CMR INSTITUTE OF TECHNOLOGY	USN	CMRIT

Internal Assesment Test-I

Sub:	Engineering Electromagnetics							Code:	17EC36
Date:	07/09 /2018	Duration:	90 mins	Max Marks:	50	Sem:	3rd	Branch:	ECE(A,B,C)

Answer FIVE FULL Questions

		Marks	OBE	
			CO	RBT
1.(a)	State and explain Coulomb’s law in vector form.	[06]	CO1	L1

Soln The force b/w two very small charged objects separated in vacuum or free space by a distance which is large compared to their size is proportional to the charge on each and inversely proportional to the square of the dist. b/w them.

i.e. $F = \frac{k\, Q_1 Q_2}{R^2}$

Q_1 & Q_2 → +ve or −ve quantities of charge
unit Coulomb (C)

R → separation in m.

k → constant of proportionality.

$k = \frac{1}{4\pi\epsilon_0}$

where, ϵ_0 → permittivity of free space.

$\epsilon_0 = 8.854 \times 10^{-12}$ F/m

$= \frac{1}{36\pi} \times 10^{-9}$ F/m.

F → Force in Newton.

1.(a) $\therefore\ F_1 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{21}^2}\left(-\hat{a}_{21}\right) = -\vec{F}_2$

⇒ Coulomb's law is a mutual force.

Important observations:

i) charges should be point charges and stationary in nature.

ii) Should consider the sign of the charges to decide wheather the force will be attractive or repulsive.

iii) Coulomb's law is linear.

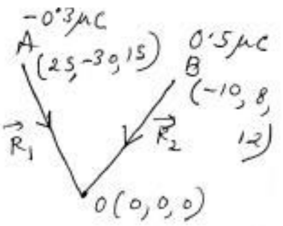
i.e. if $\vec{F_2} = -\vec{F_1}$

then, $n\vec{F_2} = -n\vec{F_1}$

where n is a scalar.

iv) Force on a charge in the presence of several other charges is the sum of the forces on that charge due to each of the other charges acting alone.

(b)	A charge of -0.3μC is located at A (25,-30,15) and a second charge of 0.5μC at B(-10,8,12). Find **E** at the origin.	[04]	CO1	L3



$\vec{R}_1 = \left(-25\,\hat{a}_x + 30\,\hat{a}_y - 15\,\hat{a}_z\right)$

$\vec{R}_2 = \left(10\,\hat{a}_x - 8\,\hat{a}_y - 12\,\hat{a}_z\right)$

$|\vec{R}_1| = \sqrt{(-25)^2 + (30)^2 + (-15)^2}$

$= 41.83$

$|\vec{R}_2| = \sqrt{100+64+144} = 17.54$

$$\vec{E}_1 = \frac{-0.3\times10^{-6}\times\left(-25\,\hat{a}_x + 30\,\hat{a}_y - 15\,\hat{a}_z\right)}{4\pi\times8.854\times10^{-12}\times(41.83)^2\times41.83}$$

$= 0.92\,\hat{a}_x - 1.04\,\hat{a}_y + 0.552\,\hat{a}_z$ V/m

$$\vec{E}_2 = \frac{0.5\times10^{-6}\times\left(10\,\hat{a}_x - 8\,\hat{a}_y - 12\,\hat{a}_z\right)}{4\pi\times8.854\times10^{-12}\times(17.54)^2\times(17.54)}$$

$= \left(8.33\,\hat{a}_x - 6.664\,\hat{a}_y - 9.996\,\hat{a}_z\right)$ V/m

$\therefore\ \vec{E} = \vec{E}_1 + \vec{E}_2$

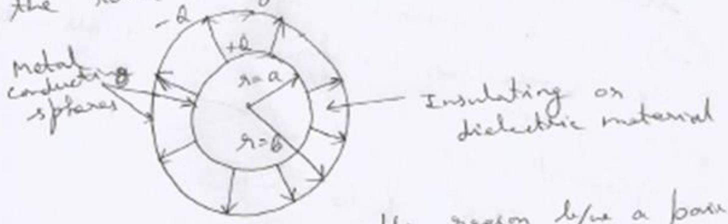
$= \left(9.25\,\hat{a}_x - 7.768\,\hat{a}_y - 9.446\,\hat{a}_z\right)$ V/m

2.(a)	Define electric flux density. Derive the relation between electric flux density and electric field intensity.	[04]	CO1	L1

Electric flux density is defined as the electric flux per unit area where the area is perpendicular to the surface.

soln.

Gauss's law states that the electric flux passing through any closed surface is equal to the total charge enclosed by that surface.



The electric flux in the region b/w a pair of charged concentric spheres. The direction and mag. of D are not functions of the dielectric b/w the spheres.

The direction of D at a pt. is the direction of the flux lines at that point, and the mag. is given by the no. of flux lines crossing a surface normal to the lines divided by the surface area

$$\therefore \vec{D}\Big|_{r=a} = \frac{Q}{4\pi a^2}\,\hat{a}_r \quad \text{(inner sphere)}$$

$$\vec{D}\Big|_{r=b} = \frac{-Q}{4\pi b^2}\,\hat{a}_r \quad \text{(outer sphere)}$$

$a \le r \le b$

$$\vec{D} = \frac{Q}{4\pi r^2}\,\hat{a}_r$$

Shrink inner sphere, smaller and smaller, we reach point charge

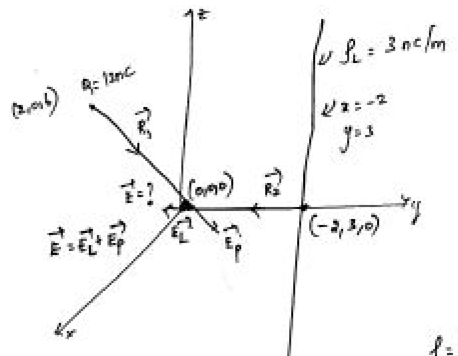
$$\vec{D} = \frac{Q}{4\pi r^2}\,\hat{a}_r .$$

Q lines of flux are symmetrically directed outward from the pt. and pass through an imaginary spherical surface of area $4\pi r^2$.

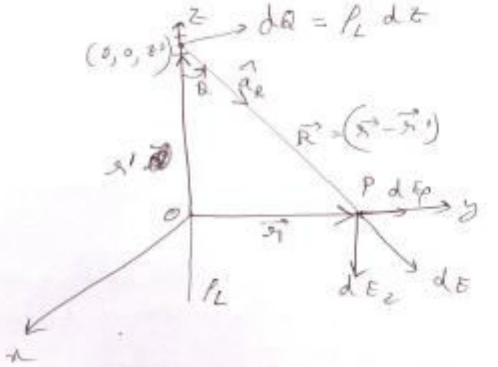
$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r .$$

$$\therefore \boxed{D = \epsilon_0 E} \rightarrow \text{free space.}$$

(b)	Find **E** at origin due to a point charge 12nC at (2, 0, 6) and a uniform line charge 3nC/m at x = -2, y = 3.	[06]	CO1	L3

	 $\vec{R_1} = -2\vec{a_x} - 6\vec{a_z}$ $\lvert\vec{R_1}\rvert = \sqrt{4+36} = \sqrt{40}$ $\rho = \lvert\vec{R_2}\rvert$ $\vec{a_\rho} = \vec{a_{R_2}} = \frac{\vec{R_2}}{\lvert\vec{R_2}\rvert}$ $\vec{E} = \vec{E_L} + \vec{E_P}$ $\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho}\vec{a_\rho} + \frac{Q}{4\pi\varepsilon_0 \lvert\vec{R_1}\rvert^3}\vec{R_1}$ $\vec{R_2} = 2\vec{a_x} - 3\vec{a_y}$ $\lvert\vec{R_2}\rvert = \sqrt{4+9} = \sqrt{13}$ $= \frac{3\times10^{-9} \times (2\vec{a_x} - 3\vec{a_y})}{2\pi \times \varepsilon_0 \times \sqrt{13} \times \sqrt{13}} + \frac{12\times10^{-9}}{4\pi\times\varepsilon_0} \times \frac{(-2\vec{a_x} - 6\vec{a_z})}{(\sqrt{40})^3}$ $= \frac{1}{4\pi\varepsilon_0}\left[\frac{3\times10^{-9} \times 2\,(2\vec{a_x} - 3\vec{a_y})}{13} + \frac{12\times10^{-9}(-2\vec{a_x} - 6\vec{a_z})}{(\sqrt{40})^3}\right]$ $= 9\times10^{9} \times 6\times10^{-9}\left[\frac{2}{13}\vec{a_x} - \frac{3}{13}\vec{a_y} - \frac{4}{(\sqrt{40})^3}\vec{a_x} - \frac{12}{(\sqrt{40})^3}\vec{a_z}\right]$ $= 54\left[0.138\,\vec{a_x} - 0.230\,\vec{a_y} - 0.0474\,\vec{a_z}\right]$ $\boxed{\vec{E} = 7.452\,\vec{a_x} - 12.42\,\vec{a_y} - 2.5596\,\vec{a_z}}$ V/m			
3.	Define electric field intensity. Obtain an expression for electric field intensity due to an infinitely long uniform line charge distribution.	[02+08]	CO1	L1

	9) <u>Electric field Intensity</u> - consider ~~the~~ one charge fixed in position say Q_1. - we move a charge slowly around. There exists a force everywhere on this 2nd charge. i.e. force field. - let the 2nd charge be Q_t. Then the force on it is: $\vec{F}_t = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{1t}^2} \hat{a}_{1t}$ $\therefore$ Force per unit charge: $\frac{F_t}{Q_t} = \frac{Q_1}{4\pi\epsilon_0 R_{1t}^2} \hat{a}_{1t}$. R.H.S. f^n of Q_1 and the directed line segment from Q_1 to the position of the test charge. - This describes a vector field and is called the electric field intensity. Def^n: Electric field intensity is the vector force on a unit +ve test charge when the charge placed in a electric field. $\boxed{\text{Unit: } N/C}$ But $V = \frac{J}{C} = \frac{N.m}{C}$. $\Rightarrow \frac{V}{m} = \frac{N}{C}$ $\therefore$ E practically expressed in $\frac{V}{m}$. Finally, $\boxed{\vec{E} = \frac{F_t}{Q_t}}$ $\boxed{\vec{E} = \frac{Q_1}{4\pi\epsilon_0 R_{1t}^2} \hat{a}_{1t}}$			



– consider a general point $P(0, y, 0)$ on the y-axis to determine the field.

– We consider incremental charge

$$dQ = \rho_L dz'.$$

– Aim to find incremental field.

$$\therefore d\vec{E} = \frac{\rho_L dz' (\vec{r} - \vec{r}\,')}{4\pi\epsilon_0 |r - r'|^3}.$$

where, $\vec{r} = y\hat{a}_y = \rho\hat{a}_\rho$

$\vec{r}\,' = z'\hat{a}_z$.

$$\therefore \vec{r} - r' = (\rho\hat{a}_\rho - z'\hat{a}_z)$$

$$\therefore d\vec{E} = \frac{\rho_L dz' (\rho\hat{a}_\rho - z'\hat{a}_z)}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}$$

We know only ρ component is present.

$$dE_\rho = \int_{-\infty}^{\infty} \frac{\rho_L \rho\, dz'}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}$$

$$E_\rho = \int_{-\infty}^{\infty} \frac{\rho_L \rho \, dz'}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}.$$

Let, $z' = \rho \cot\theta$

$\therefore \; dz' = -\rho \, \mathrm{cosec}^2\theta \, d\theta$

@ $z' = \rho \frac{\cos\theta}{\sin\theta}$.

$\infty = \frac{\cos 0}{\sin 0}$. $\Rightarrow$ for $z' = \infty$, $\theta = 0^\circ$

$z' = -\frac{\cos(\pi)}{\sin(\pi)} \Rightarrow z' = -\infty$, $\theta = \pi$.

$$E_\rho = \frac{\rho_L}{4\pi\epsilon_0} \int_{\pi}^{0} \frac{\rho . \; (-\rho \, \mathrm{cosec}^2\theta \, d\theta)}{(\rho^2 + \rho^2 \cot^2\theta)^{3/2}}$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \int_{\pi}^{0} \frac{-\rho^2 \, \mathrm{cosec}^2\theta \, d\theta}{(\rho^2)^{3/2} (1 + \cot^2\theta)^{3/2}}.$$

$$= \frac{\rho_L}{4\pi\epsilon_0 \rho} \cdot \int_{\pi}^{0} \frac{-\mathrm{cosec}^2\theta \, d\theta}{\mathrm{cosec}^3\theta}$$

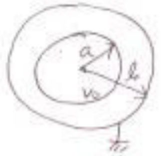
$$= \frac{\rho_L}{4\pi\epsilon_0 \rho} \cdot \int_{\pi}^{0} -\sin\theta \, d\theta = \frac{\rho_L}{4\pi\epsilon_0 \rho} [+\cos\theta]_{\pi}^{0}$$

$$= \frac{\rho_L}{4\pi\epsilon_0 \rho} \cdot [1 + 1] = \frac{2\rho_L}{4\pi\epsilon_0 \rho}$$

$$= \frac{\rho_L}{2\pi\epsilon_0 \rho}$$

4.(a)	Starting from Gauss's law, derive Poisson's and Laplace's equation and write down the equation in all the three coordinate systems.	[04+03]	CO2	L1

	Soln. From Gauss's law, $\vec{\nabla}\cdot\vec{D} = \rho_v$ $\vec{D} = \epsilon\vec{E}$ and $\vec{E} = -\vec{\nabla}V$ $\therefore \vec{\nabla}\cdot\vec{D} = \vec{\nabla}\cdot(\epsilon\vec{E}) = -\vec{\nabla}(\epsilon\vec{\nabla}V) = \rho_v$ $\Rightarrow -\vec{\nabla}\cdot(\vec{\nabla}V) = \frac{\rho_v}{\epsilon}$ $\Rightarrow \vec{\nabla}\cdot(\vec{\nabla}V) = -\frac{\rho_v}{\epsilon}$ $\Rightarrow \nabla^2 V = -\frac{\rho_v}{\epsilon}$ $\boxed{\nabla^2 V = -\frac{\rho_v}{\epsilon}}$ → Poisson's equation Now, $\vec{\nabla}\cdot\vec{A} = \left(\hat{a}_x\frac{\partial}{\partial x} + \hat{a}_y\frac{\partial}{\partial y} + \hat{a}_z\frac{\partial}{\partial z}\right)\cdot(A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z)$ $= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$ $\vec{\nabla}V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$ $\vec{\nabla}\cdot\vec{\nabla}V = \frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial V}{\partial z}\right)$ $= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$ $\boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}}$ → rectangular co-ordinates If $\rho_v = 0$ then $\nabla^2 V = 0$ In cylindrical co-ordinates, $\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial \phi^2}\right) + \frac{\partial^2 V}{\partial z^2}$ $\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$			
(b)	Given vector $\mathbf{E} = (12yx^2 - 6z^2x)\,\mathbf{a_x} + (4x^3 + 18zy^2)\,\mathbf{a_y} + (6y^3 - 6zx^2)\,\mathbf{a_z}$, Verify whether volume charge density is zero in the region. $\nabla^2 V = 0$ $\vec{E} = -\vec{\nabla}V$ $(12yx^2 - 6z^2x)\hat{a}_x + (4x^3 + 18zy^2)\hat{a}_y + (6y^3 - 6zx^2)\hat{a}_z = -\nabla V$ $\nabla V = (-12yx^2 + 6z^2x)\hat{a}_x - (4x^3 + 18zy^2)\hat{a}_y + (-6y^3 + 6zx^2)\hat{a}_z$ $\nabla^2 V = \nabla\cdot\nabla V = \nabla\cdot\left[(-12yx^2 + 6z^2x)\hat{a}_x - (4x^3 + 18zy^2)\hat{a}_y + (6y^3 + 6zx^2)\hat{a}_z\right]$ $= \frac{\partial}{\partial x}(-12yx^2 + 6z^2x) + \frac{\partial}{\partial y}(-4x^3 - 18zy^2) + \frac{\partial}{\partial z}(-6y^3 + 6zx^2)$ $= -24xy + 6z^2 - 36zy + 6x^2 \neq 0$ as $\nabla^2 V \neq 0$, the Laplace equation is not satisfied. Thus there exist ρ_v as region is not free of charge and hence $\vec{E}$ represents possible electric field.	[03]	CO2	L3
5.	Derive the expression of capacitance for concentric spheres using Laplace's equation. Consider radius of inner sphere 'a' and outer sphere 'b'. Also consider the potential $V = V_0$ at $r = a$ and $V = 0$ at $r = b$.	[10]	CO2	L1

	find the capacitance b/w the two concentric spheres of radius $r=b$ and $r=a$, such that $b>a$, if the potential $V=0$ at $r=b$, and $V=V_0$ at $r=a$, using laplace equation? $(b>a)$   $\nabla^2 V = 0$ $\frac{1}{r^2}\frac{\partial}{\partial r}\left[r^2\frac{dV}{\partial r}\right]=0$ $\frac{\partial}{\partial r}\left[r^2\frac{dV}{\partial r}\right]=0$ $r^2\frac{dV}{\partial r}=C_1$ $\frac{\partial V}{\partial r}=\frac{C_1}{r^2}$ $V=\frac{-C_1}{r}+C_2$ Applying the boundary conditions, $0=\frac{-C_1}{b}+C_2 \longrightarrow ①$ $V_0=\frac{-C_1}{a}+C_2 \longrightarrow ②$ solving ① and ② $C_1=\frac{V_0}{\frac{1}{b}-\frac{1}{a}}$ and $C_2=\frac{V_0}{b\left[\frac{1}{b}-\frac{1}{a}\right]}$ $V=\frac{-V_0}{r\left(\frac{1}{b}-\frac{1}{a}\right)}+\frac{V_0}{b\left[\frac{1}{b}-\frac{1}{a}\right]}$ $\vec{E}=-\vec{\nabla}V$ $=-\frac{\partial V}{\partial r}\hat{a}_r$ $=-\frac{\partial}{\partial r}\left[\frac{-V_0}{r\left(\frac{1}{b}-\frac{1}{a}\right)}\right]\hat{a}_r$ $\vec{E}=\frac{-V_0}{r^2\left(\frac{1}{b}-\frac{1}{a}\right)}\hat{a}_r$ $\vec{D}=\epsilon_0\vec{E}$ $\vec{D}=\frac{-\epsilon_0 V_0}{r^2\left(\frac{1}{b}-\frac{1}{a}\right)}$ $Q=\vec{D}\cdot d\vec{s}$ $Q=\frac{-\epsilon_0 V_0}{r^2\left(\frac{1}{b}-\frac{1}{a}\right)}\times 4\pi r^2$ $Q=\frac{4\pi\epsilon_0 V_0}{\frac{1}{a}-\frac{1}{b}}$ $C=\frac{Q}{V_0}$ $C=\frac{4\pi\epsilon_0}{\frac{1}{a}-\frac{1}{b}}$			

6.	State and prove uniqueness theorem for solution of Laplace's equation.	[10]	CO2	L1

"If the solution of Laplace's equation satisfies the boundary condition then that solution is unique by whatever method it is obtained"

"the solution of Laplace's equation gives the field which is unique, satisfying the same boundary conditions, in a given region"

Proof:- $\nabla^2 V = 0$

we take two solution of Laplace's equation V_1 and V_2

$\nabla^2 V_1 = 0 \qquad \nabla^2 V_2 = 0 \quad \Rightarrow \nabla^2 (V_1 - V_2) = 0$

we assume V_1 and V_2 are solutions of Laplace equation. of V_1 on boundary V_{1b}

— " — V_2 — " — V_{2b}.

$V_b \rightarrow$ given potential value on boundary

$\therefore V_{1b} = V_{2b} = V_b$

$\Rightarrow V_{1b} - V_{2b} = 0$

considering the vector identity

$\vec{\nabla} \cdot (V\vec{D}) = V(\vec{\nabla} \cdot \vec{D}) + \vec{D} \cdot (\vec{\nabla} V)$

considering a scalar to be $(V_1 - V_2)$

considering the vector to be $\vec{\nabla}(V_1 - V_2)$

$\vec{\nabla} \cdot [(V_1 - V_2)\vec{\nabla}(V_1 - V_2)] = (V_1 - V_2)[\vec{\nabla} \cdot \vec{\nabla}(V_1 - V_2)] + [\vec{\nabla}(V_1 - V_2) \cdot \vec{\nabla}(V_1 - V_2)]$

taking volume integral on both sides

$$\int_{vol} \vec{\nabla} \cdot [(V_1 - V_2)\vec{\nabla}(V_1 - V_2)]\, dv = \int_{vol} (V_1 - V_2)\underbrace{\nabla^2 (V_1 - V_2)}_{0 \text{ [by hypothesis]}}\, dv + \int_{vol} [\vec{\nabla}(V_1 - V_2)]^2\, dv \quad (1)$$

$LHS = \oint_S [(V_1 - V_2)\vec{\nabla}(V_1 - V_2)] \cdot d\vec{S}$

$= \oint_S [(V_{1b} - V_{2b})\vec{\nabla}(V_{1b} - V_{2b})] \cdot d\vec{S}$

$= 0$

$\therefore \int_{vol} [\vec{\nabla}(V_1 - V_2)]^2\, dv = 0$ [Note: an integral may be zero if either the integrand is everywhere zero, or the integrand is +ve in some regions and -ve in other regions, and the contributions cancel algebraically. In this case the 1st reason holds good as $[\vec{\nabla}(V_1 - V_2)]^2$ can not be -ve.

$\Rightarrow \vec{\nabla}(V_1 - V_2) = 0$

$V_1 - V_2 = \text{Constant}$

on the boundary

$V_1 - V_2 = V_{1b} - V_{2b} = 0$

$\Rightarrow V_1 - V_2 = 0$

$\boxed{V_1 = V_2}$

7. (a)	Verify if the field given by $V = (A\rho^4 + B\rho^{-4})\sin(4\phi)$ satisfies Laplace's equation or not.	[05]	CO2	L3

	$V = (A\rho^4 + B\rho^{-4})\sin(4\phi)$ $\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial\phi^2}$ $\frac{\partial V}{\partial\rho} = (4A\rho^3 - 4B\rho^{-5})\sin(4\phi)$ $\rho\frac{\partial V}{\partial\rho} = (4A\rho^4 - 4B\rho^{-4})\sin(4\phi)$ $\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) = (16A\rho^3 + 16B\rho^{-5})\sin(4\phi)$ $\boxed{\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) = (16A\rho^2 + 16B\rho^{-6})\sin(4\phi)}$ $\frac{\partial V}{\partial\phi} = (A\rho^4 + B\rho^{-4})\,4\cos(4\phi)$ $\frac{\partial^2 V}{\partial\phi^2} = (A\rho^4 + B\rho^{-4})(-16)\sin(4\phi)$ $\boxed{\frac{1}{\rho^2}\frac{\partial^2 V}{\partial\phi^2} = (A\rho^2 + B\rho^{-6})(-16)\sin(4\phi)}$ $\therefore \nabla^2 V = 16(A\rho^2 + B\rho^{-6})\sin(4\phi) - 16(A\rho^2 + B\rho^{-6})\sin(4\phi)$ $= 0$ ∴ The given field satisfies Laplace's eqn			
(b)	Calculate the numerical values of V and ρ_v in free space for $V = \frac{4yz}{x^2+1}$ at P (1, 2, 3). $V\|_P = \frac{4.2.3}{1^2+1} = \frac{24}{2} = 12$ V $\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = \frac{\partial^2}{\partial x^2}\left(\frac{4yz}{x^2+1}\right) + \frac{\partial^2}{\partial y^2}\left(\frac{4yz}{x^2+1}\right) + \frac{\partial^2}{\partial z^2}\left(\frac{4yz}{x^2+1}\right)$ $= \frac{\partial}{\partial x}\left[4yz(-1)(x^2+1)^{-2}\cdot 2x\right]$ $= -4yz\cdot 2\frac{\partial}{\partial x}\left[\frac{x}{(x^2+1)^2}\right] = -8yz\left[\frac{1\cdot(x^2+1)^2 - x\cdot 2(x^2+1)\cdot 2x}{(x^2+1)^4}\right]$ $= -8yz\left[\frac{x^4+2x^2+1 - 4x^2(x^2+1)}{(x^2+1)^4}\right]$ $= -8yz\left[\frac{x^2+1-4x^2}{(x^2+1)^3}\right] = -8yz\left[\frac{-3x^2+1}{(x^2+1)^3}\right]$ $\therefore \rho_v$ at P, $\rho_{v_P} = 8yz\left[\frac{-3x^2+1}{(x^2+1)^3}\right]\cdot 8.854\times10^{-12}$ $= \cancel{8}\cdot 2.3\left[\frac{-3+1}{\cancel{8}}\right]\times 8.854\times10^{-12}$ $= -106.2$ pC/m³	[05]	CO2	L3
8.	Define volume charge density. Also find the total charge within each of the indicated volumes. i. $0 \le \rho \le 0.1,\ 0 \le \phi \le \pi,\ 2 \le z \le 4,\ \rho_v = \rho^2 z^2 \sin(0.6\phi)$ C/m³	[02+04+04]	CO1	L3

ii. Universe: $\rho_v = \dfrac{e^{-2r}}{r^2}$ C/m^3.

$$Q_{enc} = \iiint_V \rho_v \, dv = \int_{z=2}^{4}\int_{\phi=0}^{\pi}\int_{\rho=0}^{0.1} \rho^2 z^2 \sin 0.6\phi \;\; \rho \, d\rho \, d\phi \, dz$$

$$= \int_0^{0.1} \rho^3 \, d\rho \times \int_0^{\pi} \sin 0.6\phi \, d\phi \times \int_2^4 z^2 \, dz$$

$$= \left[\frac{\rho^4}{4}\right]_0^{0.1} \left[-\frac{\cos 0.6\phi}{0.6}\right]_0^{\pi} \times \left[\frac{z^3}{3}\right]_2^4$$

$$= \frac{(0.1)^4}{4} \times \left[\frac{0.309}{0.6} + \frac{1}{0.6}\right] \times \left[\frac{1}{3}\left(4^3 - 2^3\right)\right]$$

$$\boxed{Q_{enc} = 1.018 \text{ mC}}$$

$$Q_{enc} = \iiint_V \rho_v \, dv$$

$$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{\infty} \frac{e^{-2r}}{r^2} \, r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= \int_0^{\infty} e^{-2r} \, dr \times \int_0^{\pi} \sin\theta \, d\theta \times \int_0^{2\pi} d\phi$$

$$= \left[\frac{e^{-2r}}{-2}\right]_0^{\infty} \left[-\cos\theta\right]_0^{\pi} \left[\phi\right]_0^{2\pi}$$

$$= -\frac{1}{2}(0 - 1) \times (1+1) \times 2\pi$$

$$= 2\pi$$

$$\boxed{Q_{enc} = 6.28 \text{ C}}$$