For the channel diagram shown in fig Q1 calculate $H(X)$, $H(Y)$, $H(X,Y)$, $H(X|Y)$, $H(Y|X)$ and $I(X:Y)$ if $P(x_1)=p(x_2)=p(x_3)$

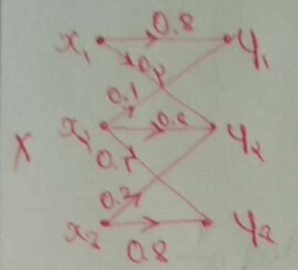

Fig Q1. Noise diagram

Sol^n The channel matrix

$$P(Y|X)=\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array}\begin{array}{c} \begin{matrix} y_1 & y_2 & y_3 \end{matrix} \\ \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.2 & 0.8 \end{bmatrix}\end{array}$$

$p(x_1)=P(x_2)=p(x_3)=\frac{1}{3}$

Joint Probability matrix

$$P(X,Y)=\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array}\begin{bmatrix} (0.8)/3 & (0.2)/3 & 0 \\ (0.1)/3 & (0.8)/3 & (0.1)/3 \\ 0 & (0.2)/3 & (0.8)/3 \end{bmatrix}$$

$P(y_1)=0.9/3 ; P(y_2)=1.2/3 ; P(y_3)=0.9/3$

i). $H(X)=\sum_{i=1}^{3} P(x_i)\log_2\frac{1}{p(x_i)}$

$=3\left(\frac{1}{3}\log_2 3\right)$

$=1.5849$ bits/sym

(ii) $H(Y)=\sum_{j=1}^{3} p(y_j)\log_2\frac{1}{p(y_j)}$

$=2\left(0.9/3\log_2\frac{3}{0.9}\right)+\frac{1.2}{3}\log_2\frac{3}{1.2}$

$=1.5709$ bits/sym

(iii) $H(X,Y)=\sum_{i=1}^{3}\sum_{j=1}^{3} p(x_i,y_j)\log_2\frac{1}{p(x_i,y_j)}$

$=2\left(\frac{0.8}{3}\log_2\frac{3}{0.8}\right)+2\left(\frac{0.2}{3}\log_2\frac{3}{0.2}\right)$

$+2\left(\frac{0.1}{3}\log_2\frac{3}{0.1}\right)$

$=2.374$ bits/sym

(iv) $H(Y|X)=H(X,Y)-H(X)$

$=2.374-1.5849$

$=0.7891$ bits/sym

(v) $H(X|Y)=H(X,Y)-H(Y)$

$=2.374-1.5709$

$=0.8031$ bits/sym

(vi) $I(X;Y)=H(X)-H(X|Y)$

$=1.5849-0.8031$

$=0.7818$ bits/sym

Derive an expression for the channel capacity of binary symmetric channel.

The case is as shown below

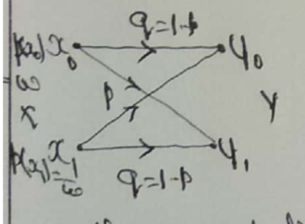

fig. Channel diagram

Channel matrix

$$P(Y|X)=\begin{bmatrix} q & p \\ p & q \end{bmatrix}$$

The joint probability matrix

$$P(X,Y)=\begin{bmatrix} \omega q & \omega p \\ \bar{\omega} p & \bar{\omega} q \end{bmatrix}$$

$\therefore P(y_0)=\omega q+\bar{\omega}p$

$P(y_1)=\omega p+\bar{\omega}q$

W.K.T. $C=\max\{I(X;Y)\}r_s$ bits/sec

$I(X:Y)=H(Y)-H(Y|X)$

$H(Y)=\sum_{j=0}^{1} P(y_j)\log_2\frac{1}{p(y_j)}$

$=(\omega q+\bar{\omega}p)\log_2\frac{1}{(\omega q+\bar{\omega}p)}+(\omega p+\bar{\omega}q)\log_2\frac{1}{(\omega p+\bar{\omega}q)}$

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~~Joint entropy~~

Conditional entropy $H(Y|X)$ is

$$H(Y|X) = \sum_{j=0}^{1}\sum_{i=0}^{1} p(x_i, y_j) \log_2 \frac{1}{p(y_j|x_i)}$$

$$= \omega q \log_2 \frac{1}{q} + \bar{\omega} q \log_2 \frac{1}{q} + \omega p \log_2 \frac{1}{p} + \bar{\omega} p \log_2 \frac{1}{p}$$

$$= (\omega + \bar{\omega}) q \log_2 \frac{1}{q} + (\omega + \bar{\omega}) p \log_2 \frac{1}{p}$$

$$= p \log_2 \frac{1}{p} + q \log_2 \frac{1}{q}$$

$$\therefore I(X;Y) = \left[(\omega q + \bar{\omega} p) \log_2 \frac{1}{(\omega q + \bar{\omega} p)} + (\omega p + \bar{\omega} q) \log_2 \frac{1}{(\omega p + \bar{\omega} q)}\right] - \left[p \log_2 \frac{1}{p} + q \log_2 \frac{1}{q}\right]$$

$\max\{I(X;Y)\} \Rightarrow \omega = \bar{\omega} = \frac{1}{2}$

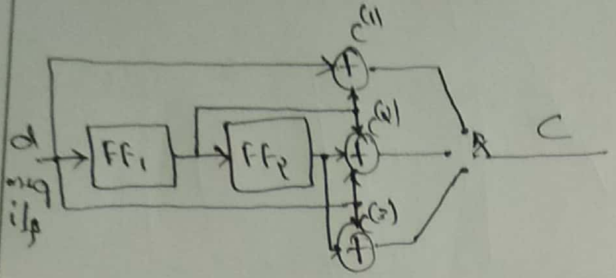
$$\therefore \max I(X;Y) = \left\{\frac{1}{2}\log_2 2 + \frac{1}{2}\log_2 2\right\} - p\log_2 1/p + q\log_2 1/q$$

$$I(X;Y) = 1 - (p\log_2 1/p + q\log_2 1/q) \text{ bits/sym}$$

$$\therefore C = [1 - (p\log_2 1/p + q\log_2 1/q)] r_s \text{ bits/sec}$$

4. Consider the (3,1,2) Convolutional code with $g^{(1)} = (110)$, $g^{(2)} = (111)$, $g^{(3)} = (101)$

(a) Convolution Encoder Circuit



Encoding Circuit.

(b) Obtain the generator matrix

$$[G]_{L\times n(L+m)} = \begin{bmatrix} g_1^{(1)} g_1^{(2)} g_1^{(3)} & g_2^{(1)} g_2^{(2)} g_2^{(3)} & g_3^{(1)} g_3^{(2)} g_3^{(3)} & 000 \\ 000 & g_1^{(1)} g_1^{(2)} g_1^{(3)} & g_2^{(1)} g_2^{(2)} g_2^{(3)} & g_3^{(1)} g_3^{(2)} g_3^{(3)} \\ & & & \end{bmatrix}$$

5. Encode d = 1011 using time domain approach

Given d = 1011 ∴ L = 4 bits

o/p bits = $n(L+m) = 3(4+2) = 12$ bits

$C = [d] * [G]$

$$[C] = [1011]\begin{bmatrix} 111 & 110 & 011 & 000 & 000 & 000 \\ 000 & 111 & 110 & 011 & 000 & 000 \\ 000 & 000 & 111 & 110 & 011 & 000 \\ 000 & 000 & 000 & 111 & 110 & 011 \end{bmatrix}$$

$= [111 \quad 110 \quad 100 \quad 001 \quad 101 \quad 011]$

Transform domain approach.

$c(x) = c^{(1)}(x^n) + x\, c^{(2)}(x^n) + x^2 c^{(3)}(x^n)$

$d(x) = 1 + x^2 + x^3$

$g^{(1)}(x) = 1 + x$

$g^{(2)}(x) = 1 + x + x^2$

$g^{(3)}(x) = 1 + x^2$

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CMR

Output from top adder

$c^{(1)}(x) = d(x) * g^{(1)}(x)$

$= (1+x^2+x^3) * (1+x)$

$= 1 + x^2 + x^3$
$\quad x + x^3 + x^4$

$= 1 + x + x^2 + x^4$

$\therefore c^{(1)}(x^3) = c^{(1)}(x^3)$

$c^{(1)}(x^3) = 1 + x^3 + x^6 + x^{12}$

Output from middle adder

$c^{(2)}(x) = d(x) * g^{(2)}(x)$

$= (1+x^2+x^3)(1+x+x^2)$

$= 1 + x^2 + x^3$
$\quad + x + x^3 + x^4$
$\quad + x^2 + x^4 + x^5$

$= 1 + x + x^5$

$\therefore c^{(2)}(x^3) = 1 + x^3 + x^{15}$

$x\,c^{(2)}(x^3) = x + x^4 + x^{16}$

Output from bottom adder

$c^{(3)}(x) = d(x) * g^{(3)}(x)$

$= (1+x^2+x^3)(1+x^2)$

$= 1 + x^2 + x^3$
$\quad + x^2 + x^4 + x^5$

$= 1 + x^3 + x^4 + x^5$

$\therefore c^{(3)}(x^3) = 1 + x^9 + x^{12} + x^{15}$

$x^2 c^{(3)}(x^3) = x^2 + x^{11} + x^{14} + x^{17}$

$\therefore C(x) = 1 + x + x^2 + x^3 + x^4 + x^6 + x^{11} + x^{12} + x^{14} + x^{16} + x^{17}$

c = [111 110 100 001 101 011]

Consider the binary Convolution Encoder shown in fig Q6.

message $D = 101$ using code tree.

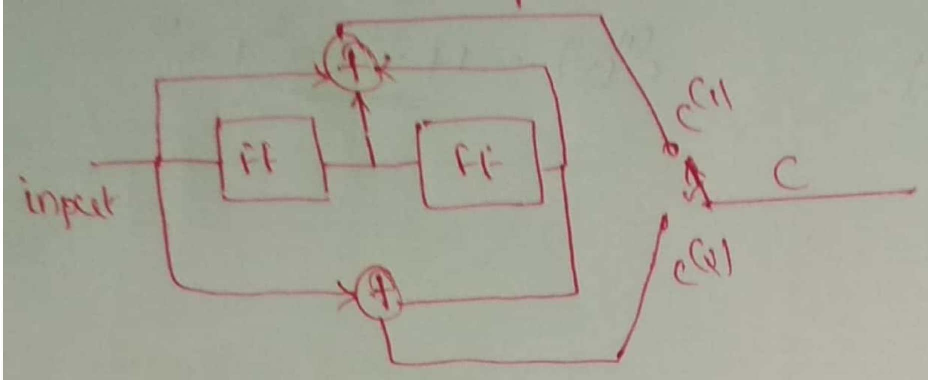


fig Q6. (2,1,2) Convolutional Encoder

No of states possible $= 2^m = 2^2 = 4$ states

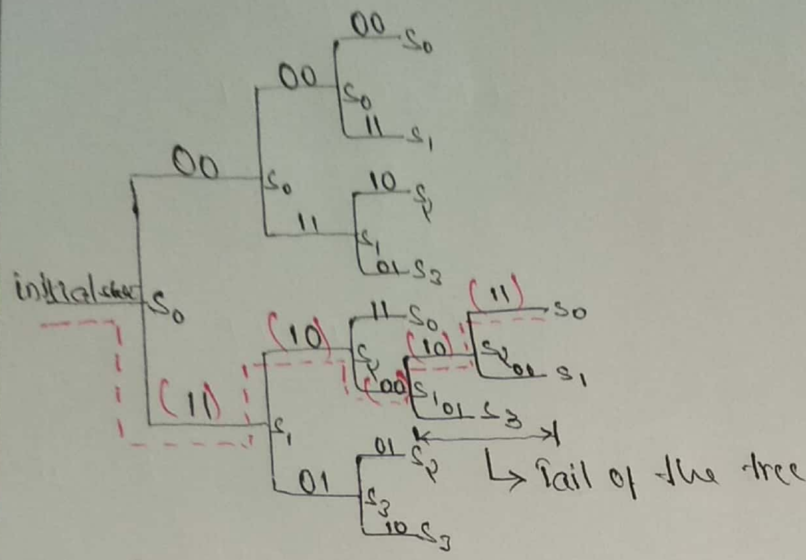
state table:

state	S_0	S_1	S_2	S_3
BD	00	10	01	11

State transition table

PS	BD	i/p	NS	BD	d_l	d_{l-1}	d_{l-2}	$C^{(1)}$	$C^{(2)}$
S_0	00	0	S_0	00	0	0	0	0	0
		1	S_1	10	1	0	0	1	1
S_1	10	0	S_2	01	0	1	0	1	0
		1	S_3	11	1	1	0	0	1
S_2	01	0	S_0	00	0	0	1	1	1
		1	S_1	10	1	0	1	0	0
S_3	11	0	S_2	01	0	1	1	0	1
		1	S_2	11	1	1	1	1	0

Code tree



Msg → 1 0 1 – –

Encoded sequence → [11 10 00 10 11]

7. Find the capacity of the discrete channel shown in fig Q7 using Muroga's theorem.

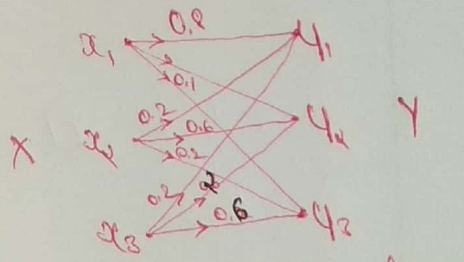


fig. Q7 Noise diagram

Sol^n: Channel capacity $C = \sum_{i=1}^{3} \log(2^{-Q_i})$ bits/sec

$$\begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.2 & 0.2 & 0.6 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} 0.8 \log_2 \frac{1}{0.8} + 2(0.1 \log_2 \frac{1}{0.1}) \\ 2(0.2 \log_2 \frac{1}{0.2}) + 0.6 \log_2 \frac{1}{0.6} \\ 2(0.2 \log_2 \frac{1}{0.2}) + 0.6 \log_2 \frac{1}{0.6} \end{bmatrix}$$

$0.8Q_1 + 0.1Q_2 + 0.1Q_3 = 0.9219$

$0.2Q_1 + 0.6Q_2 + 0.2Q_3 = 1.371$ $\Rightarrow Q_1 = 0.7722$; $Q_2 = 1.5207$; $Q_3 = 1.5207$

$0.2Q_1 + 0.2Q_2 + 0.6Q_3 = 1.371$

$\therefore C = \log_2 \left[2^{-0.7722} + 2^{-1.5207} + 2^{-1.5207} \right]$

$= \underline{0.359}$ bits/sec