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INTERNAL ASSESSMENT TEST – I

Sub:	DIGITAL SIGNAL PROCESSING							Code:	15EC52
Date:	07 / 09 / 2018	Duration:	90 mins	Max Marks:	50	Sem:	V	Branch:	ECE(D)/TCE

Answer any 5 full questions

		Marks	CO	RBT
1	Explain frequency domain sampling. Derive an expression for DFT and IDFT of a finite length sequence.	[10]	CO1	L2
2	Compute the 8-point DFT of the sequence $x[n] = [8,6,4,2]$. Plot the magnitude spectrum and the phase spectrum.	[10]	CO1	L2
3(a)	Compute the 4-point DFT of $x[n] = [1,3,5,7]$ using matrix method. Plot the magnitude spectrum and the phase spectrum.	[06]	CO1	L2
3(b)	Compute the IDFT of $X[k] = [12, -3 + j1.7321, -3 - j1.7321]$ using matrix method.	[04]	CO1	L2

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3(b)	Compute the IDFT of $X[k] = [12, -3 + j1.7321, -3 - j1.7321]$ using matrix method.	[04]	CO1	L2

4	Without explicitly determining the DFT $X[k]$ of the sequence $x[n] = [2,1,1,0,3,2,0,3,4,6]$, evaluate the following. i. $X(0)$ ii. $X(5)$ iii. $\sum_{k=0}^{9} X[k]$ iv. $\sum_{k=0}^{9} \lvert X[k] \rvert^2$ v. $\sum_{k=0}^{9} e^{j\frac{4\pi k}{5}} X[k]$	[10]	CO1	L3
5(a)	Derive the relationship between DFT and Z-transform of a finite length sequence $x[n], 0 \leq n \leq N-1$.	[06]	CO1	L2
5(b)	Compute the Z-transform of the sequence $x[n] = [0.5, 0, 0.5, 0]$. Using Z-transform compute the DFT of $x[n]$.	[04]	CO1	L2
6(a)	Compute the DFT of the sequence $x[n] = 0.5^n, 0 \leq n \leq 3$ by evaluating the DFT of $x[n] = a^n, 0 \leq n \leq N-1$ where $0 < a < 1$.	[05]	CO1	L2
6(b)	State and prove the periodicity property of DFT and IDFT.	[05]	CO1	L2
7	Show that multiplication of DFTs of two sequences results in circular convolution of their respective time domain sequences.	[10]	CO1	L2
8	Compute the 4-point circular convolution of $x[n] = [2,1,2,1]$ and $h[n] = [1,2,3,4]$ using Stockham's method (DFT-IDFT method).	[10]	CO1	L2

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Solution and Scheme of Evaluation

1 $X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$

$$X\left(\frac{2\pi}{N}k\right) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\frac{2\pi}{N}kn}$$

$0 \leq k \leq N-1$

$$= \sum_{l=-\infty}^{\infty} \sum_{n=lN}^{lN+N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} x_p(n) e^{-j\frac{2\pi}{N}kn} \quad \ldots (1) \quad (3)$$

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x_p(n) e^{-j\frac{2\pi}{N}kn} \quad \ldots (2) \quad (1)$$

$$\therefore X\left(\frac{2\pi}{N}k\right) = N a_k \quad \ldots (3) \quad (1)$$

$$\therefore x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N}kn} \quad \ldots (4) \quad (3)$$

$$X\left(\frac{2\pi}{N}k\right) = X(k)$$

$$= \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn} \quad \ldots (5) \quad (2)$$

2 $x(n) = (8, 6, 4, 2)$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}, \quad 0 \leq k \leq N-1$$

K	X(K)	$\lvert X(K)\rvert$	$\angle X(K)$
0	20	20	0
1	10.8284−j9.6569	14.509	−0.72827
2	4−4j	5.6569	−0.7854
3	5.1716−j1.6569	5.4305	−0.31005
4	4	4	0
5	5.1716+j1.6569	5.4305	0.31005
6	4+4j	5.6569	0.7854
7	10.8284−j9.6569	14.509	0.72827

(10)

3a

$$X(K) = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \end{bmatrix}$$

$$= \begin{bmatrix} 16 \\ -4+4j \\ -4 \\ -4-4j \end{bmatrix}$$

$\lvert X(K)\rvert$	$\angle X(K)$
16	0
5.6569	2.3562
4	3.1416
5.6569	−2.3562

(6)

3b

$$x(n) = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & -0.5+j0.866 & -0.5-j0.866 \\ 1 & -0.5-j0.866 & -0.5+j0.866 \end{bmatrix}\begin{bmatrix} 12 \\ -3+j1.7321 \\ -3-j1.7321 \end{bmatrix}$$
$0 \le n \le 2$

$$= \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix} \qquad (4)$$

4 $X(0) = 22$ (2)

$X(5) = -2$ (2)

$$\sum_{k=0}^{9} X(k) = 10\,x(0) = 20 \qquad (2)$$

$$\sum_{k=0}^{9} |X(k)|^2 = N \sum_{n=0}^{N-1} |x(n)|^2 \qquad (2)$$
$$= 800$$

$$\sum_{k=0}^{9} e^{j\frac{4\pi}{5}k} X(k) = N\,x(4) = 30 \qquad (2)$$

5(a) $$X(z) = \sum_{n=0}^{N-1} x(n) z^{-n} \qquad (1)$$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn} \qquad (1)$$

$$\therefore \; X(z)\Big|_{z=e^{j\frac{2\pi}{N}k}} = X(k) \qquad (1)$$

$$X(z) = \frac{1-z^{-N}}{N} \sum_{k=0}^{N-1} \frac{X(k)}{1-e^{j\frac{2\pi}{N}k} z^{-1}} \qquad (3)$$

5b $\quad x(n) = (0.5, 0, 0.5, 0)$

$$X(z) = 0.5 + 0.5 z^{-2} \qquad (1)$$

$$X(k) = 0.5 + 0.5 e^{-j\pi k} \qquad (1)$$

$$= (1, 0, 1, 0) \qquad (2)$$

6a
$$\underset{0 \le k \le N-1}{X(k)} = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} a^n e^{-j\frac{2\pi}{N}kn}$$

$$= \frac{1-a^N}{1-a e^{-j\frac{2\pi}{N}k}} \qquad (3)$$

If $x(n) = 0.5^n$,

$$X(k) = (1.875, 0.75-0.375j, 0.625, 0.75+0.375j) \qquad (2)$$

6a $X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$

$0 \le k \le N-1$

$$X(k+N) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

(2.5)

$$= X(k)$$

$$x(n) = \frac{1}{N} \sum_{n=0}^{N-1} \cancel{x}(k) e^{j\frac{2\pi}{N}kn}$$

(2.5)

$$x(n+N) = x(n).$$

7 $X_3(k) = X_1(k) X_2(k)$ (1)

$$x_3(m) = \frac{1}{N} \sum_{k=0}^{N-1} X_3(k) e^{j\frac{2\pi}{N}km}$$ (2)

$0 \le m \le N-1$

$$= \frac{1}{N} \sum_{k=0}^{N-1} \sum_{n=0}^{N-1} x_1(n) e^{-j\frac{2\pi}{N}kn} \sum_{l=0}^{N-1} x_2(l) e^{-j\frac{2\pi}{N}kl} e^{j\frac{2\pi}{N}km}$$ (3)

$$= \frac{1}{N} \sum_{n=0}^{N-1} x_1(n) \sum_{l=0}^{N-1} x_2(l) \sum_{k=0}^{N-1} e^{j\frac{2\pi}{N}k(m-l-n)}$$

$$= \frac{1}{N} \sum_{n=0}^{N-1} x_1(n) x_2((m-n)_N) N.$$ (4)

8 $X_1(K) = (6, 0, 2, 0)$ (2)

$X_2(K) = (10, -2+2j, -2, -2-2j)$ (2)

$X_1(K) X_2(K) = (60, 0, -4, 0)$ (2)

$x_3(m) = (14, 16, 14, 16)$ (4)