CMR INSTITUTE OF TECHNOLOGY

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Internal Assesment Test-I

Sub:	Engineering Electromagnetics							Code:	17EC36
Date:	17/10/2018	Duration:	90 mins	Max Marks:	50	Sem:	3rd	Branch:	ECE(D)

Answer any **FIVE FULL** Questions. Mention **units** wherever necessary.

		Marks	OBE CO	OBE RBT
1.(a)	State and explain Coulomb's law in vector form.	[06]	CO1	L1
1.(b)	Two point charges of magnitude 2mC and -7mC are located at places P_1(4,7,-5) and P_2(3,2,-9) respectively in free space. Evaluate the vector force **F** on the charge at P_2 due to the charge placed at P1.	[04]	CO1	L3
2.(a)	Derive the expression for electric field intensity due to N number of charges in the region.	[05]	CO1	L3
2.(b)	A charge of -0.3µC is located at A (25,-30,15) and a second charge of 0.5µC at B(-10,8,12). Find **E** at the origin.	[05]	CO1	L3
3.(a)	Define electric flux density. Derive the relation between electric flux density and electric field intensity.	[06]	CO1	L1
3.(b)	Find electric field intensity and electric flux density at the origin due to Q = 0.35 µC at (0, 4, 0).	[04]	CO1	L3
4.	Define electric field intensity. Obtain an expression for electric field intensity due to an infinitely long uniform line charge distribution.	[02 + 08]	CO1	L1
5.	Find **E** at origin due to a point charge 12nC at (2, 0, 6) and a uniform line charge 3nC/m at x = -2, y = 3.	[10]	CO1	L3
6.	Determine electric flux density caused at P(6,8,-10) due to i) a point charge of 30 mC at origin. ii) an infinite line charge of $\rho_L = 40\ \mu C/m$ along Z-axis.	[10]	CO1	L3
7.(a)	State and explain Gauss's law as applied to an electric field. Derive the Maxwell's first equation of electrostatics.	[05]	CO1	L1
7.(b)	Obtain the expression for Gauss's Divergence theorem from Gauss's law.	[05]	CO1	L1
8.	Given $\mathbf{D} = 4xy\,\mathbf{a_x} + 2(x^2 + z^2)\mathbf{a_y} + 4yz\,\mathbf{a_z}\ C/m^2$. Evaluate total charge in the volume enclosed by $0 \le x \le 2,\ 0 \le y \le 3,\ 0 \le z \le 5$.	[10]	CO1	L3

SCHEME OF EVALUATION

		Mark Split-Up
1(a)	Statement	2
	Expression	4
(b)	Formula	2
	Approach & Answer	2
2(a)	Diagram	1
	Derivation	3
	Final Expression	1
(b)	Formula	2
	Approach & Answer	3
3(a)	Definition	1
	Derivation	3
	Final Expression	1
(b)	Formula	2
	Approach & Answer	3
4	Definition	2
	Derivation	7
	Final Expression	1
5	Formula	2+2
	Approach	2+2
	Answer	1+1
6	Formula	2+2
	Approach	2+2
	Answer	1+1
7(a)	Derivation	2
	Final Expression	3
(b)	Derivation	2
	Final Expression	3
8	Formula	2+2
	Approach	2+2
	Answer	1+1

electricity)

1a) i) Coulomb's law:

The force between two very small objects separated in vacuum or free space by a distance which is large compared to their size is proportional to the charge on each and inversely proportional to the square of the distance between them.

$$\boxed{F = K \frac{(Q_1 Q_2)}{R^2}}$$ → positive or negative quantities of charge

K → proportionality constant

R → separation

$$\boxed{K = \frac{1}{4\pi \varepsilon_0}}$$

ε_0 → Permittivity of free space.

$$\varepsilon_0 = \frac{10^{-9}}{36\pi} \ F/m.$$

$$\varepsilon_0 = 8.854 \times 10^{-12} \ F/m$$

$$K = \frac{1}{\frac{4\pi \times 10^{-9}}{36\pi}}$$

$$K = 9 \times 10^{9}$$

$$\boxed{F = \frac{Q_1 Q_2}{4\pi \varepsilon_0 R^2}}$$

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1 C → large unit of charge

charge of an electron = -1.602×10^{-19} C

charge of a proton = 1.602×10^{-19} C

$Q_1 = Q_2 = 1C$

$R = 1m$ $\boxed{F = 9 \times 10^{9}\ N}$

electron has the rest mass $= 9.109 \times 10^{-31}$ kg → doesnot mean e^- as spherical

& has radius $= 3.8 \times 10^{-15}$ m.

→ size of the region in which the steady moving e^- has the greatest probability of being found.

All other charged particle & proton → larger mass, larger radii → probabilistic volume → larger than electron.

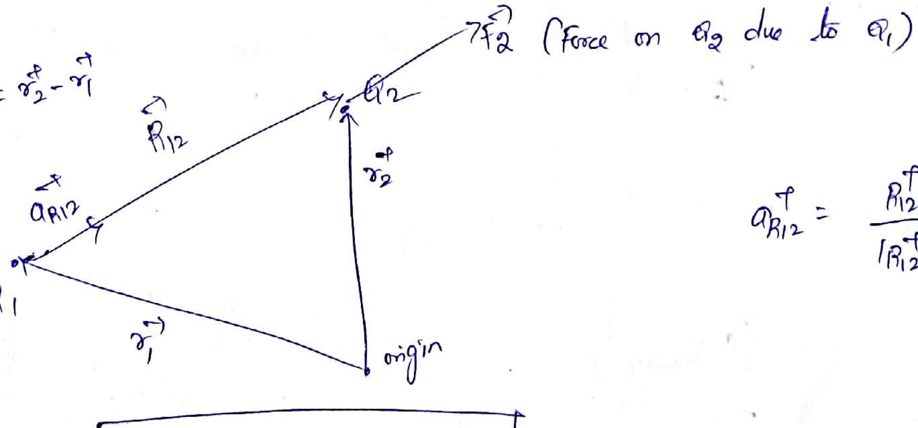
<u>Vector form:</u>

Force acts along line joining two charges.

Like charges → repulsive force

unlike charges → attractive force.

$\vec{R}_{12} = \vec{r}_2 - \vec{r}_1$



$$\vec{a}_{R_{12}} = \frac{\vec{R}_{12}}{|\vec{R}_{12}|}$$

$$\boxed{\vec{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 (R_{12})^2} \cdot \vec{a}_{R_{12}}}$$

$$\vec{F}_2 = \frac{Q_1 Q_2 \vec{R}_{12}}{4\pi\epsilon_0 |\vec{R}_{12}|^3}$$

Force expressed by Coulomb's law → mutual force (each of the charges experiences a force of same magnitude but opposite direction) (7)

$\vec{F_1} = -\vec{F_2} \Rightarrow$

$$\boxed{\vec{F_1} = \frac{Q_1 Q_2}{4\pi\epsilon_0 |\vec{R_{21}}|^2} \cdot \vec{a_{21}}}$$

$$\boxed{\vec{F_1} = -\frac{Q_1 Q_2}{4\pi\epsilon_0 |\vec{R_{12}}|^2} \cdot \vec{a_{12}}}$$

(1) Coulomb's law is linear → $\boxed{nQ_1 \Rightarrow n\vec{F_2}.}$

(2) Principle of superposition of charges:

Force on a charge in the presence of several other charges

↓

sum of the forces on that charge due to each of the other charges acting alone.

1b) Two point charges of magnitude 2 mC and −7 mC are located at places $P_1(4,7,-5)$ and $P_2(3,2,-9)$ respectively in free space. Evaluate the vector force on charge at P_2.

Soln.

$$\vec{R} = (3-4)\,\hat{a}_x + (2-7)\,\hat{a}_y + (-9+5)\,\hat{a}_z$$

$$= (-\hat{a}_x - 5\hat{a}_y - 4\hat{a}_z)$$

$$\therefore \vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2}\,\hat{a}_R$$

$$= \frac{2\times10^{-3} \times (-7)\times10^{-3}}{4\pi \times 8.854\times10^{-12}\,(1+25+16)^{3/2}} \cdot (-\hat{a}_x - 5\hat{a}_y - 4\hat{a}_z)$$

$$= \frac{-14\times10^{-6}}{4\pi\times8.854\times10^{-12}\,(42)^{3/2}}\,(-\hat{a}_x - 5\hat{a}_y - 4\hat{a}_z)$$

$$= .0004625 \times 10^{6}\,(-\hat{a}_x - 5\hat{a}_y - 4\hat{a}_z)\ \text{N}$$

$$= -462.5\,(\hat{a}_x + 5\hat{a}_y + 4\hat{a}_z)\ \text{N}.$$

2a) since

Coulomb's force is linear,

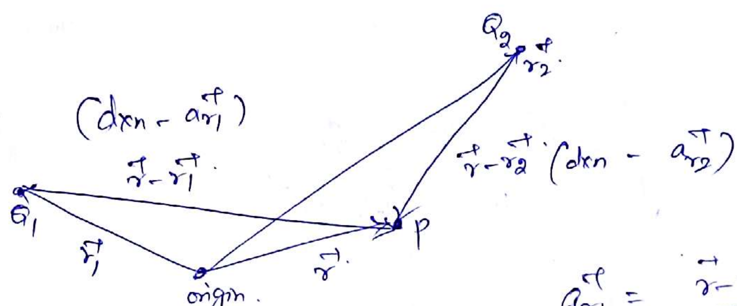
(8)

$\vec{E}$ due to two point charges, Q_1 at $\vec{r_1}$ and Q_2 at $\vec{r_2}$.

$\Uparrow$

Sum of forces on Q_t caused by Q_1 and Q_2 acting alone.

$$\vec{E}(r) = \frac{Q_1}{4\pi\epsilon_0 |\vec{r}-\vec{r_1}|^2} \cdot \vec{a_{r1}} + \frac{Q_2}{4\pi\epsilon_0 |\vec{r}-\vec{r_2}|^2} \cdot \vec{a_{r2}}.$$



$$\vec{a_{r1}} = \frac{\vec{r}-\vec{r_1}}{|\vec{r}-\vec{r_1}|} \quad \& \quad \vec{a_{r2}} = \frac{\vec{r}-\vec{r_2}}{|\vec{r}-\vec{r_2}|}.$$

'n' number of charges

$$\boxed{\vec{E}(r) = \sum_{m=1}^{n} \frac{Q_m}{4\pi\epsilon_0 |\vec{r}-\vec{r_m}|^2} \cdot \vec{a_{rm}}.}$$ V/m (or) N/C

2b)

1. Two point charges of magnitude 2 mC and −7 mC are located at places $P_1(4,7,-5)$ and $P_2(3,2,-9)$ respectively in free space. Evaluate the vector force on charge at P_2.

Soln.

$$\vec{R} = (3-4)\hat{a}_x + (2-7)\hat{a}_y + (-9+5)\hat{a}_z$$

$$= (-\hat{a}_x - 5\hat{a}_y - 4\hat{a}_z)$$

$$\therefore \vec{F} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2}\hat{a}_R$$

$$= \frac{2\times10^{-3} \times (-7)\times10^{-3}}{4\pi \times 8.854\times10^{-12} (1+25+16)^{3/2}} \cdot (-\hat{a}_x - 5\hat{a}_y - 4\hat{a}_z)$$

$$= \frac{-14\times10^{-6}}{4\pi\times8.854\times10^{-12}(42)^{3/2}}(-\hat{a}_x - 5\hat{a}_y - 4\hat{a}_z)$$

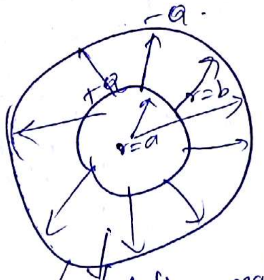
$$= .0004625\times10^{6}(-\hat{a}_x - 5\hat{a}_y - 4\hat{a}_z)\ N$$

$$= -462.5(\hat{a}_x + 5\hat{a}_y + 4\hat{a}_z)\ N.$$

3a)

Electric Flux Density, Gauss's law and Divergence.

1837, Director of Royal society in London → Michael Faraday.



Dielectric material.
→ displacement from inner sphere to outer sphere (independent of the medium).

→ This flux as displacement flux (or electric flux.

Larger positive charge on inner sphere → induce larger negative charge on outer sphere

Faraday's experiment.

(Electric flux). $\boxed{\Psi = Q}$. (Total charge on inner sphere)

surface area $4\pi a^2$.

⇒ Density of flux at the surface $= \frac{Q}{4\pi a^2}$ C/m^2

$Q = \Psi$ ← flux is generated at the surface

Electric flux density, $\vec{D}$:- Displacement flux density (or) Displacement density.

→ drn → direction of flux lines at a point

magnitude → no of flux lines crossing a surface normal to the lines divided by surface area.

(i) $\vec{D}\Big|_{r=a} = \frac{Q}{4\pi a^2}\,\vec{a}_r$ (inner sphere).

(ii) $\vec{D}\Big|_{r=b} = \frac{Q}{4\pi b^2}\,\vec{a}_r$ (outer sphere).

(iii) At $a<r<b$

$\vec{D} = \frac{Q}{4\pi r^2}\,\vec{a}_r$.

Inner sphere $\rightarrow$ $r \rightarrow 0$ (becomes a point charge). ← same charge 'Q'

Electric flux density at a point r metres

$$\boxed{\vec{D} = \frac{Q}{4\pi r^2}\,\vec{a_r}} \quad C/m^2$$

Q lines of flux are passing through surface area $4\pi r^2$ radially outward.

Radial electric field intensity,

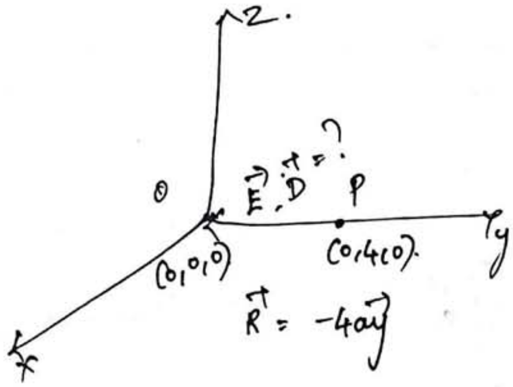
$$\boxed{\vec{E} = \frac{Q}{4\pi \epsilon_0 r^2}\,\vec{a_r}} \quad V/m$$

$\therefore$ $\boxed{\vec{D} = \epsilon_0 \vec{E}}$ (free space only)

not restricted to point charge.

Problem:

3.b) Find the electric field intensity and flux density at the origin due to $Q = 0.35\,\mu C$ at $(0,4,0)$.



$$\vec{E} = \frac{Q}{4\pi\varepsilon_0} \cdot \frac{(\vec{R})}{|\vec{R}|^3}$$

$$= 0.35 \times 10^{-6} \times 9 \times 10^{9} \times \frac{(-4\vec{a_y})}{(4)^3}$$

$$= 0.196875 \times 10^{3} (-\vec{a_y})$$

$$= -0.196875 \times 10^{3}\, \vec{a_y}$$

$$\boxed{\vec{E} = -196.875\, a_y} \; V/m$$

$$\vec{D} = \varepsilon_0 \vec{E}$$

$$= -1743.13 \times 10^{-12}\; c/m^2\; \vec{a_y}$$

$$= -1.743 \times 10^{-9}\; c/m^2\, \vec{a_y}$$

$$\boxed{\vec{D} = -1.743}\; nc/m^2\, \vec{a_y}$$

4)

Electric Field intensity:

One charge fixed in position (say) Q_1.

Move a second charge around, force everywhere around, on this second charge.

(ie) Q_2 is displaying the existence of force field.

Let $Q_2 = Q_t$ (test charge).

$$\boxed{\vec{F_t} = \frac{Q_1 Q_t}{4\pi\epsilon_0 |\vec{R_{1t}}|^2} \cdot \vec{a_{R_{1t}}}}$$

Force per unit test charge (ie) $Q_t = 1C$

(ie) $$\boxed{\frac{\vec{F_t}}{Q_t} = \frac{Q_1}{4\pi\epsilon_0 (\vec{R_{1t}})^2} \cdot \vec{a_{R_{1t}}}}$$ → vector field.

↓

Electric field intensity.

$\vec{E}$

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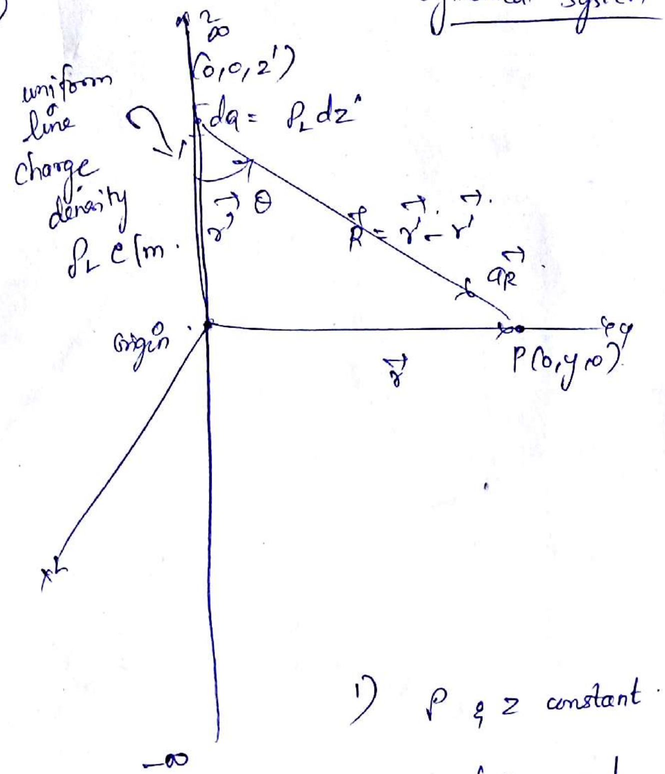
Cylindrical system



$\vec{E}$ at any & every point = ?

Symmetry is considered to determine two factors

1) with which co-ordinates the field does not vary

2) which components of the field are not present.

1) ρ & z constant.

ϕ is varied $\rightarrow$ line charge appears same from every angle.

Azimuthal symmetry.

2) ρ and ϕ constant

z is varied $\rightarrow$ $-\infty < z < \infty \Rightarrow$ problem is unchanged.

Axial symmetry.

3) ϕ and z constant

ρ is varied $\rightarrow$ problem changes

Coulomb's law $\rightarrow$ field becomes weaker as ρ increases.

$\boxed{\vec{E} \text{ varies only with } \rho}$

$\therefore$ $E_\phi = 0$

$E_z = 0$

E_ρ ✓.

Choose a point $P(0,y,0)$ to determine the field.

(where ϕ and z does not vary)

Incremental field at P due to incremental charge $dq = \rho_L dz'$.

$$d\vec{E} = \frac{\rho_L dz' (\vec{r} - \vec{r}')}{4\pi\epsilon_0 |\vec{r} - \vec{r}'|^3}.$$

$\vec{r} = y\vec{a_y}$

$\vec{r} = \rho\vec{a_\rho}$

$\vec{r}' = z\vec{a_z}$

$\vec{R} = \rho\vec{a_\rho} - z'\vec{a_z}$

$$\boxed{d\vec{E} = \frac{\rho_L dz' (\rho\vec{a_\rho} - z'\vec{a_z})}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}}$$

Only E_ρ component is present

$$\boxed{dE_\rho = \frac{\rho_L dz' \cdot \rho}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}}.$$

Integrating by change of variables, $z' = \rho \cot\theta$

$dz' = -\rho \operatorname{cosec}^2\theta \, d\theta$

$$E_\rho = -\int_{z=-\infty}^{\infty} \frac{\rho_L \rho^2 \operatorname{cosec}^2\theta \, d\theta}{4\pi\epsilon_0 (\rho^2 + \rho^2\cot^2\theta)^{3/2}}.$$

z	$-\infty$	∞
θ	π	0

$$= -\int_{z=-\infty}^{\infty} \frac{\rho_L \rho^2}{4\pi\epsilon_0 \rho^2 \cdot \rho} \cdot \frac{\operatorname{cosec}^2\theta \, d\theta}{(1+\cot^2\theta)^{3/2}}$$

$$= -\int_{\theta=\pi}^{0} \frac{\rho_L \operatorname{cosec}^2\theta \, d\theta}{4\pi\epsilon_0 \rho \operatorname{cosec}^3\theta}$$

$$= \frac{\rho_L}{4\pi\epsilon_0\rho} \cdot \left[\cos\theta\right]_{\pi}^{0}$$

$$E_\rho = \frac{\rho_L}{2\pi\epsilon_0\rho}$$

$$\boxed{\vec{E} = \frac{\rho_L}{2\pi\epsilon_0\rho}\vec{a_\rho}}.$$

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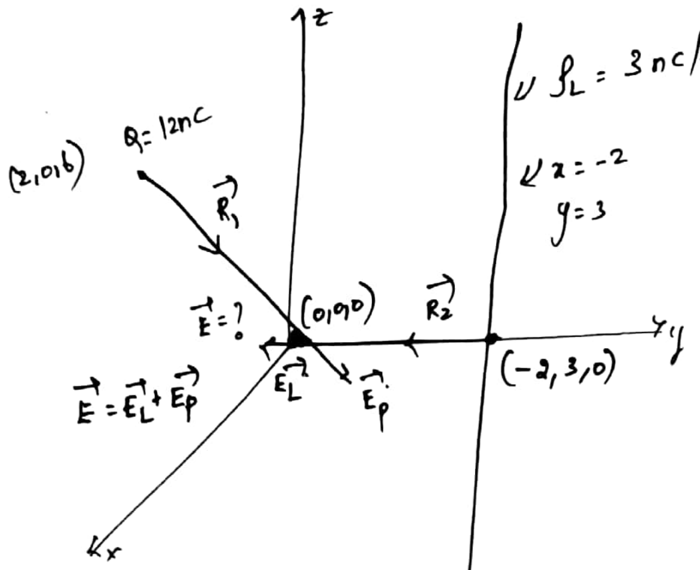
5) ② Find $\vec{E}$ at origin due to point charge 12 nC at (2,0,6) and uniform line charge 3 nC/m at $x=-2, y=3$.



$\vec{R_1} = -2\vec{a_x} - 6\vec{a_z}$

$|\vec{R_1}| = \sqrt{4+36} = \sqrt{40}$

$\rho = |\vec{R_2}|$

$\vec{a_\rho} = \vec{a_{R_2}} = \dfrac{\vec{R_2}}{|\vec{R_2}|}$

$\vec{R_2} = 2\vec{a_x} - 3\vec{a_y}$

$|\vec{R_2}| = \sqrt{4+9} = \sqrt{13}$

$\vec{E} = \vec{E_L} + \vec{E_p}$

$$\vec{E} = \frac{\rho_L}{2\pi\varepsilon_0 \rho}\vec{a_\rho} + \frac{Q}{4\pi\varepsilon_0 |\vec{R_1}|^3}\vec{R_1}$$

$$= \frac{3\times10^{-9} \times (2\vec{a_x} - 3\vec{a_y})}{2\pi\times\varepsilon_0\times\sqrt{13}\times\sqrt{13}} + \frac{12\times10^{-9}}{4\pi\times\varepsilon_0}\times\frac{(-2\vec{a_x} - 6\vec{a_z})}{(\sqrt{40})^3}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{3\times10^{-9}\times 2\,(2\vec{a_x} - 3\vec{a_y})}{13} + \frac{12\times10^{-9}(-2\vec{a_x} - 6\vec{a_z})}{(\sqrt{40})^3}\right]$$

$$= 9\times10^{9}\times 6\times10^{-9}\left[\frac{2}{13}\vec{a_x} - \frac{3}{13}\vec{a_y} - \frac{4}{(\sqrt{40})^3}\vec{a_x} - \frac{12}{(\sqrt{40})^3}\vec{a_z}\right]$$

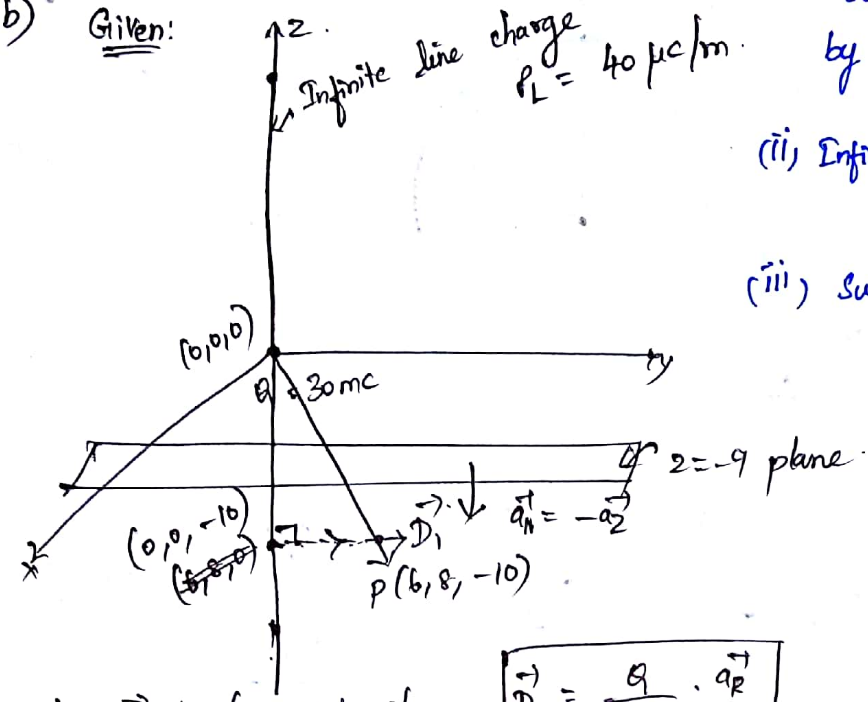
$$= 54\left[0.138\,\vec{a_x} - 0.230\,\vec{a_y} - 0.0474\,\vec{a_z}\right]$$

$$\boxed{\vec{E} = 7.452\,\vec{a_x} - 12.42\,\vec{a_y} - 2.5596\,\vec{a_z}}\ \text{V/m}$$

6)

1) b) Given:

① b) Determine electric flux density $\vec{D}$ caused at $P(6,8,-10)$ by (i) a point charge of 30 mC at origin
(ii) Infinite line charge $\rho_L = 40\ \mu C/m$ along z-axis
(iii) Surface charge with $\rho_s = 57.2\ \mu C/m^2$ on a plane $z=-9$ m.



i) $\vec{D}$ due to point charge $\boxed{\vec{D_1} = \frac{Q}{4\pi |\vec{R}|^2} \cdot \vec{a_R}}$

$\vec{R} = 6\vec{a_x} + 8\vec{a_y} - 10\vec{a_z}$

$|\vec{R}| = \sqrt{36+64+100} = 10\sqrt{2}$

$$= \frac{30\times10^{-3}}{4\pi} \times \frac{[6\vec{a_x} + 8\vec{a_y} - 10\vec{a_z}]}{(10\sqrt{2})^3}$$

$$\boxed{\vec{D} = 5.06\,\vec{a_x} + 6.75\,\vec{a_y} - 8.44\,\vec{a_z}}\ \mu C/m^2$$

(ii) $\vec{D}$ due to infinite line charge, $\boxed{\vec{D_2} = \frac{\rho_L}{2\pi\rho}\vec{a_\rho}}$

$\vec{R} = 6\vec{a_x} + 8\vec{a_y}$

$|\vec{R}| = \sqrt{6^2+8^2} = 10$

$$= \frac{40\times10^{-6}}{2\pi\times10} \times \frac{(6\vec{a_x} + 8\vec{a_y})}{10}$$

$$\boxed{\vec{D} = 0.381\,\vec{a_x} + 0.509\,\vec{a_y}}\ \mu C/m^2$$

(iii) $\vec{D}$ due to surface charge, $\vec{D_3} = \frac{\rho_s}{2}\vec{a_N}$

$$= \frac{57.2\times10^{-6}}{2}(-\vec{a_z})$$

$$\boxed{\vec{D_3} = -28.6\,\vec{a_z}}\ \mu C/m^2$$

7 a)

Divergence of $\vec{D}$ = $\text{div}\,\vec{D} = \lim_{\Delta v \to 0} \frac{\oint_S \vec{D}\cdot d\vec{s}}{\Delta v}$

Divergence of the vector flux density $\vec{A}$ is the outflow of flux from a small closed surface per unit volume as the volume shrinks to zero.

$\boxed{\text{div}\,\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}}$ Rectangular

$\boxed{\text{div}\,\vec{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}}$ cylindrical

$\boxed{\text{div}\,\vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, D_\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial \phi}}$ Spherical

$$\left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) = \frac{\oint_s \vec{D}\cdot d\vec{s}}{\Delta v} = \frac{Q}{\Delta v}.$$

$$\left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) = \lim_{\Delta v \to 0} \frac{\oint_s \vec{D}\cdot d\vec{s}}{\Delta v} = \left(\lim_{\Delta v \to 0} \frac{Q}{\Delta v}\right) \rightarrow \text{volume charge density}$$

$$\boxed{\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \lim_{\Delta v \to 0} \frac{\oint_s \vec{D}\cdot d\vec{s}}{\Delta v}} \rightarrow (1)$$

&

$$\boxed{\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho_v.} \rightarrow (2)$$

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(7.) MAXWELL's FIRST EQUATION OF ELECTROSTATICS:

1) $\operatorname{div} \vec{D} = \lim_{\Delta v \to 0} \dfrac{\oint_s \vec{D} \cdot d\vec{s}}{\Delta v}$ (Definition of divergence)

2) $\operatorname{div} \vec{D} = \dfrac{\partial D_x}{\partial x} + \dfrac{\partial D_y}{\partial y} + \dfrac{\partial D_z}{\partial z}$ (Result of applying the definition to a differential volume element (in Rectangular co-ordinates))

3) $\operatorname{div} \vec{D} = \rho_v$

Gauss's law flux leaving any closed surface

$$\boxed{\oint_s \vec{D} \cdot d\vec{s} = Q}$$ ← charge enclosed

Gauss' law per unit volume,

$$\frac{\oint_s \vec{D} \cdot d\vec{s}}{\Delta v} = \frac{Q}{\Delta v}$$

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As the volume shrinks to zero,

$$\lim_{\Delta v \to 0} \frac{\oint_S \vec{D}\cdot d\vec{s}}{\Delta v} = \lim_{\Delta v \to 0} \frac{Q}{\Delta v}$$

Divergence

volume charge density

(i) $$\boxed{\operatorname{div} \vec{D} = \rho_v.}$$

First of Maxwell's four equations

Point form of Gauss's law (or) Maxwell's first equation

Statement:

The electric flux per unit volume leaving a vanishingly small volume unit is exactly equal to the volume charge density there.

Gauss's law → $$\boxed{\oint_S \vec{D}\cdot d\vec{s} = Q = \int_V \rho_v \, dv.}$$

(ii) Integral form of Maxwell's first equation

7 b)

Divergence theorem: (For Electric flux density).

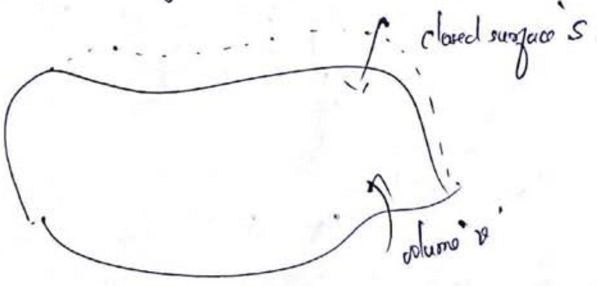
Gauss' law:

$$\oint_S \vec{D}\cdot d\vec{s} = Q = \int_V \rho_v \, dv.$$

$$\boxed{\oint_S \vec{D}\cdot d\vec{s} = \int_V \nabla\cdot\vec{D} \, dv.}$$ ← Divergence theorem.

Statement:

The integral of the normal component of any vector field over a closed surface is equal to the integral of the divergence of this vector field throughout the volume enclosed by the closed surface.



Divergence of flux density throughout a volume

=

net flux crossing the enclosing surface.

8. $D = 4xy\,\vec{a_x} + 2(x^2+z^2)\,\vec{a_y} + 4yz\,a_z$ c/m².

$0 \le x \le 2,\ 0 \le y \le 3,\ 0 \le z \le 5$

$$Q = \iiint \cancel{D \cdot dv}\ \vec{\nabla} \cdot \vec{D}\, dv$$

$$dv = dx\ dy\ dz$$

$$\vec{\nabla} \cdot \vec{D} = \frac{\partial}{\partial x}(4xy) + \frac{\partial}{\partial y}\left[2(x^2+z^2)\right] + \frac{\partial}{\partial z}(4yz)$$

$$\vec{\nabla} \cdot \vec{D} = 4y + 0 + 4y = 8y \ \text{c/m}^3$$

$$Q = \int_{x=0}^{2} \int_{y=0}^{3} \int_{z=0}^{5} 8y\ dx\ dy\ dz$$

$$Q = \int_{y=0}^{3} \int_{z=0}^{5} \left[8xy\right]_0^2\ dy\ dz$$

$$Q = \int_{y=0}^{3} \int_{z=0}^{5} (16y - 0)\ dy\ dz$$

$$= \int_{y=0}^{3} \int_{z=0}^{5} 16y\ dy\ dz$$

$$= \int_{x=0}^{5} \frac{16y^2}{2} \Big]_0^3 \, dx$$

$$= \int_{x=0}^{5} 8y^2 \Big]_0^3 \, dx = \int_0^5 \left[(8 \times 9) - (8 \times 0)\right] dx$$

$$= \int_0^5 72 \, dx = \boxed{\underline{360}}$$ c