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CMRIT

Improvement Test

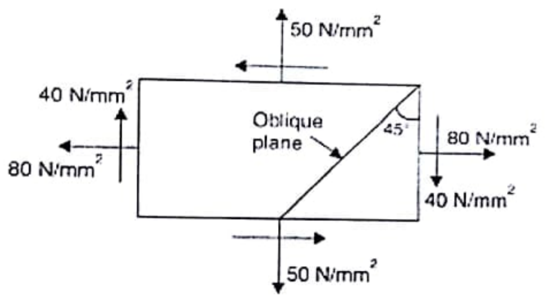
Sub: Mechanics of Materials

Code: 15ME34

Date: 18/11/2017 | Duration: 90 mins | Max Marks: 50 | Sem: 3 | Branch (sections): ME (A,B)

Answer any five questions. Good luck!

	PART A	Marks	OBE CO	OBE RBT
1	A simply supported beam of span L carries a point load W at mid-span. Find the strain energy stored by the beam.	[10]	CO1	L3
2	A beam of length L is simply supported at its ends. The beam carries a uniformly distributed load of w per unit length run over the whole span. Find the strain energy stored by the beam.	[10]	CO1	L3
3	Derive an expression for circumferential and longitudinal stress for thin cylinder.	[10]	CO1	L3
4	A thin cylindrical shell 1.2 m in diameter and 3m long has a metal wall thickness of 12mm. It is subjected to an internal fluid pressure of 3.2 MPa. Find the circumferential and longitudinal stress in the wall. Determine change in length, diameter and volume of the cylinder. Assume E= 210GPa and μ=0.3.	[10]	CO1	L4
5	The internal and external diameters of a thick cylinder are 300mm and 500mm respectively. It is subjected to an external pressure of 4MPa. Find the internal pressure that can be applied if the permissible stress in cylinder is limited to 13MPa. Sketch radial and hoop stresses distribution across the section.	[10]	CO1	L4

6	A thick cylinder of 500 mm inner diameter is subjected to an internal pressure of 9 MPa. Taking the allowable stress for the material of the cylinder as 40 MPa, determine the wall thickness of the cylinder. Also plot the stress distribution across the wall thickness of the cylinder.	[10]	CO1	L4
7	A point in a strained material is subjected to stresses shown in Fig. below. Using Mohr's circle method, determine the normal, tangential and resultant stresses across the oblique plane. 	[10]	CO2	L4
8	A rectangular block of material is subjected to a tensile stress of 110 N/mm^2 on one plane and a tensile stress of 47 N/mm^2 on the plane at right angles to the former. Each of the above stresses is accompanied by a shear stress of 63 N/mm^2 and that associated with the former tensile stress tends to rotate the block anticlockwise. Find: i) The direction and magnitude of each of the principal stress and (ii) Magnitude of the greatest shear stress.	[10]	CO2	L4

CI

CIE

HOD

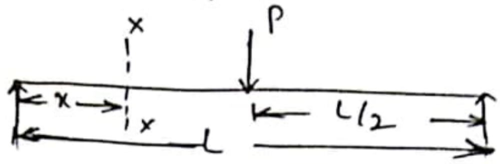
Mechanics of Materials [15ME34]
Improvement Test
Solutions

18-11-2017

SEM: 3^{rd} A & B
Max Marks: 50

1. Fig. below shows a simply supported beam AB of span length 'l' carries a point load 'P' at its midspan 'C'.

Reactions at supports, $R_A = R_B = P/2$



Consider section x-x at distance 'x' from A i.e., between 'A' and 'C'.

Bending moment at x-x, $M = \frac{P}{2} \cdot x$

Strain energy stored by the part AC $= \int_0^{l/2} \frac{M^2 dx}{2EI}$

$$= \int_0^{l/2} \frac{\left(\frac{P}{2}x\right)^2 dx}{2EI}$$

$$= \frac{P^2}{8EI} \int \left(\frac{x^3}{3}\right)_0^{l/2}$$

$$= \frac{P^2 l^3}{192EI}$$

Since length, AC = CB and $R_A = R_B = P/2$

Strain energy stored by the part BC = Strain energy stored by the part AC

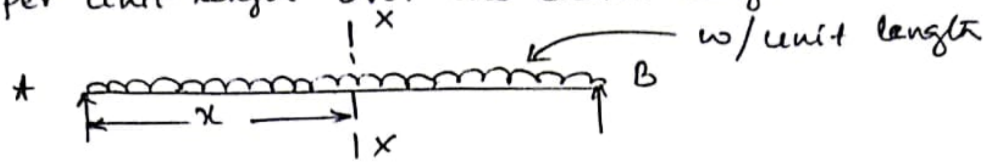
$$= \frac{P^2 l^3}{192EI}$$

∴ Total strain energy stored by the beam

$$U = \frac{P^2 l^3}{192EI} + \frac{P^2 l^3}{192EI}$$

$$\therefore \boxed{U = \frac{P^2 l^3}{96EI}} \text{ [Ans]}$$

2. Fig. below shows a' ~~Cantilever b~~ simply supported beam AB of span length 'l' carrying uniformly distributed load 'w' per unit length over the entire length.



Reaction at A = Reaction at B = $\frac{wl}{2}$

Consider Section X – X at distance 'x' from 'A'.

$\therefore$ Bending moment at X – X, $M = \frac{wl}{2}x - wx \cdot \frac{x}{2}$

$$= \frac{wl}{2} \cdot x - \frac{wx^2}{2}$$

$$= \frac{wx}{2}(l - x)$$

Strain energy stored by beam $U = \int_0^l \frac{M^2 dx}{2EI}$

$$= \int_0^l \frac{\left\{\frac{wx}{2}(l-x)\right\}^2 dx}{2EI} = \frac{w^2}{8EI}\int_0^l x^2(l^2 - 2lx + x^2)\,dx$$

$$= \frac{w^2}{8EI}\int_0^l (l^2x^2 - 2lx^3 + x^4)\,dx$$

$$= \frac{w^2}{8EI}\left[l^2 \cdot \frac{x^3}{3} - 2l\frac{x^4}{4} + \frac{x^5}{5}\right]_0^l$$

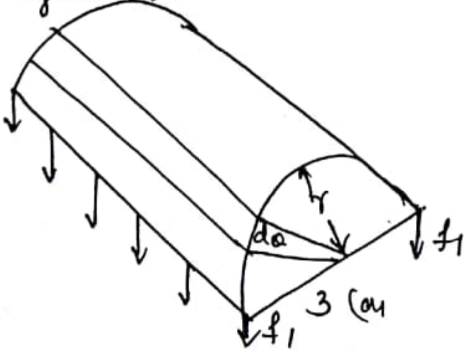
$$= \frac{w^2}{8EI}\left[l^2 \cdot \frac{l^3}{3} - l \cdot \frac{l^4}{4} + \frac{l^5}{5}\right]$$

$$= \frac{w^2 l^5}{8EI}\left[\frac{10 - 15 + 6}{30}\right]$$

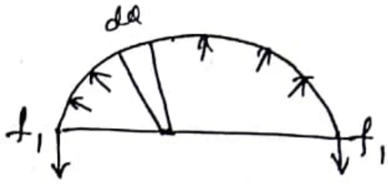
$\therefore$ Strain energy stored by the beam, $U = \frac{w^2 l^5}{240EI}$

3. Circumferential stress: This is also called as hoop stress.

Consider the longitudinal and diametrical section x-x of the cylinder.



3 (a)



3 (b)

Let 'r' be the radius. Then normal pressure on elemental length shown in fig. b = $\underline{prl\,d\theta}$

Bursting force on the element normal to the section x-x

$$= \underline{prl\,d\theta\cos\theta}$$

∴ Total bursting force normal to the section

$$2\int_0^{\pi/2} prL\cos\theta\,d\theta = 2prL[\sin\theta]_0^{\theta/2}$$

$$= 2prl\,[1] = 2prl$$

$$= \underline{Pdl} \quad -(1)$$

Resisting force for this bursting force is $= f_1 \times t \times L \times 2$

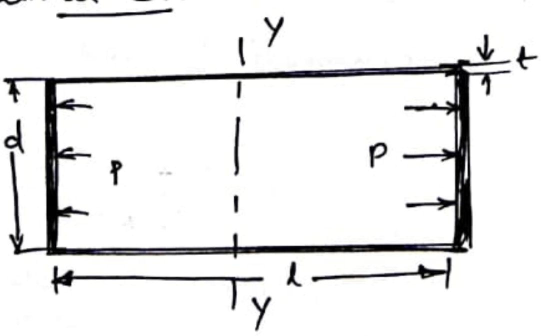
$$= 2f_1tL \quad -(2)$$

Equating (1) and (2) we get,

$$2f_1tL = Pdl$$

(or) $f_1 = \dfrac{Pd}{2t}$ (Ans)

Longitudinal stress:



3 (c)

Consider a thin cylindrical shell subjected to an internal pressure as shown in fig. As a result of the internal pressure, the cylinder also has tendency to split up into two cylinders along section Y-Y as shown in fig. 3(c)

* Bursting will take place along the section Y-Y, if the force due to internal pressure acting at the ends of the cylinder is more than the resisting force due to longitudinal stress developed in the material.

Force due to internal pressure at the ends of the cylinder or Bursting force

= Internal pressure × Area on which the pressure is acting

$= P \times \frac{\pi d^2}{4}$

Force due to longitudinal stress or Resisting force

= Longitudinal stress × Area on which the stress is acting

$= \sigma_2 \times \pi d t$

For equilibrium, Resisting force = Bursting force

i.e. $\sigma_2 \times \pi d t = P \times \frac{\pi d^2}{4}$

$\therefore$ Longitudinal stress, $\sigma_2 = \frac{Pd}{4t}$ (Ans)

* Longitudinal stress is one half of circumferential stress

i.e., $\boxed{\sigma_2 = \frac{\sigma_1}{2}}$

4. **Data:**

$d = 1.2\,m = 1200\,mm$

$l = 3\,m = 3000\,mm$

$t = 12\,mm$

$P_i = 3.2\,MPa$

σ_1 or $\sigma_c = ?$

σ_2 or $\sigma_L = ?$

$E = 210\,GPa$

$\mu = 0.3$

(i) We know that,

Circumferential stress, $\sigma_1 = \frac{Pd}{2t}$

$= \frac{3.2 \times 1200}{2 \times 12}$

$\therefore \sigma_1$ or $\sigma_c = 160\,N/mm^2$

Longitudinal stress, σ_2 or $\sigma_L = \frac{Pd}{4t} = \frac{3.2 \times 1200}{4 \times 12}$

$= \underline{80 \, N/mm^2}$ (Ans)

(ii) Circumferential, longitudinal and volumetric strain

Circumferential strain, $e_1 = \frac{\sigma_1}{E} - \frac{1}{m}\frac{\sigma_2}{E}$

$$= \frac{160}{210 \times 10^3} - 0.3 \times \frac{80}{210 \times 10^3}$$

$$= \underline{6.476 \times 10^{-4}}$$

Longitudinal strain, $e_2 = \frac{\sigma_2}{E} - \frac{1}{m}\frac{\sigma_1}{E}$

$$= \frac{80}{210 \times 10^3} - \frac{0.3 \times 160}{210 \times 10^3}$$

$$= \underline{\underline{1.523 \times 10^{-4}}}$$

Volumetric strain, $e_v = 2e_1 + e_2 = (2 \times 6.476 + 1.523) \times 10^{-4}$

$= \underline{14.475 \times 10^{-4}}$

Change in length, diameter and volume.

Longitudinal strain, $e_2 = \frac{\delta l}{l}$

$$1.523 \times 10^{-4} = \frac{\delta l}{3000}$$

$\therefore$ Change in length, $\delta l = \underline{0.4569 \, mm}$ (Ans)

Circumferential strain, $e_1 = \frac{\delta d}{d}$

$$6.476 \times 10^{-4} = \frac{\delta d}{1200}$$

$\therefore$ Change in diameter, $\delta d = \underline{\underline{0.77712 \, mm}}$

Volumetric strain, $e_v = \frac{\delta v}{V}$ where, $V = \frac{\pi d^2}{4} \times l$

$$= \frac{\pi}{4} \times 1200^2 \times 3000$$

$$14.475 \times 10^{-4} = \frac{\delta v}{\frac{\pi}{4} \times 1200^2 \times 3000}$$

$$\therefore \delta v = \underline{\underline{49.1125 \times 10^5 \ mm^3}} \ (\text{Ans})$$

(iii) Max. shear stress, $\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{160 - 80}{2}$

$$= \underline{\underline{40 \ N/mm^2}}$$

5. $d_2 = 300mm$; $r_2 = 150mm$ (Inner radius)

$d_1 = 500mm$; $r_1 = 250mm$ (outer radius)

$t = r_1 - r_2 = 250 - 150 = 100m$

$r_m = \frac{r_1 + r_2}{2} = \frac{250 + 150}{2} = \underline{200mm}$ (Mean radius)

~~P/6/=13~~ MPa = ~~P~~ $\underline{= 4MPa = P_{250}} = P_o$

$f_{r_2} = f_{150} = 13MPa = 13N/mm^2$

From Lame's theorem

Radial pressure at any radius x, $P_x = \frac{b}{x^2} - a$

when $x = r_1$, $P_x = P_{r_1} = P_{250} = 4 \ 13MPa = 13N/mm^2$

$$\therefore 4 = \frac{b}{250^2} - a \quad -(i)$$

Circumferential stress at any radius x, $f_x = \frac{b}{x^2} + a$

when, $x = r_2$, $f_x = f_{r_2} = f_{150} = 13 N/mm^2$

$$\therefore f_{150} = \frac{b}{x^2} + a$$

$$13 = \frac{b}{150^2} + a \quad -(ii)$$

From (i) and (ii)

$$17 = \frac{b}{150^2} + \frac{b}{250^2}$$

$$\therefore b = \underline{281250} \ (\text{Ans})$$

From eqn (i) $\quad a = \underline{0.5}$

when $x = r_2 = 150$ mm, p_x = Internal pressure

$\therefore$ Internal pressure $p_{r_2} = p_{150} = \frac{281250}{150^2} - 0.5$

$= 12$ N/mm^2 (Ans)

when $x = r_m = 200$ mm, $p_x = p_{200} = p_{rm}$

$p_{rm} = p_{200} = \frac{281250}{200^2} - 0.5$

$\therefore p_{200} = 6.53125$ N/mm^2

Radial pressure

Circumferential stress

when $x = r_1 = 250$ mm, $f_x = f_{250} = f_{r_1}$

$\therefore f_{250} = \frac{281250}{250^2} + 0.5$

$= 5$ N/mm^2

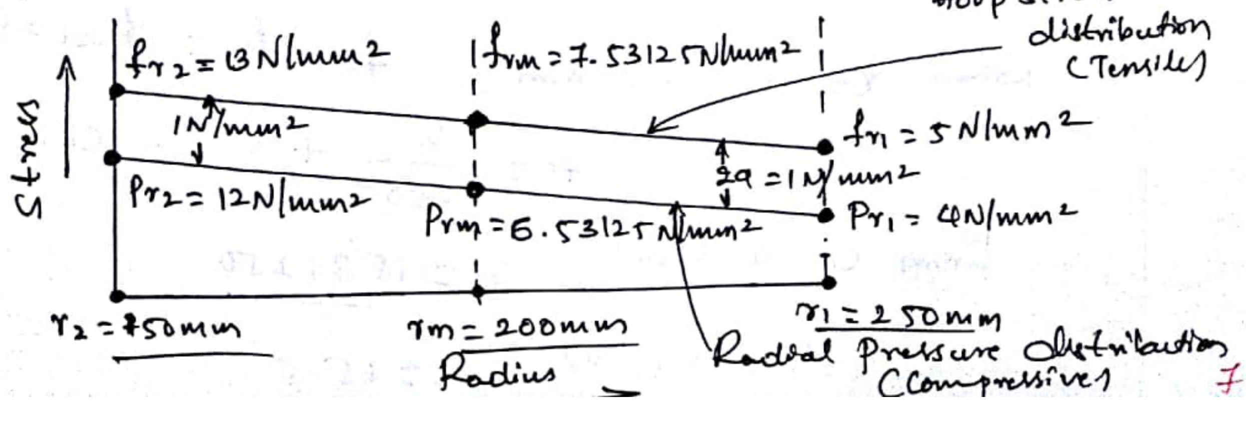
when $x = r_m = 200$ mm, $f_x = f_m = f_{200}$

$\therefore f_{200} = \frac{281250}{200^2} + 0.5$

$= 7.53125$ N/mm^2

$p_{r_2} = 12$ N/mm^2 ; $p_{rm} = 6.53125$ N/mm^2 $p_{r_1} = 4$ N/mm^2

$f_{r_2} = 13$ N/mm^2 ; $f_{rm} = 7.53125$ N/mm^2 $f_{r_1} = 5$ N/mm^2



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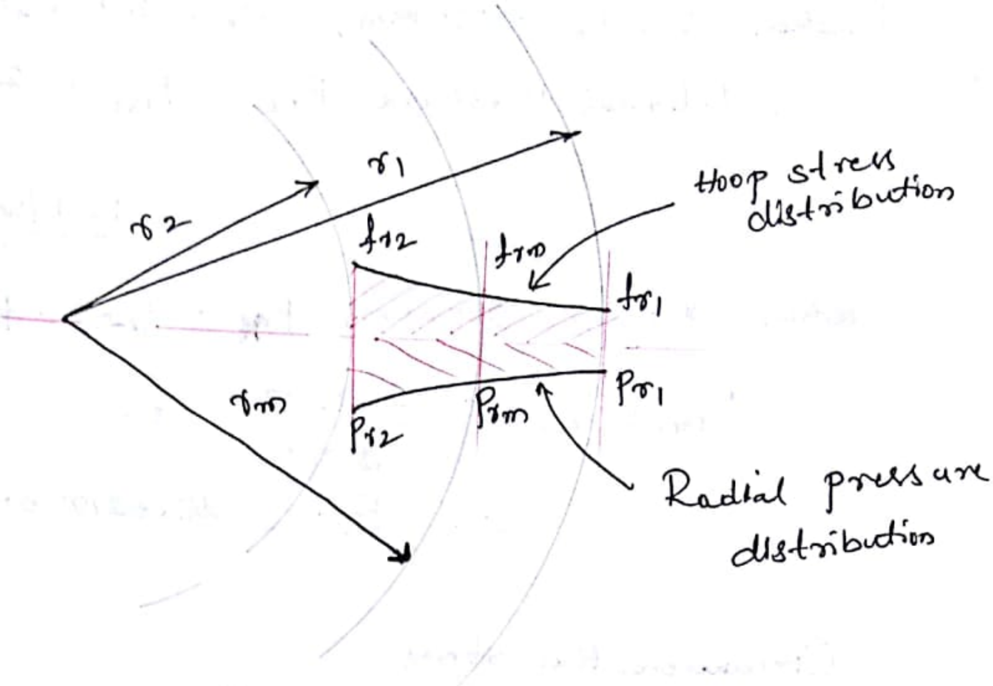


6. Data: $d_2 = 500mm$; $r_2 = 250mm$

$P_i = 9MPa = P_{r_2} = P_{250}$

$f_{r_2} = f_{250} = 40MPa = 40N/mm^2$

thickness, $t = ?$

From Lame's theorem

Radial pressure at any radius, $P_x = \frac{b}{x^2} - a$

& Circumferential stress at any radius, $f_x = \frac{b}{x^2} + a$

When $x = r_2 = 250mm$; $P_x = P_{r_2} = P_{250} = 9MPa$

$$9 = \frac{b}{250^2} - a \quad -(i)$$

When $x = r_2 = 250mm$, $f_x = f_{r_2} = f_{250} = 40MPa$

$$40 = \frac{b}{250^2} + a \quad -(ii)$$

From (i) and (ii) $\underline{b = 1531250}$

From eqn (i) $\underline{\underline{a = 15.5}}$

At $x = r_1$, $p_{r_1} = 0$

$$\therefore \quad p_{r_1} = \frac{b}{r_1^2} - a$$

$$0 = \frac{1531250}{r_1^2} - 15.5$$

$$\frac{1531250}{r_1^2} = 15.5$$

$$r_1^2 = 98790.3226$$

$$\therefore \quad r_1 = 314.309 \simeq \underline{\underline{315\ mm}}$$

At $x = r_1$, $f_{r_1} = f_{315}$

$$f_{315} = \frac{b}{315^2} + a$$

$$= \frac{1531250}{315^2} + 15.5$$

$$\therefore \quad f_{315} = \underline{31\ N/mm^2}$$

Mean radius, $r_m = \dfrac{r_1 + r_2}{2} = \dfrac{315 + 250}{2} = \underline{\underline{282.5\ mm}}$

At $x = r_m$; $p_x = p_{r_m} = p_{282.5}$

$$p_{282.5} = \frac{1531250}{282.5^2} - 15.5$$

$$= 3.\underline{687}\ N/mm^2$$

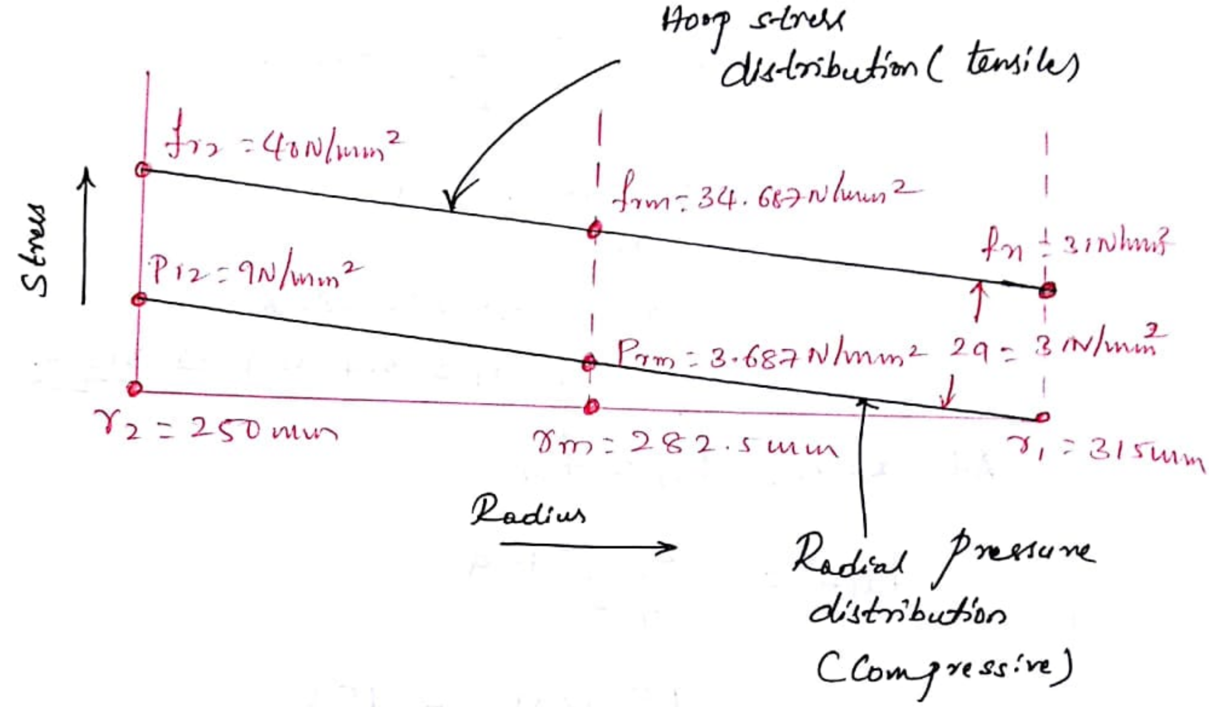
At $x = r_m$; $f_x = f_{r_m} = f_{282.5}$

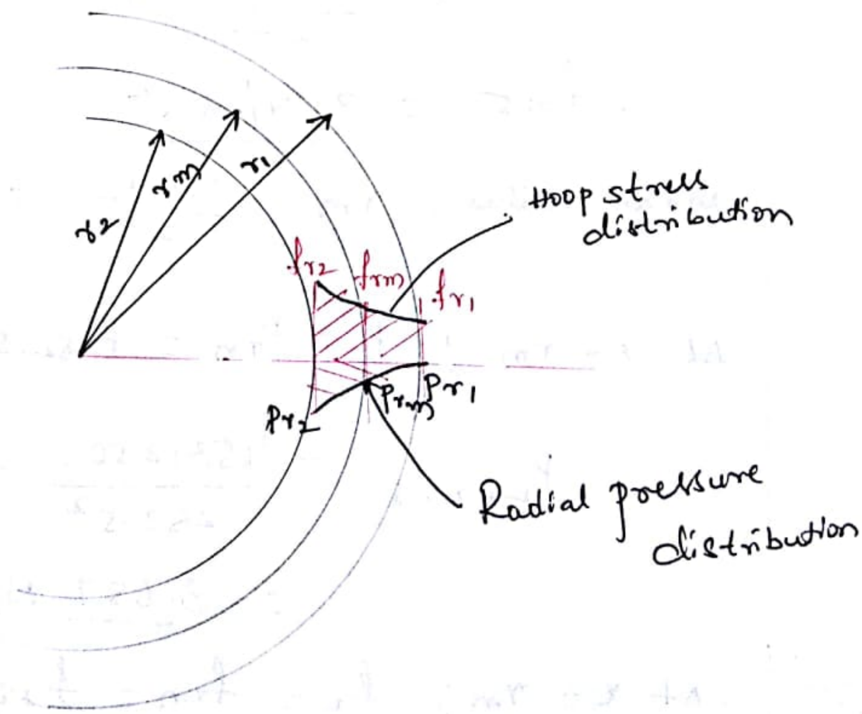
$$f_{282.5} = \frac{1531250}{282.5^2} + 15.5$$

$$= \underline{\underline{34.687\ N/mm^2}}$$

$P_{r_2} = 9\,N/mm^2$; $P_{rm} = 3.687\,N/mm^2$; $P_{r_1} = 0$

$f_{r_2} = 40\,N/mm^2$; $f_{rm} = 34.687\,N/mm^2$; $f_{r_1} = 31\,N/mm^2$





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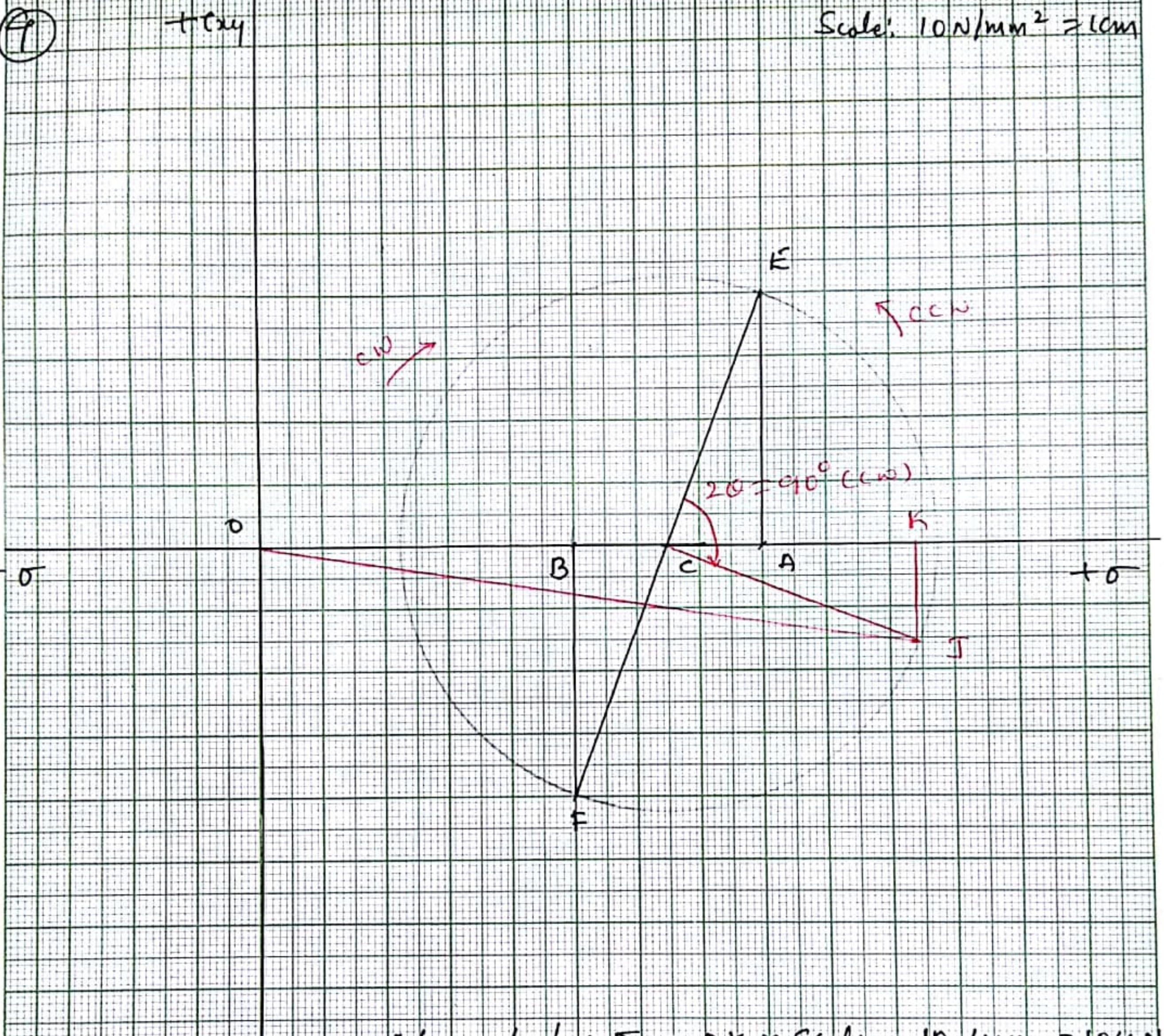
(1)

Scale: 10 N/mm² = 1 cm



Normal stress, σ_n = OK × Scale = 10.4 × 10 = 104 N/mm²

Tangential stress, σ_t = JK × Scale = 1.6 × 10 = 16 N/mm²

Resultant stress, σ_R = OJ × Scale = 10.5 × 10 = 105 N/mm²

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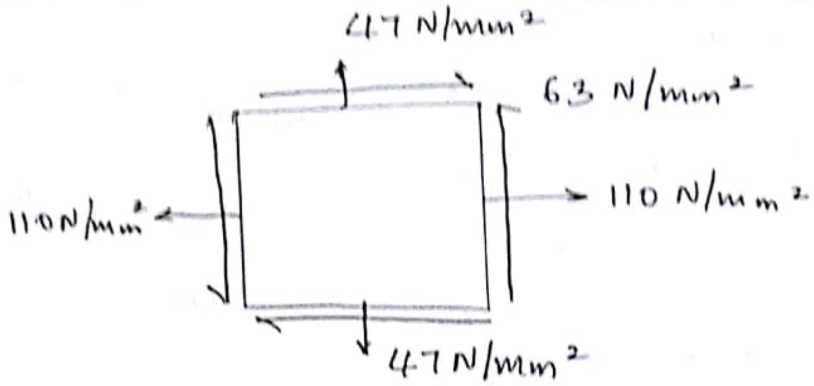
8.



$\sigma_x = +110\ N/mm^2$, $\sigma_y = 47\ N/mm^2$, $\tau_{xy} = +63\ N/mm^2$

(i) Direction and magnitude of principal stress

Direction of principal planes:

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times 63}{110 - 47}$$

$$2\theta = \tan^{-1}\left(\frac{2 \times 63}{110 - 47}\right)$$

$$\therefore \theta = 31.71^\circ = \theta_1$$

$$\theta_2 = \theta_1 + 90^\circ = 121.71^\circ$$

Major principal stress

$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{110 + 47}{2} + \sqrt{\left(\frac{110 - 47}{2}\right)^2 + 63^2}$$

$$\therefore \sigma_1 = 148.936\ N/mm^2 \simeq 149\ N/mm^2$$

Minor principal stress

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= \frac{110 + 47}{2} - \sqrt{\left(\frac{110 - 47}{2}\right)^2 + 63^2}$$

$$\therefore \sigma_2 = 8.064\ N/mm^2$$

(ii) Maximum Shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{110-47}{2}\right)^2 + 63^2}$$

$$= 70.436 \ N/mm^2$$

or

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{148.936 - 8.064}{2}$$

$$\therefore \quad \tau_{max} = 70.436 \ N/mm^2$$

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