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Internal Assesment Test – II

Sub:	Information Theory and Coding			Sec	All			Code:	15EC54
Date:	17/ 10 /18	Duration:	90 mins	Max Marks:	50	Sem:	V	Branch:	ECE/TCE

ANSWER ANY FIVE FULL QUESTIONS — MARKS — OBE: CO, RBT

		MARKS	CO	RBT
1	For the channel diagram given in Fig. Q1, calculate $H(X)$, $H(Y)$, $H(X,Y)$, $H(X\|Y)$, $H(Y\|X)$ and $I(X,Y)$. (Assume $p(x_1) = p(x_2) = p(x_3)$)	[10]	C504.3	L3

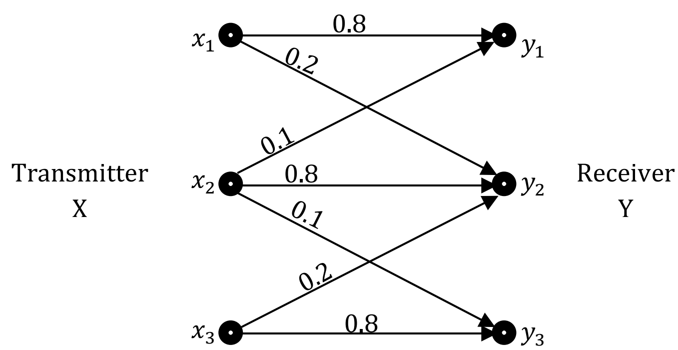

Fig. Q1. Channel diagram

		MARKS	CO	RBT
2	Derive an expression for the channel capacity of a binary symmetric channel.	[10]	C504.3	L3
3	Show that Mutual information is always non negative. Prove that $I(X;Y) = H(Y) - H(Y\|X)$	[10]	C504.3	L1
4	Consider the (3, 1, 2) convolutional code with $g^{(1)} = (110)$, $g^{(2)} = (111)\ and\ g^{(3)} = (101)$. Draw the encoder circuit. Obtain the generator Matrix.	[10]	C504.5	L2
5	Consider the (3, 1, 2) convolutional code with $g^{(1)} = (110)$, $g^{(2)} = (111)$ and $g^{(3)} = (101)$, Encode the message $d = 1011$ using, Time domain approach and Transform domain approach.	[10]	C504.5	L2
6	Consider the binary convolution encoder shown in Fig. Q6. Encode the message D=101 using code tree.	[10]	C504.5	L3

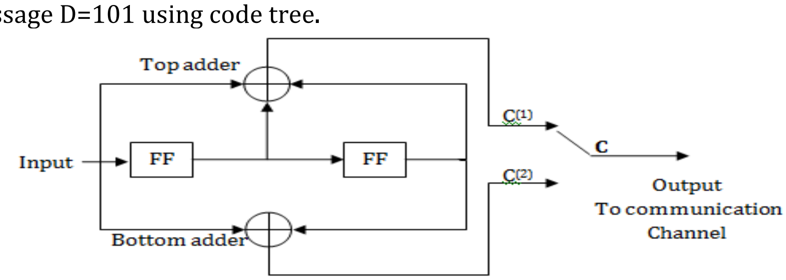

Fig. Q6. Convolutional Encoder

		MARKS	CO	RBT
7	Find the capacity of the discrete channel shown in Fig. Q7 using Muroga's theorem.			

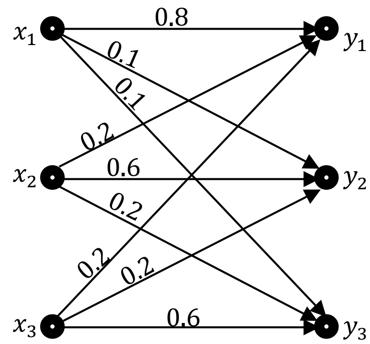

Fig. Q7. Channel diagram

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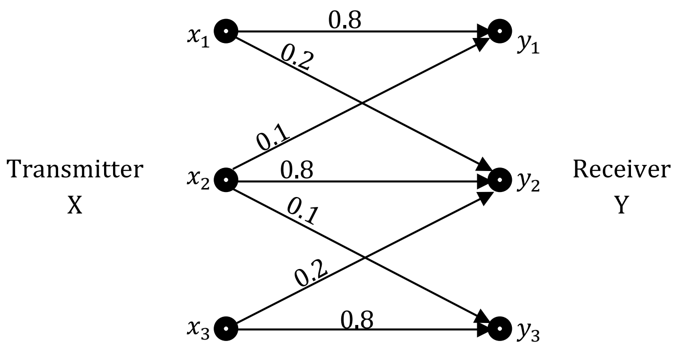


Fig. Q1. Channel diagram

$H(X) = 1.5849\ bits/sym$ 1
$H(Y) = 1.5709\ bits/sym$ 1
$H(X,Y) = 2.374\ bits/sym$ 2
$H(Y|X) = 0.7891\ bits/sym$ 2
$H(X|Y) = 0.8031\ bits/sym$ 2
$I(X;Y) = 0.7818\ bits/sym$ 2

		MARKS	CO	RBT
2	Derive an expression for the channel capacity of a binary symmetric channel.	[10]	C504.3	L3

Derivation for C 10

$$C = \left(1 - \left(plog_2\frac{1}{p} + qlog_2\frac{1}{q}\right)\right) \text{bits/sec}$$

		MARKS	CO	RBT
3	Show that Mutual information is always non negative. Prove that $I(X;Y) = H(Y) - H(Y\|X)$	[10]	C504.3	L1

Proof for $\boldsymbol{I(X;Y) \geq 0}$ 5
Proof for $\boldsymbol{I(X;Y) = H(Y) - H(Y|X)}$ 5

		MARKS	CO	RBT
4	Consider the (3, 1, 2) convolutional code with $g^{(1)} = (110)$, $g^{(2)} = (111)$ and $g^{(3)} = (101)$. Draw the encoder circuit. Obtain the generator Matrix.	[10]	C504.5	L2

Encoder circuit 5
Generator matrix $[\boldsymbol{G}]_{\boldsymbol{LXn(L+m)}}$ 5

		MARKS	CO	RBT
5	Consider the (3, 1, 2) convolutional code with $g^{(1)} = (110)$, $g^{(2)} = (111)$ and $g^{(3)} = (101)$, Encode the message $d = 1011$ using, Time domain approach and Transform domain approach.	[10]	C504.5	L2

Obtaining $\boldsymbol{C} = [\mathbf{111\ 110\ 100\ 001\ 101\ 011}]$ 5

Obtaining $\boldsymbol{C(x)} = \mathbf{1} + \boldsymbol{x} + \boldsymbol{x^2} + \boldsymbol{x^3} + \boldsymbol{x^4} + \boldsymbol{x^6} + \boldsymbol{x^{11}} + \boldsymbol{x^{12}} + \boldsymbol{x^{14}} + \boldsymbol{x^{16}} + \boldsymbol{x^{17}}$ 5

6 Consider the binary convolution encoder shown in Fig. Q6. Encode the message D=101 using code tree. [10] C504.5 L3

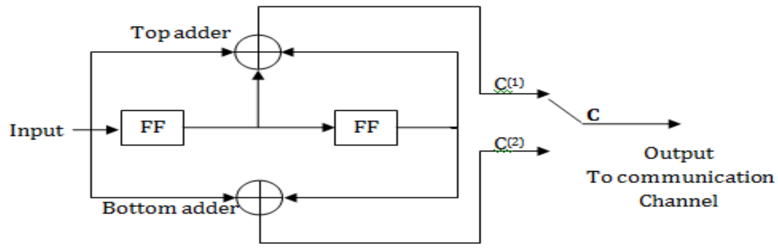


Fig. Q6. Convolutional Encoder

Drawing the code tree using either state transition table or state diagram 5

Obtaining $\boldsymbol{C} = [\mathbf{11\ 10\ 00\ 10\ 11}]$ 5

7 Find the capacity of the discrete channel shown in Fig. Q7 using Muroga's theorem. [10] C504.3 L1

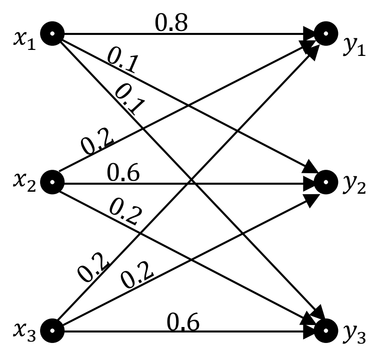


Fig. Q7. Channel diagram

Channel capacity using Muroga's theorem is given by 1

$$\boldsymbol{C} = \sum_{i=1}^{m} \log_2(2^{-Q_i})\ \boldsymbol{bits/sec}$$

Obtaining $\boldsymbol{C} = \mathbf{0.359}\ \boldsymbol{bits/sec}$ 9

For the channel diagram shown in fig Q1 calculate $H(X)$, $H(Y)$, $H(X,Y)$, $H(X|Y)$, $H(Y|X)$ and $I(X:Y)$ if $P(x_1) = p(x_2) = p(x_3)$

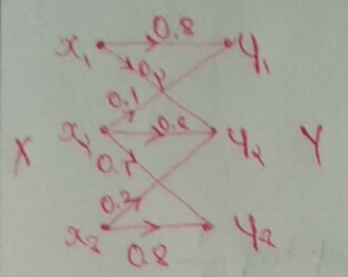


Fig Q1. Noise diagram

The channel matrix

Sol^n
$$P(Y|X) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array}\begin{array}{c} \begin{array}{ccc} y_1 & y_2 & y_3 \end{array} \\ \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.1 & 0.8 & 0.1 \\ 0 & 0.2 & 0.8 \end{bmatrix} \end{array}$$

$$p(x_1) = p(x_2) = p(x_3) = \frac{1}{3}$$

Joint Probability matrix

$$P(X,Y) = \begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array}\begin{bmatrix} (0.8)/3 & (0.2)/3 & 0 \\ (0.1)/3 & (0.8)/3 & (0.1)/3 \\ 0 & (0.2)/3 & (0.8)/3 \end{bmatrix}$$

$P(y_1) = 0.9/3 ; P(y_2) = 1.2/3 ; P(y_3) = 0.9/3$

i). $H(X) = \sum_{i=1}^{3} P(x_i) \log_2 \frac{1}{p(x_i)}$

$= 3\left(\frac{1}{3} \log_2 3\right)$

$= 1.5849$ bits/sym

(ii) $H(Y) = \sum_{j=1}^{3} p(y_j) \log_2 \frac{1}{p(y_j)}$

$= 2\left(0.9/3 \log_2 \frac{3}{0.9}\right) + \frac{1.2}{3} \log_2 \frac{3}{1.2}$

$= 1.5709$ bits/sym

(iii) $H(X,Y) = \sum_{i=1}^{3}\sum_{j=1}^{3} p(x_i, y_j) \log_2 \frac{1}{p(x_i,y_j)}$

$= 2\left(\frac{0.8}{3}\log_2 \frac{3}{0.8}\right) + 2\left(\frac{0.2}{3} \log_2 \frac{3}{0.2}\right)$

$+ 2\left(\frac{0.1}{3} \log_2 \frac{3}{0.1}\right)$

$= 2.374$ bits/sym

(iv) $H(Y|X) = H(X,Y) - H(X)$

$= 2.374 - 1.5849$

$= 0.7891$ bits/sym

(v) $H(X|Y) = H(X,Y) - H(Y)$

$= 2.374 - 1.5709$

$= 0.8031$ bits/sym

(vi) $I(X;Y) = H(X) - H(X|Y)$

$= 1.5849 - 0.8031$

$= 0.7818$ bits/sym

Derive an expression for the channel capacity of binary symmetric channel.

The BSC is as shown below

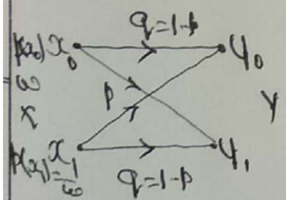


fig. Channel diagram

Channel matrix

$$P(Y|X) = \begin{bmatrix} q & p \\ p & q \end{bmatrix}$$

The joint probability matrix

$$P(X,Y) = \begin{bmatrix} \omega q & \omega p \\ \bar{\omega} p & \bar{\omega} q \end{bmatrix}$$

$\therefore p(y_0) = \omega q + \bar{\omega} p$

$p(y_1) = \omega p + \bar{\omega} q$

W.K.T. $C = \max \{I(X;Y)\} r_s$ bits/sec

$I(X:Y) = H(Y) - H(Y|X)$

$H(Y) = \sum_{j=0}^{1} p(y_j) \log_2 \frac{1}{p(y_j)}$

$= (\omega q + \bar{\omega} p) \log_2 \frac{1}{(\omega q + \bar{\omega} p)} + (\omega p + \bar{\omega} q) \log_2 \frac{1}{(\omega p + \bar{\omega} q)}$

~~Joint entropy~~

Conditional entropy $H(Y|X)$ is

$$H(Y|X) = \sum_{j=0}^{1}\sum_{i=0}^{1} p(x_i, y_j) \log_2 \frac{1}{p(y_j|x_i)}$$

$$= \omega q \log_2 \frac{1}{q} + \bar{\omega} q \log_2 \frac{1}{q} + \omega p \log_2 \frac{1}{p} + \bar{\omega} p \log_2 \frac{1}{p}$$

$$= \cancel{(\omega+\bar{\omega})}^{1} q \log_2 \frac{1}{q} + \cancel{(\omega+\bar{\omega})}^{1} p \log_2 \frac{1}{p}$$

$$= p \log_2 \frac{1}{p} + q \log_2 \frac{1}{q}$$

$$\therefore I(X;Y) = \left[(\omega q+\bar{\omega} p) \log_2 \frac{1}{(\omega q+\bar{\omega} p)} + (\omega p+\bar{\omega} q) \log_2 \frac{1}{(\omega p+\bar{\omega} q)}\right] - \left[p \log_2 \frac{1}{p} + q \log_2 \frac{1}{q}\right]$$

$\max\{I(X;Y)\} \Rightarrow \omega = \bar{\omega} = \frac{1}{2}$

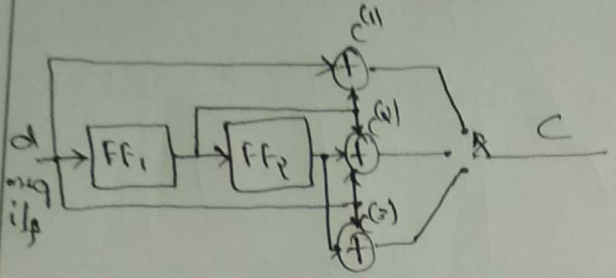
$$\therefore \max I(X:Y) = \left\{\frac{1}{2}\log_2 2 + \frac{1}{2}\log_2 2\right\} - p\log_2 1/p + q \log_2 1/q$$

$$I(X;Y) = 1 - (p\log_2 1/p + q\log_2 1/q) \text{ bits/sym}$$

$$\therefore C = [1-(p\log_2 1/p + q\log_2 1/q)]\, r_s \text{ bits/sec}$$

4. Consider the (3,1,2) Convolutional Code with $g^{(1)} = (110)$, $g^{(2)} = (111)$ $g^{(3)} = (101)$

(a) Convolution Encoder Circuit



Encoding Circuit.

(b) Obtain the generator matrix

$$[G]_{L\times n(L+m)} = \begin{bmatrix} g_1^{(1)} g_1^{(2)} g_1^{(3)} & g_2^{(1)} g_2^{(2)} g_2^{(3)} & g_3^{(1)} g_3^{(2)} g_3^{(3)} & 000 \\ 000 & g_1^{(1)} g_1^{(2)} g_1^{(3)} & g_2^{(1)} g_2^{(2)} g_2^{(3)} & g_3^{(1)} g_3^{(2)} g_3^{(3)} \\ & & & \end{bmatrix}$$

5. Encode $d = 1011$ using time domain approach

Given $d = 1011$ $\therefore L = 4$ bits

o/p bits $= n(L+m) = 3(4+2) = 12$ bits

$C = [d] * [G]$

$$[C] = [1011] \begin{bmatrix} 111 & 110 & 011 & 000 & 000 & 000 \\ 000 & 111 & 110 & 011 & 000 & 000 \\ 000 & 000 & 111 & 110 & 011 & 000 \\ 000 & 000 & 000 & 111 & 110 & 011 \end{bmatrix}$$

$= [111 \quad 110 \quad 100 \quad 001 \quad 101 \quad 011]$

Transform domain approach.

$c(x) = c^{(1)}(x^n) + x\, c^{(2)}(x^n) + x^2 c^{(3)}(x^n)$

$d(x) = 1 + x^2 + x^3$

$g^{(1)}(x) = 1 + x$

$g^{(2)}(x) = 1 + x + x^2$

$g^{(3)}(x) = 1 + x^2$

Output from top adder

$c^{(1)}(x) = d(x) * g^{(1)}(x)$

$= (1+x^2+x^3) * (1+x)$

$= 1 + x^2 + \cancel{x^3}$

$\quad x + \quad + \cancel{x^3} + x^4$

$= 1 + x + x^2 + x^4$

$\therefore c^{(1)}(x^3) = c^{(1)}(x^3)$

$c^{(1)}(x^3) = 1 + x^3 + x^6 + x^{12}$

Output from middle adder

$c^{(2)}(x) = d(x) * g^{(2)}(x)$

$= (1+x^2+x^3)(1+x+x^2)$

$= 1 + \quad + \cancel{x^2} + \cancel{x^3}$

$\quad + x + \quad + \cancel{x^2} + \cancel{x^4}$

$\quad + \cancel{x^2} + \quad + \cancel{x^4} + x^5$

$= 1 + x + x^5$

$\therefore c^{(2)}(x^3) = x + x^3 + x^{15}$

$x c^{(2)}(x^3) = x^2 + x^4 + x^{16}$

Output from bottom adder

$c^{(3)}(x) = d(x) * g^{(3)}(x)$

$= (1+x^2+x^3)(1+x^2)$

$= 1 + \cancel{x^2} + x^3$

$\quad + \cancel{x^2} + \quad + x^4 + x^5$

$= 1 + x^3 + x^4 + x^5$

$\therefore c^{(3)}(x^3) = 1 + x^9 + x^{12} + x^{15}$

$x^2 c^{(3)}(x^3) = x^2 + x^{11} + x^{14} + x^{17}$

$\therefore C(x) = 1 + x + x^2 + x^3 + x^4 + x^6 + x^{11} + x^{12} + x^{14} + x^{16} + x^{17}$

c = [111 110 100 001 101 011]

Consider the binary convolution Encoder shown in fig Q6.

message D=101 using code tree.

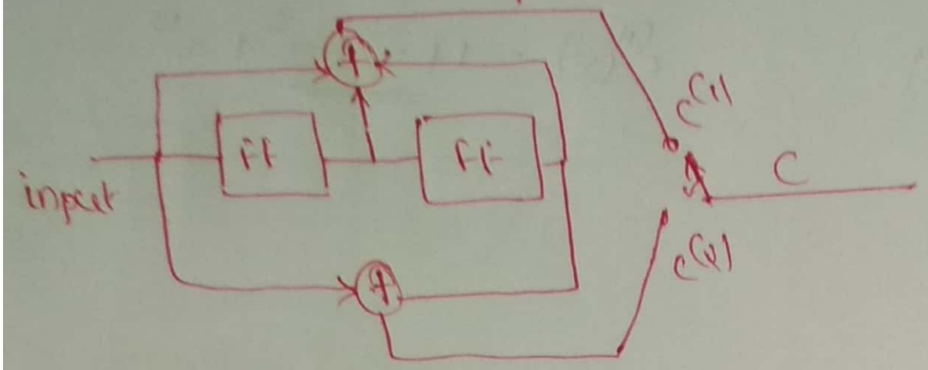


fig Q6. (2,1,2) Convolutional Encoder

No of states possible $= 2^m = 2^2 = 4$ states

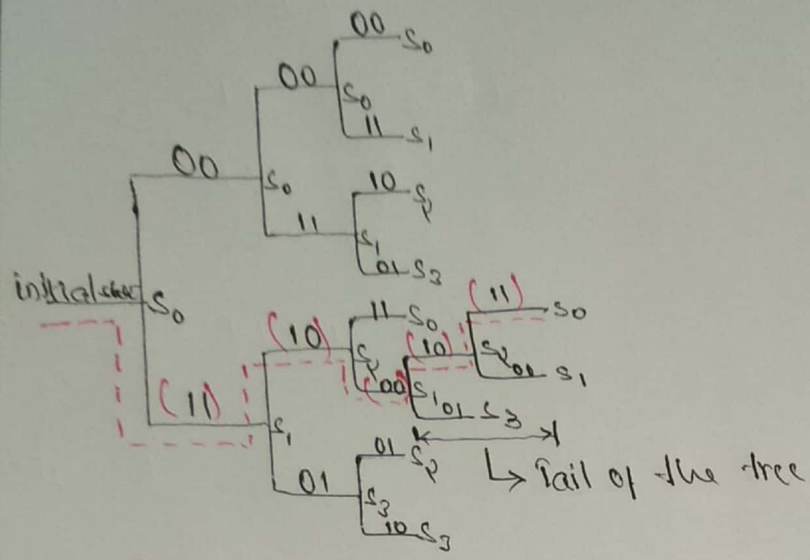
state table:

state	S_0	S_1	S_2	S_3
BD	00	10	01	11

State transition table

PS	BD	i/p	NS	BD	d_1	$d_{1,1}$	$d_{1,2}$	$c^{(1)}$	$c^{(2)}$
S_0	00	0	S_0	00	0	0	0	0	0
		1	S_1	10	1	0	0	1	1
S_1	10	0	S_2	01	0	1	0	1	0
		1	S_3	11	1	1	0	0	1
S_2	01	0	S_0	00	0	0	1	1	1
		1	S_1	10	1	0	1	0	0
S_3	11	0	S_2	01	0	1	1	0	1
		1	S_2	11	1	1	1	1	0

Code tree



Msg $\longrightarrow$	1	0	1	–	–
Encoded sequence $\longrightarrow$	[11	10	00	10	11]

7. Find the capacity of the discrete channel shown in fig Q7 using Muroga's theorem.

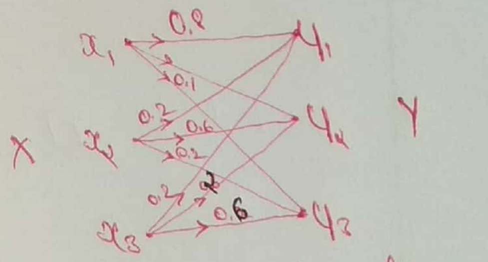


fig. Q7 Noise diagram

Sol^n: Channel capacity $C = \sum_{i=1}^{3} \log(2^{-Q_i})$ bits/sec

$$\begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.2 \\ 0.2 & 0.2 & 0.6 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} 0.8 \log_2 1/0.8 + 2(0.1 \log_2 1/0.1) \\ 2(0.2 \log_2 1/0.2) + 0.6 \log_2 1/0.6 \\ 2(0.2 \log_2 1/0.2) + 0.6 \log_2 1/0.6 \end{bmatrix}$$

$0.8Q_1 + 0.1Q_2 + 0.1Q_3 = 0.9219$

$0.2Q_1 + 0.6Q_2 + 0.2Q_3 = 1.371$ $\Rightarrow Q_1 = 0.7722$; $Q_2 = 1.5207$; $Q_3 = 1.5207$

$0.2Q_1 + 0.2Q_2 + 0.6Q_3 = 1.371$

$\therefore C = \log_2 [2^{-0.7722} + 2^{-1.5207} + 2^{-1.5207}]$

$= \underline{0.359}$ bits/sec