CMR
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USN										

Internal Assesment Test – II

Sub:	Digitsl Image Processing							Code:	15EC72
Date:	/ 10 / 2018	Duration:	90 mins	Max Marks:	50	Sem:	VII	Branch:	ECE: A,B,C & D

Answer any 5 questions.

1. Discuss all the noise probability density functions.
2. Write a note on smoothing and sharpening filters for image enhancement.
3. Derive an expression of the linear degradation model in presence of additive noise.
4. What are the methods of estimating the degradation function? Explain each of them.
5. Explain Weiner filtering and inverse filtering in image processing.
6. Discuss the following color models a) CMYK Color Model b) HSI color model.
7. Explain briefly about Pseudo color image processing.
8. For R= 0.5, G=0.3, B=0.2 obtain RGB to HSI color model and when S= 0.65, I=0.29, with theta= 430 degree, obtain HSI to RGB color model.

Solution

1. The Gaussian distribution is often used to describe, at least approximately, any variable that tends to cluster around the mean $\bar{z}$ Gaussian distribution can be completely characterized by its mean $\bar{z}$ and the standard deviation σ.

The Gaussian function has certain very useful mathematical properties. It is symmetric around the point $z = \bar{z}$. The PDF of a gausian random variable, z , is given by

$$p(z) = \frac{1}{\sqrt{2\pi\sigma}} e^{-(z-\bar{z})^2/2\sigma^2}$$

Where z represents the intensity, $\bar{z}$ is the mean(average) value of z, σ is its standard deviation. The standard deviation squared, σ^2 is called the variance of z. approximately 70% of p(z) values will be in the range $[(\bar{z} - \sigma),(\bar{z} + \sigma)]$ and 95% will be in the range $[(\bar{z} - 2\sigma),(\bar{z} + 2\sigma)]$.

The Gaussian model is suitable to model the electronic circuitry noise in image acquisition systems. It is also useful to characterize the sensor noise which can be due to factors like poor illumination or high temperature.

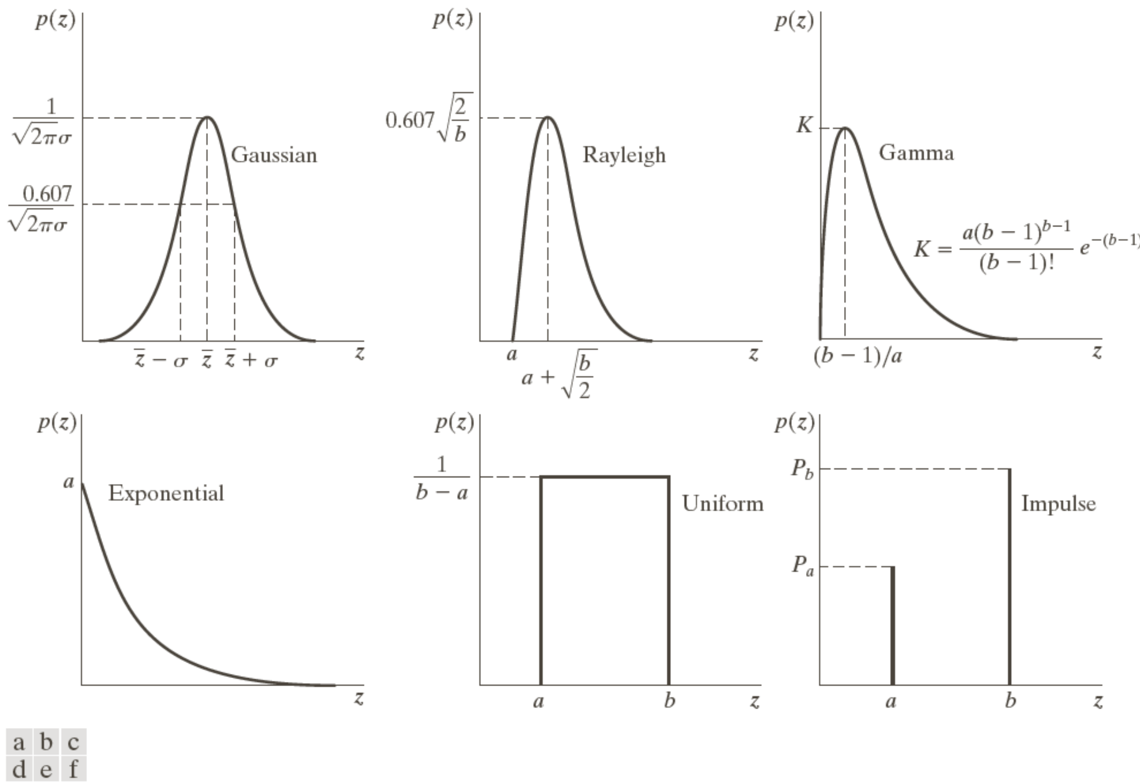


FIGURE 5.2 Some important probability density functions.

The Rayleigh Distribution is left skewed distribution with light tails. An attractive feature of the Rayleigh distribution is that the mode can be estimated from the mean. The range is determined by the scale parameter b. Its skewness is constant.
The PDF of Rayleigh noise is given by

$$p(z) = \begin{cases} \frac{2}{b}(z-b)e^{-\frac{(z-a)^2}{b}} & for\ z \geq a \\ 0 & for\ z < a \end{cases}$$

The mean and variance of this density are given by
$\bar{z} = a + \sqrt{\pi b/4}$
And

$$\sigma^2 = \frac{b(4-\pi)}{4}$$

The formula for Rayleigh distribution has 2 factors, the first one (z-a) is a linearly increasing term and the second one $e^{-\frac{(z-a)^2}{b}}$ is an exponentially decaying term like the one in Gaussian. The second term also indicates that the parameter b plays a role similar to the variance.
The Rayleigh distribution is useful for modeling skewed distributions. The Rayleigh distribution is suitable for characterizing noise in range imaging.

The Erlang distribution is also skewed like the Rayleigh distribution. Similar to the Rayleigh distribution, its formula shows two factors, the term z^{b-1} is responsible for the increase and the other term e^{-az} is responsible for the exponential decay. The exponential decay in Erlang distribution is slower compared to Rayleigh because Rayleigh has a quadratic decay term.

- The Erlang distribution is suitable for characterizing noise in range imaging.
  The PDF of Erlang noise is given by

$$p(z) = \begin{cases} \dfrac{a^b z^{b-1}}{(b-1)!} e^{-az} & for\ z \geq 0 \\ 0\ for\ z < 0 \end{cases}$$

  Where a>0, bis positive integer and "!" indicates factorial.
  The mean and variance of this density are given by

$$\bar{z} = \frac{b}{a}$$

$$\sigma^2 = \frac{b}{a^2}$$

  Exponential noise is a special case of Erlang distribution with the parameter b = 1
  The PDF is given by

$$p(z) = \begin{cases} ae^{-az} & for\ z \geq 0 \\ 0\ for\ z < 0 \end{cases}$$

  Wher a>0. The mean and variance of this density function are given by

$$\bar{z} = \frac{1}{a}$$

  And $\sigma^2 = \frac{1}{a^2}$

  The uniform noise has a PDF which remains constant for specified bounds$a \leq z \leq b$ of the noise amplitude. The constant value of probability is pegged at $\frac{1}{b-1}$ because the total area under the pdf curve is 1. This noise is less practical and is used for random number generator.
  The PDF is given by

$$p(z) = \begin{cases} \dfrac{1}{b-a} & if\ a \leq z \leq b \\ o & otherwise \end{cases}$$

  The mean and variance of this density function are given by

$$\bar{z} = \frac{a+b}{2}$$

  And $\sigma^2 = \frac{(b-a)^2}{12}$

  Impulse noise generally corresponds to extreme values (intensity 0 for dark and 255 for bright) in the image. The noise has only two allowable values the negative impulse causing dark points a = 0 and the positive impulse causing the bright points b = 255. The probabilities of the two types of noise impulses can be either same or different. If one of the probabilities is zero, the noise will be either a salt noise or a pepper noise.
  The PDF is given by

$$p(z) = \begin{cases} P_a & \text{for } z = a \\ P_b & \text{for } z = b \\ 0 & \text{otherwise} \end{cases}$$

  - bipolar if $P_a \neq 0,\ P_b \neq 0$
  - unipolar if one of P_a and P_b is 0

- noise looks like salt-and-pepper granules if $P_a \approx P_b$
- negative or positive; scaling is often necessary to form digital images
- extreme values occur (e.g. $a = 0$, $b = 255$)

- The Salt and Pepper noise is suitable for characterizing noise due to electrical or illumination transients during imaging or communication.

Periodic noise is a spatially dependent noise. This can be in the form of spatially sinusoidal noise corrupting the image. The Fourier transform of a pure sinusoid is a pair of conjugate impulses located at the conjugate frequencies of the sinusoid. Hence the Fourier spectrum of the noisy image would indicate a pair of impulse for each frequency in the periodic noise. The impulses will be more pronounced if the sinusoid amplitude is large enough.

2. Smoothing Spatial Filters

Smoothing filters are used for blurring and for noise reduction.

a. Blurring is used in preprocessing steps, such as removal of small details from an image prior to object extraction, and bridging of small gaps in lines or curves
b. Noise reduction can be accomplished by blurring

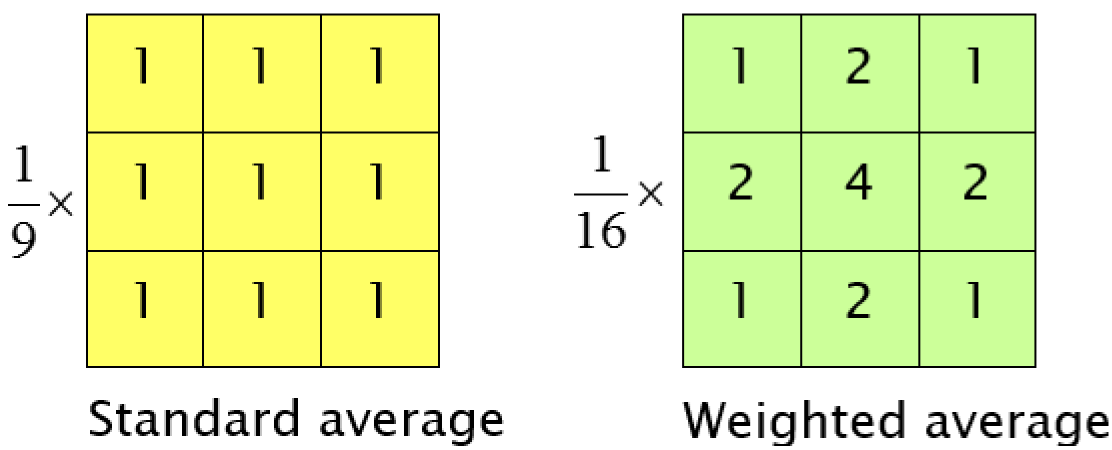
There are 2 way of smoothing spatial filters

c. Smoothing Linear Filters
d. Order-Statistics Filters(Non Linear)

Linear spatial filter is simply the average of the pixels contained in the neighborhood of the filter mask.

Sometimes called "averaging filters".

The idea is replacing the value of every pixel in an image by the average of the gray levels in the neighborhood defined by the filter mask.



- The general implementation for filtering an MxN image with a weighted averaging filter of size mxn is given by the expression

$$g(x,y)=\frac{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)\,f(x+s,y+t)}{\sum_{s=-a}^{a}\sum_{t=-b}^{b} w(s,t)}$$

- Order-statistics filters are nonlinear spatial filters whose response is based on ordering (ranking) the pixels contained in the image area encompassed by the filter, and then replacing the value of the center pixel with the value determined by the ranking result.
- Egs: Median filter, Min Filter, Max Filter.
- Median Filter reduces both Salt and Pepper Noise.
- Min Filter reduces Salt noise.
- Max Filter reduces Pepper noise.

Sharpening Spatial Filters

The principal objective of sharpening is to highlight

- transitions in intensity(edge sharpening).
- fine detail in an image
- to enhance detail that has been blurred, either in error or as an natural effect of a particular method of image acquisition.
- The image blurring is accomplished in the spatial domain by pixel averaging in a neighborhood.
- Since averaging is analogous to integration.

Sharpening could be accomplished by *spatial differentiation(first and second derivatives).*

- We are interested in the behavior of these derivatives in areas of constant gray level(flat segments), at the onset and end of discontinuities(step and ramp discontinuities), and along gray-level ramps.
- These types of discontinuities can be noise points, lines, and edges.
- First let us consider 1D Digital signals.
- The derivatives of a digital function are defined in terms of differences.

- A basic definition of the first-order derivative of a one-dimensional function f(x) is

$$\frac{\partial f}{\partial x} = f(x+1) - f(x)$$

- Must be zero in flat segments
- Must be nonzero at the onset of a gray-level step or ramp; and
- Must be nonzero along ramps.

- We define a second-order derivative as the difference

$$\frac{\partial^2 f}{\partial x^2} = f(x+1) + f(x-1) - 2f(x).$$

- Must be zero in flat areas;
- Must be nonzero at the onset and end of a gray-level step or ramp;
- Must be zero along ramps of constant slope

The Laplacian Mask is

0	1	0
1	-4	1
0	1	0

1	0	1
0	-4	0
1	0	1

The other forms of Laplacian

0	-1	0
-1	4	-1
0	-1	0

-1	0	-1
0	4	0
-1	0	-1

-1	-1	-1
-1	8	-1
-1	-1	-1

$$g(x,y) = \begin{cases} f(x,y) - \nabla^2 f(x,y) & \text{When centre co-efficient is negative} \\ f(x,y) + \nabla^2 f(x,y) & \text{When centre co-efficient is positive} \end{cases}$$

Where f(x,y) is the original image
$\nabla^2 f(x,y)$ is Laplacian filtered image
g(x,y) is the sharpen image

3.

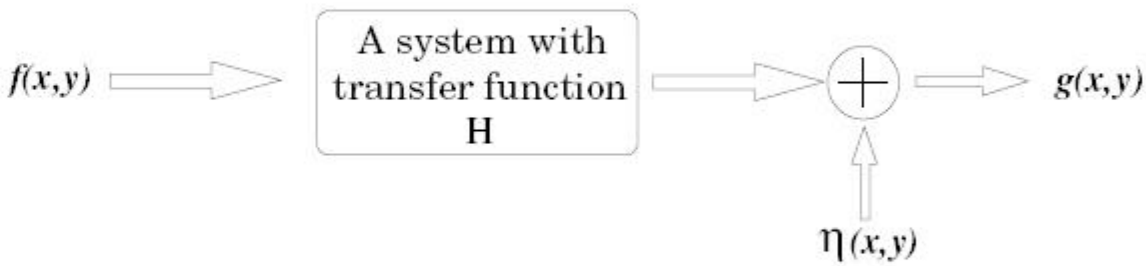


Figure 2: A model of the image degradation process

$$g(x,y) = H[f(x,y)] + \eta(x,y)$$

Considering the noise term to be zero

$$g(x,y) = Hf(x,y)$$

Linear

The assumption that H is linear means that H satisfies the properties of additivity and homogeneity.

- Additivity

$$H[af_1(x,y) + bf_2(x,y)] = aH[f_1(x,y)] + bH[f_2(x,y)]$$

- Homogeneity

$$H[af_1(x,y)] = aH[f_1(x,y)]$$

Position Invariant

The assumption that H is position invariant implies that for any spatial shift (α, β)

$$H[f(x-\alpha, y-\beta)] = g(x-\alpha, y-\beta)$$

To examine this we reformulate $f(x,y)$ as

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\delta(x-\alpha,y-\beta)d\alpha d\beta$$

since the convolution of a function with an impulse would give the same function. Substituting this in $g(x,y) = H[f(x,y)]$ gives

$$\begin{aligned} g(x,y) &= H[f(x,y)] \\ &= H\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\delta(x-\alpha,y-\beta)d\alpha d\beta\right] \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} H[f(\alpha,\beta)\delta(x-\alpha,y-\beta)]d\alpha d\beta \quad \because H \text{ is a linear operator} \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)\, H[\delta(x-\alpha,y-\beta)]d\alpha d\beta \quad \because H \text{ follows homogeneity property} \end{aligned}$$

Let $h(x,\alpha,y,\beta) = H\ [\delta(x-\alpha,y-\beta)]$ The term $h(x,\alpha,y,\beta)$ is the impulse response of the degradation system, i.e. the response of the system H to an impulse at at location (α,β).

Since H is linear position invariant

$$H\ [\delta(x-\alpha,y-\beta)] = h(x-\alpha,y-\beta)$$

This gives us

$$\begin{aligned} g(x,y) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha,y-\beta)d\alpha d\beta \\ &= h(x,y)\bigstar f(x,y) \\ &= h(x,y)\bigstar f(x,y) \\ G(u,v) &= H(u,v)F(u,v) \end{aligned}$$

The important conclusion of this analysis is that

> If we consider H to be linear and a position invariant operator, then the degradation process can be modelled as the convolution of the given image $f(x,y)$ and the impulse response $h(x,y)$ of the system which introduces the degradation.

Consider non-zero noise $\eta(x,y)$

Since the noise is considered as additive to the degradation process, we can easily incorporate the noise term as follows:

$$g(x,y) = h(x,y) \star f(x,y) + \eta(x,y)$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha,y-\beta)\,d\alpha\, d\beta + \eta(x,y)$$

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

Restoration can be done by convolving the degraded image with a filter which can reverse the effect of convolving the original one with $h(x,y)$. This process is known as image deconvolution and the restoration filters used are called as deconvolution filters.

4. There are 3 principal ways to estimate the degradation function: 1) Observation. 2) Experimentation 3) Mathematical Modeling. The process of restoring an image by using degradation function that has been estimated in some way is called blind deconvolution.

Estimation by Image Observation: This method of estimating the degradation function is used when we have absolutely no clue of what caused the image degradation. We just have the degraded image given to us. In order to restore the image we must have some idea of what the original image could be looking like. On the given degraded image we select a small patch which has relatively less noise and has good contrast. Following our guesswork, we attempt to restore this patch by applying image operations like sharpening, contrast or brightness adjustment, etc. Our objective here is to get the restored patch. It does not depend on what operations we apply and in what sequence. Let the Fourier transform of the degraded patch be $G_s(u,v)$ and that of the restored patch be $\hat{F}_s(u,v)$ Then the Fourier transform of the degradation function $H_s(u,v)$ can be estimated as:

$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

Following our assumption that $H(u,v)$ is position invariant, the degradation function $H(u,v)$ will have the same basic shape as $H_s(u,v)$. However the scale of $H(u,v)$ will be larger compared to that of $H_s(u,v)$.

Estimation by Experimentation: If the image acquisition system which was used to acquire the degraded image is available to us, then we can tune the system settings so that we get an image (not necessarily of the same scene/object) of similar degradation. The idea is to recover the same system settings which were responsible for producing the degradation which we want to estimate. Once we are able to achieve those system settings we need to know the response of the system to an impulse signal. An impulse can be simulated using a small bright dot of light. We record the system's response for this impulse as G_δ in frequency domain. Since the Fourier transform of an impulse is a constant say A the frequency domain representation of the system transfer function, i.e. the degradation $H(u,v)$ is given as:

$$H(u,v) = \frac{G_\delta(u,v)}{A}$$

Estimation by Modeling: We can mathematically model the physical phenomena or the imaging conditions which lead to degraded images. This requires extensive research. For example, it has been possible to model the different type of blurring effects (low-pass filtering) due to various degrees of severity of atmospheric turbulence conditions. For simple cases of blurring due to image motion, it is possible to mathematically derive the degradation function

When we acquire the image of a moving object we generally get a blurred image because of the relative motion between the sensor and the object. In this section we consider how to mathematically model the blur due to image motion. To simplify the modeling we assume that the image moves along a plane and the time varying displacement in x and y directions for every pixel is given as $x_0(t)$ and $y_0(t)$ respectively. If T is the duration for which the camera shutter is open, the intensity at each pixel on the blurred image $g(x,y)$ is computed as an integration of the (true, unblurred) image . $f(x,y)$ intensities over the period T .

Relation between Fourier transform of $f(x,y)$ and $g(x,y)$

The Fourier transform of $g(x,y)$ can be written as:

$$G(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\, e^{-j2\pi(ux+vy)}dxdy$$

$$G(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)\, e^{-j2\pi(ux+vy)}dxdy$$

$$\begin{aligned} G(u,v) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left[\int_0^T f[x-x_0(t), y-y_0(t)]\,dt\right] e^{-j2\pi(ux+vy)}dxdy \\ &= \int_0^T \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f[x-x_0(t), y-y_0(t)]\, e^{-j2\pi(ux+vy)}dxdy\right] dt \end{aligned}$$

Using the Fourier transform shift property $\mathscr{F}[f(x-a,y-b)] = F(u,v)\, e^{-j2\pi(ua+vb)}$ we have

$$\begin{aligned} G(u,v) &= \int_0^T F(u,v) e^{-j2\pi[ux_0(t)+vy_0(t)]}dt \\ &= F(u,v)\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt \\ &= F(u,v)\int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt \\ &= F(u,v)\, H(u,v) \end{aligned}$$

since $F(u,v)$ does not depend on t

We find that the Fourier transform of degradation due to motion blurring can be formulated as:

$$H(u,v) = \int_0^T e^{-j2\pi[ux_0(t)+vy_0(t)]}dt$$

As we notice from this formulation, the degradation function can be estimated only when the image motion is planar and the time-varying displacements $x_0(t)$ and $y_0(t)$ are known. For general objects which can have articulated motion, it is very difficult to estimate the values of $x_0(t)$ and $y_0(t)$ for the different parts of the objects.

Once we have estimated the degradation function $H(u,v)$ the next step is to use it to restore the image. This process is called as filtering the degraded image so as to get the restored image as the output. In the next 3 subsections we discuss the filtering methods for restoration.

5.

The Wiener filter solves the signal estimation problem for stationary signals. The filter was introduced by Norbert Wiener in the 1940's. A major contribution was the use of a statistical model for the estimated signal. The noise present in the signal is reduced by comparison with an estimation of the desired noiseless signal. The filter is optimal in the sense of the minimum mean square error. The Wiener filtering approach takes into account both the degradation function and the noise characteristics for estimating the undegraded image. The Wiener filter computes an optimal estimate of the undegraded image.

Assumptions:

1. The power spectrum $S_\eta(u,v)$ of the noise is available.
2. The power spectrum $S_f(u,v)$ of the original image is available.
3. The image signal and the noise signal are uncorrelated.
4. Either the image signal or the noise must have zero mean.
5. The intensity levels in the restored image are a linear function of the levels in the degraded image.

Optimality Criteria

The Wiener filter is a minimum mean square error filter. The optimality criteria is to minimize the expected value (i.e. the mean) of the square error between the original image f and the estimate of the un-degradedimage $\hat{f}$. $e^2 = E\{(f-\hat{f})^2\}$

Here the $E\{\circ\}$ denotes the expected value.

The Wiener Filter expression

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)S_f(u,v)}{S_f(u,v)|H(u,v)|^2 + S_\eta(u,v)}\right] G(u,v)$$

$$= \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \frac{S_\eta(u,v)}{S_f(u,v)}}\right] G(u,v)$$

$$= \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + \frac{S_\eta(u,v)}{S_f(u,v)}}\right] G(u,v)$$

where $H(u,v)$ = degradation function

$H^*(u,v)$ = complex conjugate of $H(u,v)$

$|H(u,v)|^2$ = $H^*(u,v)H(u,v)$

$S_\eta(u,v)$ = $|N(u,v)|^2$ = power spectrum of noise

$S_f(u,v)$ = $|F(u,v)|^2$ = power spectrum of undegraded image

- If the $H(u,v)$ is zero, the denominator will remain non-zero unless the noise power spectrum is also zero. This is an advantage over the inverse filter.
- The problem with the Wiener filter is that it requires an estimate of $S_\eta(u,v)$ and $S_f(u,v)$. The latter quantity is difficult to guess because we don't have access to the original image $f(x,y)$.
- In a simplified expression of the Wiener filter we assume that the ratio $\frac{S_\eta(u,v)}{S_f(u,v)}$ is a constant K.

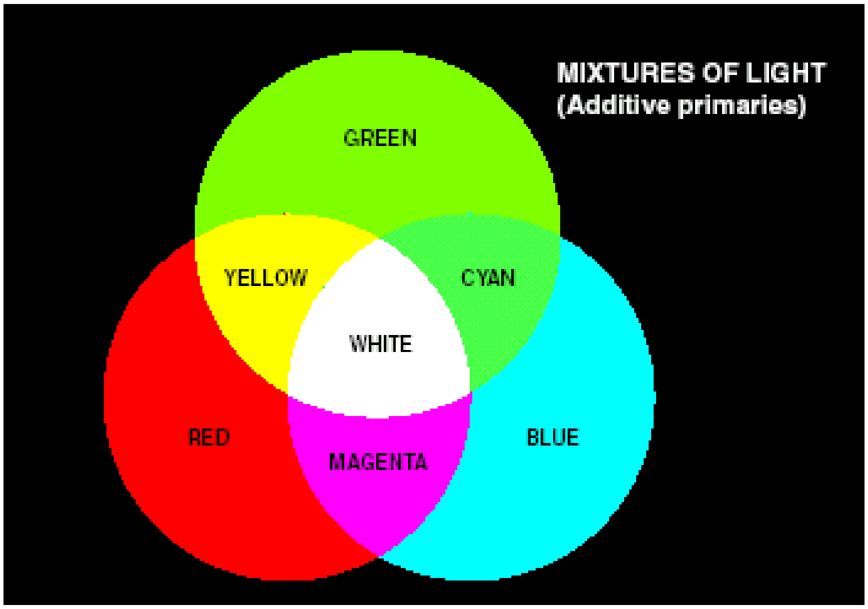
$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + K}\right] G(u,v)$$

-
- When restoring a degraded image using the Wiener filter, we can interactively adjust the value of K as per our visual assessment and obtain the most satisfactory restored image.

6. CMY: secondary colors of light, or primary colors of pigments(defined as one that subtracts /absorbs a primary color of light and reflects the other two.)

Used to generate hardcopy output

$$\begin{bmatrix} C \\ M \\ Y \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - \begin{bmatrix} R \\ G \\ B \end{bmatrix}$$



CMY model (+Black = CMYK)

- Usually this color model is used for printing.
- Ideally equal amounts of cyan, magenta and yellow should produce BLACK. But combining these colors for printing produces a muddy looking Black.
- So, in order to produce true black, a fourth color black is added giving rise to the CMYK color model.
- CMYK color model refers to three colors of the CMY color model plus Black.

HSI color model

- The RGB and CMY color models are well suited for hardware implementations.
- The RGB and CMY color models are not suited for describing colors in terms that are practical for human interpretation.

When human view a color it is described by

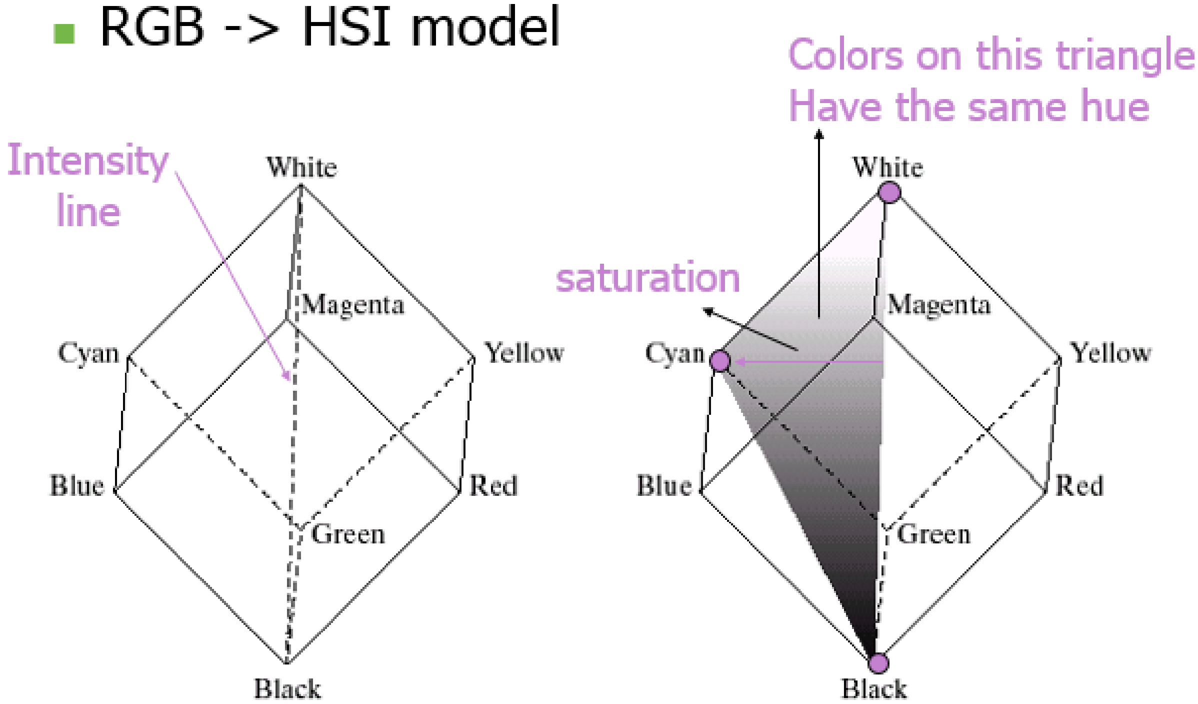
Color carrying information

- Hue: Dominant color
- Saturation: Relative purity(inversely proportional to amount of white light added)
- Intensity: Brightness

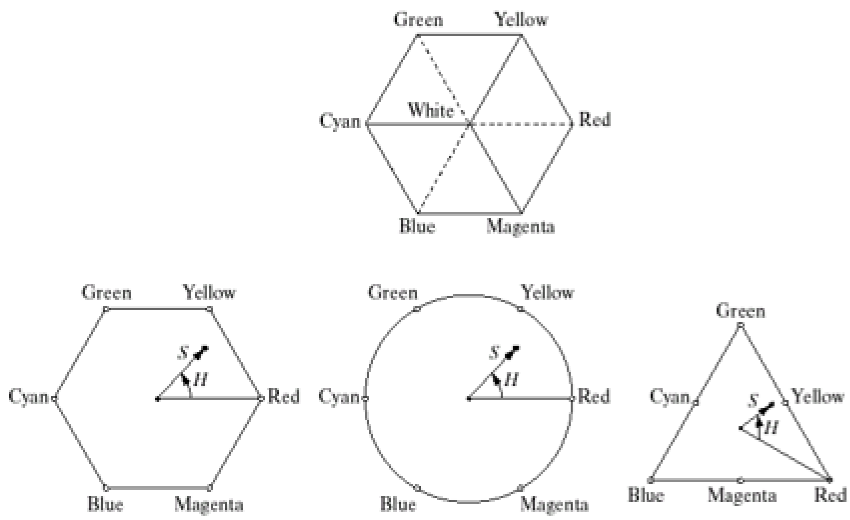
- The HSI color model decouples the intensity component from color carrying information(hue and saturation) in a color image.
- RGB model is ideal for image ***color generation.***
- HSI model is ideal for image ***color description.***

HSI color model (cont.)

- RGB -> HSI model



The HSI Color Models



a
b c d

FIGURE 6.13 Hue and saturation in the HSI color model. The dot is an arbitrary color point. The angle from the red axis gives the hue, and the length of the vector is the saturation. The intensity of all colors in any of these planes is given by the position of the plane on the vertical intensity axis.

7. Pseudocolor image processing

• Also known as indexed color

• Assign colors to gray values based on a fixed criteria

– 216 index entries from 8-bit RGB color system as a 6 × 6 × 6 cube in a direct color system

– Gives an integer in the range 0 to 5 for each component of RGB

– Requires less data to encode an image

– Some graphics file formats, such as GIF and TIFF add an index colormap to the image with gamma-corrected RGB entries

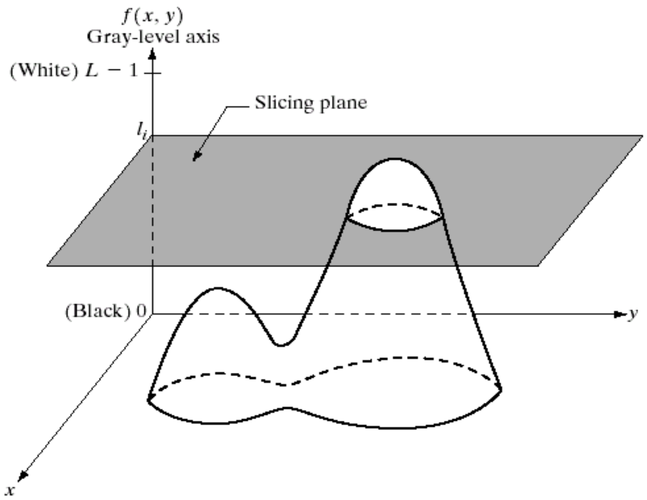
• Used as an aid to human visualization and interpretation of gray-scale events in an image or sequence of images, such as visualizing population density in different areas on a map

• May have nothing to do with processing of true color images

• Intensity slicing

– Also called density slicing or color coding

– Slicing planes parallel to horizontal plane in 3D space, with the intensity of image providing the third dimension on image plane



-Plane at f(x, y) = li to slice the image function into two levels ∗

- Assign different colors to area on different sides of the slicing plane

- Relative appearance of the resulting image manipulated by moving the slicing plane up and down the gray-level axis

– Technique summary

∗ Gray scale representation – [0, L − 1]

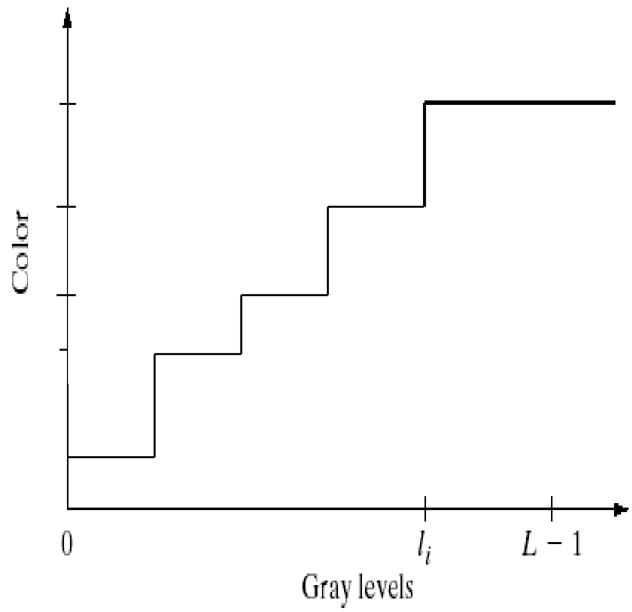
∗ Black represented by l0, [f(x, y) = 0]

∗ White represented by [lL−1], [f(x, y) = L − 1]

∗ Define P planes perpendicular to intensity axis at levels l1, l2, . . . , lP

∗ 0 < P < L − 1 ∗ P planes partition the gray scale into P + 1 intervals as V1, V2, . . . , VP +1

∗ Make gray-level to color assignment as f(x, y) = ck if f(x, y) ∈ Vk where ck is the color associated with kth intensity interval Vk defined by partitioning planes at l = k − 1 and l = k



Gray level to color transformations

– Separate independent transformation of gray level inputs to three colors

– Figure 6.23

– Composite image with color content modulated by nature of transformation function

– Piecewise linear functions of gray levels

Assigning colors to gray levels based on specific mapping functions

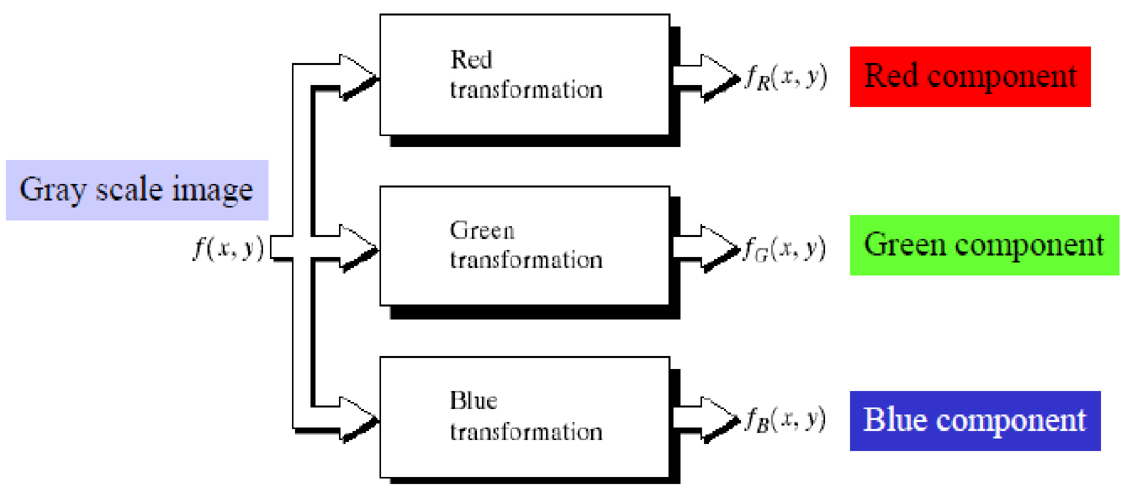


FIGURE 6.23 Functional block diagram for pseudocolor image processing. f_R, f_G, and f_B are fed into the corresponding red, green, and blue inputs of an RGB color monitor.

Combining several monochrome images into a single color composite

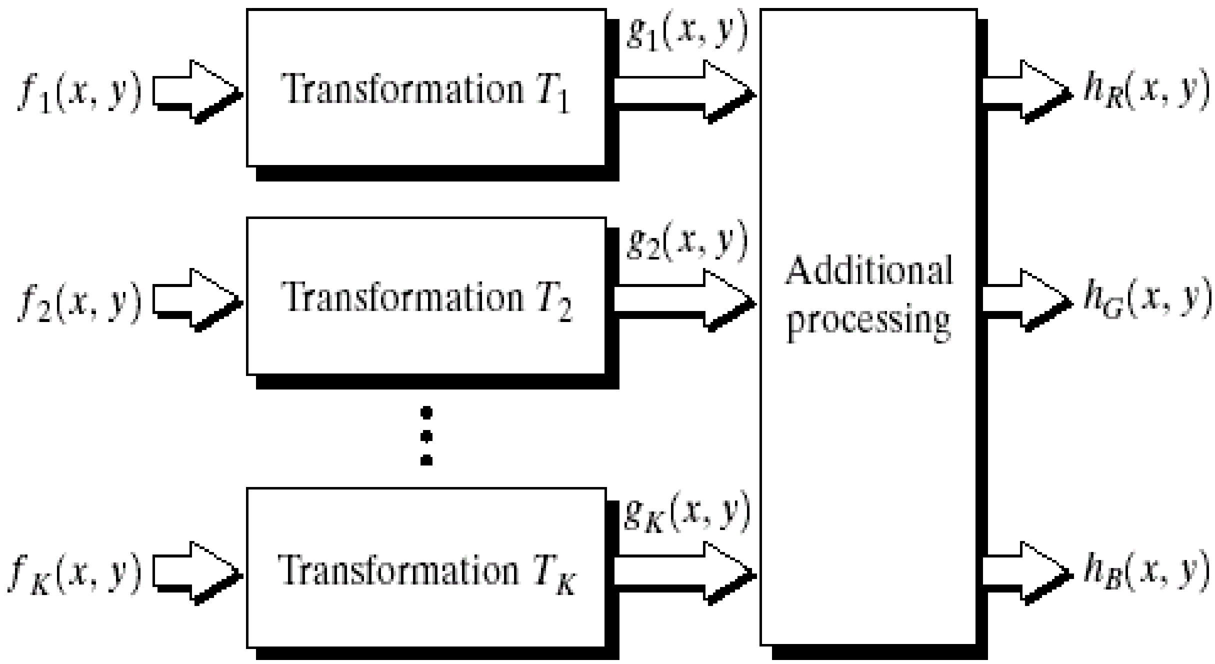


Fig: pseudo color approach used when several monochrome images are available.

Used in multispectral image processing, with different sensors producing individual monochrome images in different spectral bands

8.

HSI to RGB

$S = 0.65$, $I = 0.29$, $\theta = 430$

$\theta = 430 - 360°$

$\theta = 70°$

$H = \theta = 70°$

$0 < H < 120$

RG sector.

$B = I(1 - S)$

$\boxed{B = 0.1015}$

$$R = I\left[1 + \frac{S \cos H}{\cos(60 - H)}\right]$$

$\boxed{R = 0.355}$

$G = 3I - (B + R)$

$\boxed{G = 0.4135}$

RGB to HSI

$R = 0.5$

$G = 0.3$

$B = 0.2$

$$\theta = \cos^{-1}\left\{ \frac{\frac{1}{2}\left[(R-G)+(R-B)\right]}{\left[(R-G)^2 + (R-B)(G-B)\right]^{1/2}} \right\}$$

$\theta = 19.10.$

$$H = \begin{cases} \theta & , \text{If } B \le G \\ 360 - \theta & , \text{If } B > G \end{cases}$$

$\therefore H = 19.10$

$$S = 1 - \frac{3}{R+G+B}\left[\min(R, G, B)\right] = 0.4$$

$$I = \frac{1}{3}\left[R + G + B\right] = 0.33$$