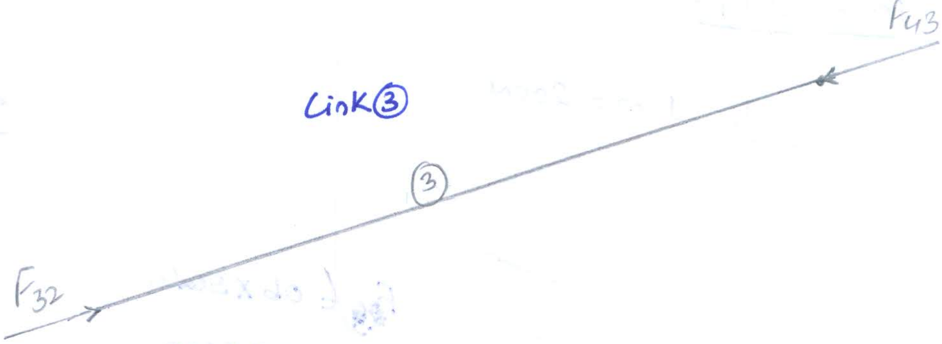
$F_{34} = 50N$

$F_{14} = 61N$

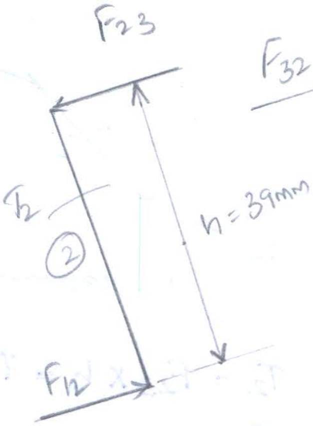
$F_{34} = -F_{43}$

$-F_{32} = +F_{23}$

Link ③



Link ②



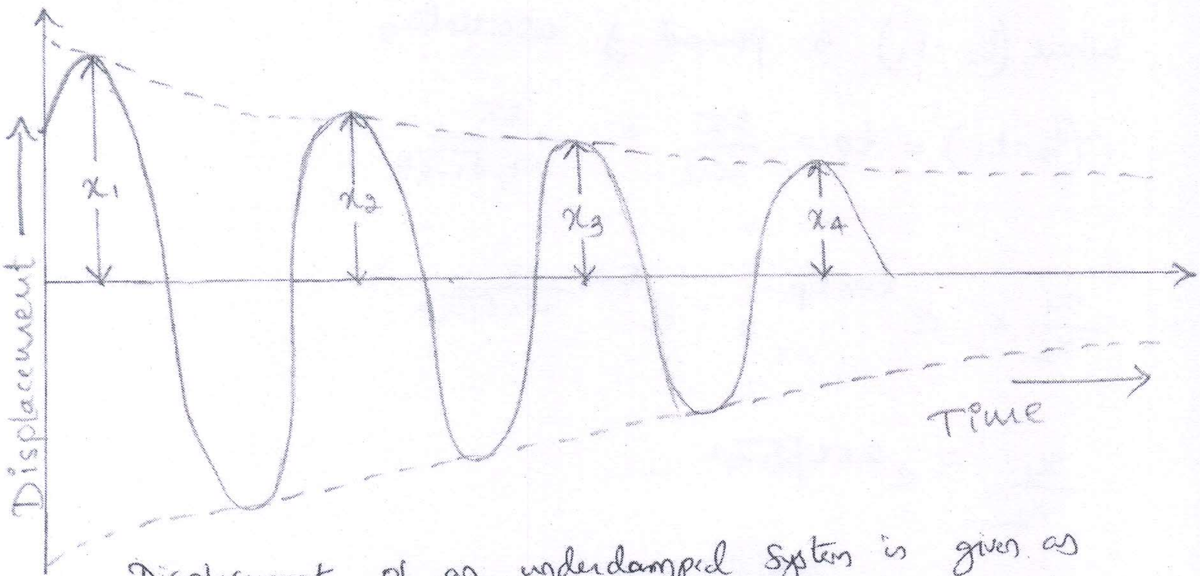
$T_2 = F_{23} \times h$

$= 50 \times 39$

$T_2 = 1.9 \text{ N-m}$

4. Logarithmic Decrement

It is defined as the natural log. of the ratio of any two successive amplitudes on the same side of the mean position in an underdamped system.



Displacement of an underdamped system is given as

$$x = X e^{-\xi \omega_n t} \sin(\omega_d t + \theta)$$

where $X e^{-\xi \omega_n t}$ – Amplitude

ω_d – Angular frequency

When $\sin(\omega_d t + \phi) = 1$, the amplitude is maximum

Now, Max' amplitude $x = X e^{-\xi \omega_n t}$

Let x_1 be max' amplitude when the time is t_1

x_2 be max' amplitude when the time is t_2

$$\therefore \quad x_1 = X e^{-\xi \omega_n t_1}$$

$$x_2 = X e^{-\xi \omega_n t_2}$$

$$\frac{x_1}{x_2} = \frac{Xe^{-\xi\omega_n t_1}}{Xe^{-\xi\omega_n t_2}} = e^{-\xi\omega_n t_1 - (\xi\omega_n t_2)} = e^{\xi\omega_n(t_2 - t_1)}$$

where $(t_2 - t_1)$ is period of oscillation

$$(t_2 - t_1) = t_p = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\xi^2}}$$

$$\frac{x_1}{x_2} = e^{\xi\omega_n t_p} = e^{\xi\omega_n \frac{2\pi}{\omega_n\sqrt{1-\xi^2}}}$$

$$\frac{x_1}{x_2} = e^{2\pi\xi/\sqrt{1-\xi^2}}$$

lly $\frac{x_2}{x_3} = e^{2\pi\xi/\sqrt{1-\xi^2}}$

$$\therefore \frac{x_1}{x_2} = \frac{x_2}{x_3} = \frac{x_3}{x_4} = \cdots\cdots \frac{x_n}{x_{n+1}} = e^{\frac{2\pi\xi}{\sqrt{1-\xi^2}}}$$

Taking natural log on both sides

$$\ln\left(\frac{x_1}{x_2}\right) = 2\pi\xi/\sqrt{1-\xi^2}$$

$$\delta = \ln\left(\frac{x_1}{x_2}\right) = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$$

when ξ is very small $\delta \simeq 2\pi\xi$

$$\frac{x_0}{x_n} = \left(\frac{x_0}{x_1}\right)^n \; ; \quad \left(\frac{x_0}{x_1}\right) = \left(\frac{x_0}{x_n}\right)^{1/n} \Rightarrow \delta = \ln\left(\frac{x_0}{x_1}\right)$$

$$\boxed{\delta = \frac{1}{n}\ln\left(\frac{x_0}{x_n}\right)}$$

(5) Given $K = 30{,}000$ N/m ; $N = 1000$ rpm

$m = 25$ kg ; $C = 400$ N-s/m

$Y = 0.1$ mm ; No of isolators = 5

Equivalent Stiffness

$$K_e = 5 \times 30{,}000 = 15 \times 10^4 \text{ N/m}$$

Equivalent damping Coeff.

$$C = 5 \times 400 = 2000 \text{ N-s/m}$$

Circular freq. $\omega = \frac{2\pi N}{60} = \frac{2\pi (1000)}{60}$

$$\omega = 104.72 \text{ r/s}$$

Circular freq. of undamped Vib.

$$\omega_n = \sqrt{\frac{K_e}{m}} = \sqrt{\frac{15 \times 10^4}{25}} = 77.46 \text{ r/s}$$

Freq. ratio

$$\frac{\omega}{\omega_n} = \frac{104.72}{77.46} = 1.35$$

Damping ratio $\xi = \frac{C}{C_c} = \frac{2000}{2 \times 25 \times 77.46}$ $\because C_c = 2m\omega_n$

$$\xi = 0.52$$

i) **Amplitude of Vibration**

$$\frac{X}{Y} = \sqrt{\frac{1+\left(2\xi \frac{\omega}{\omega_n}\right)^2}{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi \frac{\omega}{\omega_n}\right)^2}}$$

$$\frac{X}{0.1} = \sqrt{\frac{1+(2\times 0.52\times 1.35)^2}{(1-1.35^2)^2+(2\times 0.52\times 1.35)^2}}$$

$$X = 0.1059\,mm$$

ii) **Dynamic load on isolator (F_D)**

$$F_D = Z\sqrt{K_e^2+(C_e\omega)^2}$$

$$\frac{Z}{Y} = \frac{(\omega/\omega_n)^2}{\sqrt{\left[1-\left(\frac{\omega}{\omega_n}\right)^2\right]^2+\left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$\frac{Z}{0.1} = \frac{(1.35)^2}{\sqrt{(1-1.35^2)^2+(2\times 0.52\times 1.35)^2}}$$

$$Z = 0.112\,mm$$

$$F_D = 0.112\times 10^{-3}\sqrt{(15\times 10^4)^2+(2000\times 104.72)^2}$$

$$F_D = 29\,N$$

Dynamic load on each isolator $= \frac{29}{5} = 5.8\,N$

⑥ Given :- $m = 3 Kg$

$K = 100 \, N/m$

$c = 3 N\text{-}s/m$

Circular freq.

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{100}{3}} = 5.77 \, r/s.$$

Critical damping

$$C_c = 2m\omega_n = 2 \times 3 \times 5.77 = 34.64 \, N\text{-}s/m$$

i) Damping factor

$$\xi = \frac{c}{C_c} = \frac{3}{34.64} = 0.086$$

$$\xi = 0.086$$

ii) Natural freq. of damped system

$$\omega_d = \omega_n \sqrt{1 - \xi^2}$$

$$= 5.77 \sqrt{1 - 0.086^2}$$

$$\omega_d = 5.75 \, r/s$$

frequency $f = \frac{1}{2\pi} \cdot \omega_d = \frac{1}{2\pi} \cdot (5.75)$

$$f = 0.92 \, HZ$$

iii) Logarithmic decrement

$$\delta = \frac{2\pi \xi}{\sqrt{1-\xi^2}}$$

$$= \frac{2\pi(0.086)}{\sqrt{1-0.086^2}}$$

$$\delta = 0.542$$

iv) Ratio of two Consecutive amplitudes

$$\delta = \ln\left(\frac{x_1}{x_2}\right)$$

$$e^{\delta} = \frac{x_1}{x_2} = e^{0.542}$$

$$\frac{x_1}{x_2} = 1.72$$

v) No. of cycles after original amp' is reduced by 20%.

$$x_n = 0.8\,x_0$$

$$\delta = \frac{1}{n}\ln\left(\frac{x_0}{x_n}\right)$$

$$0.542 = \frac{1}{n}\ln\left(\frac{x_0}{0.8x_0}\right)$$

$$n = 0.412 \simeq 1 \text{ cycle}$$

(7) Given :- $m = 100 kg$; $K = 19,600\ N/m$; $C = 100 N\text{-}s/m$

$F_0 = 39N$

$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{19600}{100}} = 14\ r/s$$

$$C_c = 2m\omega_n = 2 \times 100 \times 14 = 2800\ N\text{-}s/m$$

Damping ratio $\xi = \frac{C}{C_c} = \frac{100}{2800}$

$$\xi = 0.0357$$

At undamped natural freq. $\omega = \omega_n$

i) Max' amp. $$x_{max} = \frac{x_0}{\sqrt{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi\frac{\omega}{\omega_n}\right)^2}}$$

$$x_0 = \frac{F_0}{K} = \frac{39}{19600} = 1.989 \times 10^{-3}\ m$$

$$x_{max} = \frac{x_0}{2\xi} = \frac{1.989 \times 10^{-3}}{2 \times 0.0357}$$

$$x_{max} = 0.028 m$$

ii) Phase difference

$$\theta \text{ or } \phi = \tan^{-1}\left[\frac{2\xi\frac{\omega}{\omega_n}}{1 - \left(\frac{\omega}{\omega_n}\right)^2}\right]$$

θ or $\phi = 90°$ or $0°$ or $180°$

iii) Transmissibility

$$\epsilon = \sqrt{\frac{1 + \left(2\xi \frac{\omega}{\omega_n}\right)^2}{\left[1 - \left(\frac{\omega}{\omega_n}\right)^2\right]^2 + \left(2\xi \frac{\omega}{\omega_n}\right)^2}}$$

$$= \frac{\sqrt{1 + (2\xi)^2}}{2\xi}$$

$$= \frac{\sqrt{1 + (2 \times 0.035)^2}}{2 \times 0.035}$$

$$\epsilon = 14.04$$