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## Internal Assesment Test - III

| Sub: | Microwave and Antennas | | | | | | | Code: | 15EC71 |
|---|---|---|---|---|---|---|---|---|---|
| Date: | 22/11/2018 | Duration: | 90 mins | Max Marks: | 50 | Sem: | 7th | Branch: | ECE (A,B,C,D) |
| Answer Any FIVE FULL Questions | | | | | | | | | |

| | | Marks | OBE CO | OBE RBT |
|---|---|---|---|---|
| 1. | Draw the polar diagram (field pattern) of a broadside array with no. of elements = 5 and spacing between the array elements $=\frac{\lambda}{2}$. | [10] | CO4 | L3 |

$n=5$ (No. of elements)

$d=\frac{\lambda}{2}$ (spacing between the array elements)

- Total far-field pattern of n-isotropic point sources

$$E_t = E_0 \cdot \frac{\sin(n\psi/2)}{\sin(\psi/2)}$$

- For broadside array, sources have same amplitude and in-phase
- Maximum radiation occurs at 90° and 270°.

$$\psi = \beta . d . \cos\theta + \delta \qquad \because \delta = 0 \text{ and } \psi = 0$$

$$0 = \beta d . \cos\theta$$

$$\cos\theta = 0$$

$$\theta = 90^\circ \text{ and } 270^\circ.$$

- Direction of pattern maxima occurs

$$\sin\left(\frac{n\psi}{2}\right) = \pm 1 \qquad \because \sin(\psi/2) \neq 0$$

$$\frac{n\psi}{2} = \pm (2N+1) . \frac{\pi}{2}$$

$$\psi = \pm (2N+1) . \frac{\pi}{n}$$

$$\beta . d . \cos(\theta_{max})_{min} + \delta = \pm (2N+1) . \frac{\pi}{n}$$

$$\cos(\theta_{max})_{min} = \frac{1}{\beta d}\left[\pm (2N+1) . \frac{\pi}{n} - \delta\right]$$

$$(\theta_{max})_{min} = \cos^{-1}\left\{\frac{1}{\beta d}\left[\pm (2N+1) . \frac{\pi}{n} - \delta\right]\right\}$$

$$\delta = 0 ;\ n = 5;\ d = \frac{\lambda}{2} ;\ \beta = \frac{2\pi}{\lambda}$$

$$(\theta_{max})_{min} = \cos^{-1}\left\{ \frac{1}{\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}} \left[ \pm (2N+1) \cdot \frac{\pi}{5} - 0 \right] \right\}$$

$$(\theta_{max})_{min} = \cos^{-1}\left\{ \frac{\pm (2N+1)}{5} \right\}$$

$N = 0$, $(\theta_{max})_{min} = \cos^{-1}\left\{ \pm \frac{1}{5} \right\} = 78.46°, 101.53°$

$N = 1$, $(\theta_{max})_{min} = \cos^{-1}\left\{ \pm \frac{3}{5} \right\} = 53.13°, 126.86°$

$N = 2$, $(\theta_{max})_{min} = \cos^{-1}\left\{ \pm \frac{5}{5} \right\} = 0°, 180°$

- Direction of pattern minima occurs at

$$\sin\left(\frac{n\psi}{2}\right) = 0$$

$$\frac{n\psi}{2} = \pm N\pi$$

$$\psi = \pm \frac{2N\pi}{n}$$

$$\beta d \cdot \cos(\theta_{min})_{min} + \delta = \pm \frac{2N\pi}{n}$$

$$\cos(\theta_{min})_{min} = \frac{1}{\beta d}\left[ \pm \frac{2N\pi}{n} - \delta \right]$$

$$(\theta_{min})_{min} = \cos^{-1}\left[ \frac{1}{\beta d} \left\{ \pm \frac{2N\pi}{n} - \delta \right\} \right]$$

$$\delta = 0 ;\ d = \frac{\lambda}{2} ;\ n = 5 ;\ \beta = \frac{2\pi}{\lambda}$$

$$(\theta_{min})_{min} = \cos^{-1}\left\{ \frac{1}{\frac{2\pi}{\lambda} \cdot \frac{\lambda}{2}} \left[ \pm \frac{2N\pi}{5} - 0 \right] \right\}$$

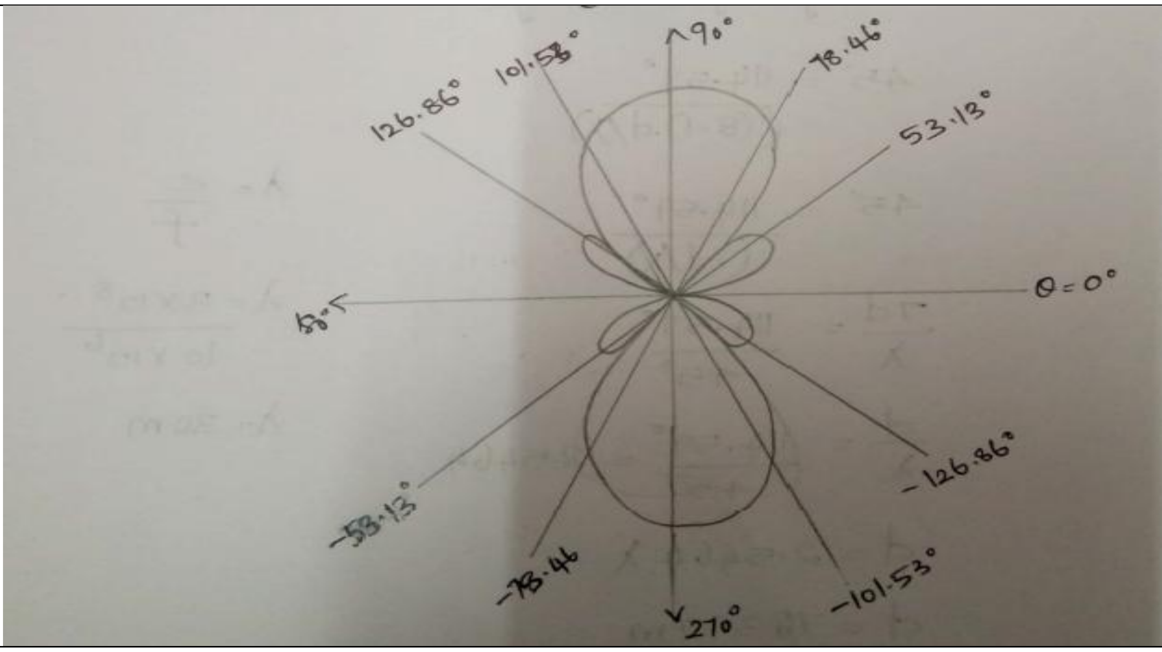
$$= \cos^{-1}\left\{ \pm \frac{2N}{5} \right\}$$

$N = 0$, $(\theta_{min})_{min} = \cos^{-1}\left\{ \pm 0 \right\} = 90°, 270°$

$N = 1$, $(\theta_{min})_{min} = \cos^{-1}\left\{ \pm \frac{2}{5} \right\} = 66.42°, 113.57°$

$N = 2$, $(\theta_{min})_{min} = \cos^{-1}\left\{ \pm \frac{4}{5} \right\} = 36.86°, 143.13°$

$N = 3$, $(\theta_{min})_{min} = \cos^{-1}\left\{ \pm \frac{6}{5} \right\} =$ Not exists.




2. (a) Explain the principle of pattern multiplication. [3] CO4 L2

> ***The field pattern of an array of nonisotropic but similar point sources is the product of the pattern of the individual source and the pattern of an array of isotropic point sources having the same locations, relative amplitudes, and phase as the nonisotropic point sources.***

This principle may be applied to arrays of any number of sources provided only that they are similar. The individual nonisotropic source or antenna may be of finite size but can be considered as a point source situated at the point in the antenna to which phase is referred. This point is said to be the "phase center."

The above discussion of pattern multiplication has been concerned only with the field pattern or magnitude of the field. If the field of the nonisotropic source and the array of isotropic sources vary in phase with space angle, i.e., have a phase pattern which is not a constant, the statement of the principle of pattern multiplication may be extended to include this more general case as follows:

> The total field pattern of an array of nonisotropic but similar sources is the product of the individual source pattern and the pattern of an array of isotropic point sources each located at the phase center of the individual source and having the same relative amplitude and phase, while the total phase pattern is the sum of the phase patterns of the individual source and the array of isotropic point sources.

The total phase pattern is referred to the phase center of the array. In symbols, the total field $E$ is then

$$E = \underbrace{f(\theta,\phi)F(\theta,\phi)}_{\text{Field pattern}} \underbrace{\big/ f_p(\theta,\phi) + F_p(\theta,\phi)}_{\text{Phase pattern}}$$

where

$f(\theta,\phi)$ = field pattern of individual source

$f_p(\theta,\phi)$ = phase pattern of individual source

$F(\theta,\phi)$ = field pattern of array of isotropic sources

$F_p(\theta,\phi)$ = phase pattern of array of isotropic sources

(b) Using exact method, calculate the distance between elements of broadside array whose Beam-width between first null is found to be $45^0$ at a freq of 10MHZ. There are 8 elements in the array. [7] CO4 L3

Given, BWFN = 45° (Beamwidth first nulls)

$f = 10\,MHz$

$n = 8$ elements.

$d = ?$ (broadside array)

For broadside array,

$$BWFN = \frac{114.59^\circ}{(L/\lambda)}$$

$L$ = length of an array = $(n-1).d \simeq nd$

$$45^\circ = \frac{114.59^\circ}{((8-1).d/\lambda)}$$

$$45^\circ = \frac{114.59^\circ}{(7d/\lambda)}$$

$$\frac{7d}{\lambda} = \frac{114.59^\circ}{45^\circ}$$

$$\frac{d}{\lambda} = \frac{114.59^\circ}{45^\circ} = 2.5464$$

$$d = 2.5464\lambda$$

$$d = 76.392\,m$$

$$\lambda = \frac{c}{f}$$

$$\lambda = \frac{3\times10^8}{10\times10^6}$$

$$\lambda = 30\,m$$

3. (a) Show that the electric field pattern of a thin linear antenna of length L=λ/2 is given by:

$$E = \frac{\cos(\frac{\pi}{2}\cos\theta)}{\sin\theta}$$

[4] CO5 L4

Far fields $H_\phi$ and $E_\theta$ of a thin linear antenna, center fed, symmetrical, of length, L

$$H_\phi = \frac{j[I_0]}{2\pi r}\left[\frac{\cos[(\beta L\cos\theta)/2] - \cos(\beta L/2)}{\sin\theta}\right]$$

$$E_\theta = \frac{j60[I_0]}{r}\left[\frac{\cos[(\beta L\cos\theta)/2] - \cos(\beta L/2)}{\sin\theta}\right]$$

When $L=\lambda/2$, the electric field pattern,

$$H_\phi = \frac{j.[I_0]}{2\pi r}\left[\frac{\cos[(\frac{2\pi}{\lambda}.\frac{\lambda}{2}.\cos\theta)/2] - \cos[(\frac{2\pi}{\lambda}.\frac{\lambda}{2})/2]}{\sin\theta}\right]$$

$$= \frac{j[I_0]}{2\pi r}\left[\frac{\cos(\pi/2\cos\theta) - \cos(\pi/2)}{\sin\theta}\right]$$

$$H_\phi = \frac{j[I_0]}{2\pi r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right]$$

then

$$E_\theta = \frac{j\,60[I_0]}{r}\left[\frac{\cos(\pi/2\cos\theta)}{\sin\theta}\right]$$

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| 4. | Derive an expression for an electric field of an array of two isotropic point sources of same amplitude and opposite phase. Also draw the field pattern and determine its maxima, minima and HPBW. | [10] | CO4 | L3 |

Array of two Isotropic point sources - Same Amplitude and opposite phase.

Total far-field strength,

$$E_t = -E_0 . e^{(j\psi/2)} + E_0 . e^{(-j\psi/2)}$$

$$E_t = -E_0 \left[ e^{j\psi/2} - e^{-j\psi/2} \right]$$

$$E_t = -E_0 . \left[ 2j . \sin(\psi/2) \right]$$

$$E_t = 2j . E_0 . \sin\left(\frac{\psi}{2}\right)$$

- phase difference between two point sources,

$$\psi = \beta . d . \cos\theta + \delta$$

$$\psi = \frac{2\pi}{\lambda} . d . \cos\theta \quad \text{(or)} \quad \psi = d_r . \cos\theta$$

where d = distance between two point sources $\left[ d_r = \frac{2\pi}{\lambda} . d \right]$

Pattern maxima occurs when

$$\sin\left(\frac{\psi}{2}\right) = \pm 1$$

$$\frac{\psi}{2} = \pm (2n+1) . \frac{\pi}{2}$$

$$\psi = \pm (2n+1) . \pi$$

$$\beta . d . \cos\theta = \pm (2n+1) . \pi$$

$$\cos\theta = \frac{1}{\beta d} \left[ \pm (2n+1) . \pi \right]$$

$$\theta_{max} = \cos^{-1}\left[\frac{1}{\beta d}\left(\pm 2n+1\right)\pi\right]$$

$$\beta = \frac{2\pi}{\lambda};\ d = \frac{\lambda}{2}\ \text{(spacing)}$$

$$\theta_{max} = \cos^{-1}\left[\frac{1}{\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}}\left(\pm 2n+1\right)\cdot\pi\right]$$

$$\theta_{max} = \cos^{-1}\left[\pm(2n+1)\right]$$

$n=0$, $\theta_{max} = \cos^{-1}\left[\pm 1\right] = 0^\circ, 180^\circ$

$n=1$, $\theta_{max} = \cos^{-1}\left[\pm 3\right]$ = Not exists

Pattern minima occurs when

$$\sin\left(\frac{\psi}{2}\right) = 0$$

$$\frac{\psi}{2} = \pm n\pi$$

$$\psi = \pm 2n\pi$$

$$\beta . d . \cos\theta = \pm 2n\pi$$

$$\cos(\theta_{min}) = \pm\frac{2n\pi}{\beta d}$$

$$(\theta_{min}) = \cos^{-1}\left[\pm\frac{2n\pi}{\beta d}\right]$$

$\beta = \frac{2\pi}{\lambda};\ d = \frac{\lambda}{2};$

$$(\theta_{min}) = \cos^{-1}\left[\pm\frac{2n\pi}{\frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}}\right]$$

$$(\theta_{min}) = \cos^{-1}[\pm 2n]$$

$n=0,\quad (\theta_{min}) = \cos^{-1}[0] = 90^\circ, 270^\circ$

$n=1,\quad (\theta_{min}) = \cos^{-1}[\pm 2] = \text{not exists}$

Half Powers beamwidth (HPBW) occurs at

$$\sin\left(\frac{\psi}{2}\right) = \pm\frac{1}{\sqrt{2}}$$

$$\frac{\psi}{2} = \pm(2n+1)\frac{\pi}{4}$$

$$\psi = \pm(2n+1).\frac{\pi}{2}$$

$$\beta d.\cos(\theta)_{HPBW} = \pm(2n+1).\frac{\pi}{2}$$

$$\cos(\theta)_{HPBW} = \frac{1}{\beta d}\left[\pm(2n+1).\frac{\pi}{2}\right]$$

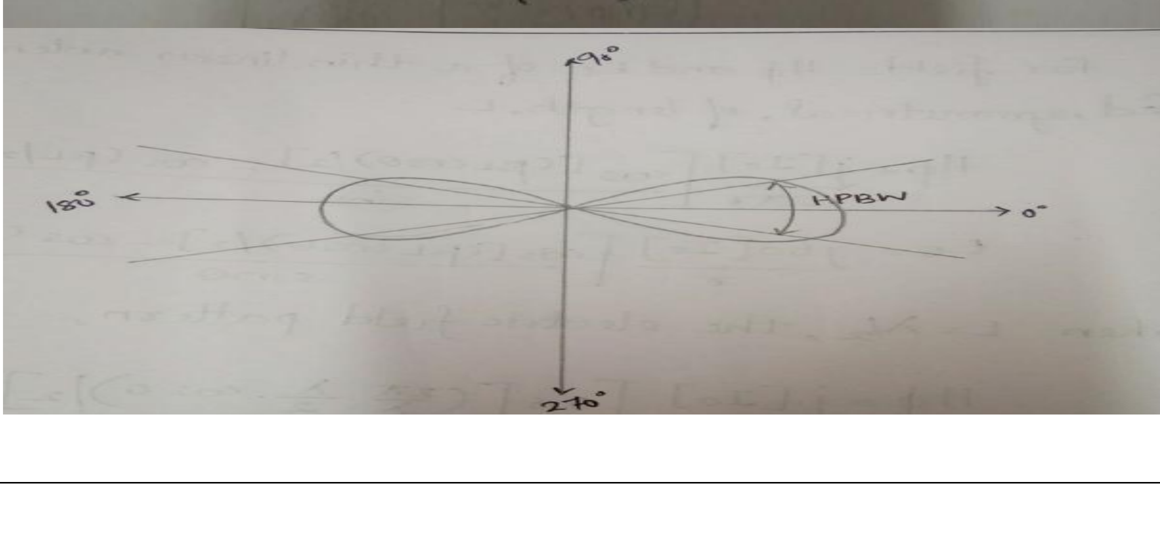
$$(\theta)_{HPBW} = \cos^{-1}\left\{\frac{1}{\beta d}\left[\pm(2n+1).\frac{\pi}{2}\right]\right\}$$

$\beta = \frac{2\pi}{\lambda};\ d = \frac{\lambda}{2}$

$$(\theta)_{HPBW} = \cos^{-1}\left\{\frac{1}{\frac{2\pi}{\lambda}.\frac{\lambda}{2}}\left[\pm(2n+1).\frac{\pi}{2}\right]\right\}$$

$$(\theta)_{HPBW} = \cos^{-1}\left\{\pm(2n+1)\frac{1}{2}\right\}$$

$n=0,\quad (\theta)_{HPBW} = \cos^{-1}\left\{\pm\frac{1}{2}\right\} = 60^\circ, 120^\circ$



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| 5. | Starting from fundamentals, derive the equation for radiation resistance of a Hertzian Dipole. | [10] | CO5 | L3 |

which implies instantaneous propagation of the effect of the current, we introduce the propagation (or retardation) time as done by Lorentz and write

$$[I] = I_0 e^{j\omega[t-(r/c)]} \qquad (2)$$

where $[I]$ is called the *retarded current*. Specifically, the retardation time $r/c$ results in a *phase retardation* $\omega r/c = 2\pi f r/c$ radians $= 360° \, fr/c = 360° \, t/T$, where $T = 1/f$ = time of one period or cycle (seconds) and $f$ = frequency (hertz, Hz = cycles per second). The brackets may be added as in (2) to indicate explicitly that the effect of the current is retarded.

Equation (2) is a statement of the fact that the disturbance at a time $t$ and at a distance $r$ from a current element is caused by a current $[I]$ that occurred at an earlier time $t - r/c$. The time difference $r/c$ is the interval required for the disturbance to travel the distance $r$, where $c$ is the velocity of light ($= 300$ Mm s$^{-1}$).

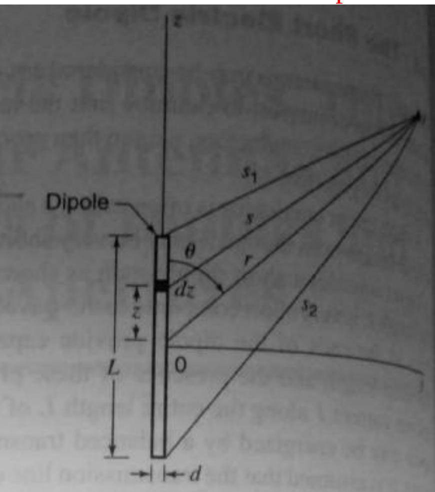



**Figure 6–3a** Geometry for short dipole

Electric and magnetic fields can be expressed in terms of vector and scalar potentials. Since we will be interested not only in the fields near the dipole but also at distances which are large compared to the wavelength we must use *retarded potentials*, i.e., expressions involving $t - r/c$. For a dipole located as in Fig. 6–2 or Fig. 6–3*a*, the retarded vector potential of the electric current has only one component, namely, $A_z$. Its value is

$$A_z = \frac{\mu_0}{4\pi} \int_{-L/2}^{L/2} \frac{[I]}{s} \, dz \qquad (3)$$

where $[I]$ is the retarded current given by

$$[I] = I_0 e^{j\omega[t-(s/c)]} \qquad (3a)$$

In (3) and (3a),

$z$ = distance to a point on the conductor
$I_0$ = peak value in time of current (uniform along dipole)
$\mu_0$ = permeability of free space $= 4\pi \times 10^{-7}$ H m$^{-1}$

If the distance from the dipole is large compared to its length ($r \gg L$) and if the wavelength is large compared to the length ($\lambda \gg L$), we can put $s = r$ and neglect the phase differences of the field contributions from different parts of the wire. The integrand in (3) can then be regarded as a constant, so that (3) becomes

$$A_z = \frac{\mu_0 L I_0 e^{j\omega[t-(r/c)]}}{4\pi r} \qquad (4)$$

The retarded scalar potential $V$ of a charge distribution is

$$V = \frac{1}{4\pi\varepsilon_0} \int_V \frac{[\rho]}{s} \, d\tau \qquad (5)$$

where $[\rho]$ is the retarded charge density given by

$$[\rho] = \rho_0 e^{j\omega[t-(s/c)]} \qquad (6)$$

and $d\tau$ = infinitesimal volume element
$\varepsilon_0$ = permittivity or dielectric constant of free space $= 8.85 \times 10^{-12}$ F m$^{-1}$

Since the region of charge in the case of the dipole being considered is confined to the points at the ends as in Fig. 6–1*b*, (5) reduces to

$$V = \frac{1}{4\pi\varepsilon_0} \left\{ \frac{[q]}{s_1} - \frac{[q]}{s_2} \right\} \qquad (7)$$

From (6–1–1) and (3a),

$$[q] = \int [I] \, dt = I_0 \int e^{j\omega[t-(s/c)]} \, dt = \frac{[I]}{j\omega} \qquad (8)$$

Substituting (8) into (7),

$$V = \frac{I_0}{4\pi\varepsilon_0 j\omega} \left[ \frac{e^{j\omega[t-(s_1/c)]}}{s_1} - \frac{e^{j\omega[t-(s_2/c)]}}{s_2} \right] \qquad (9)$$

Referring to Fig. 6–3*b*, when $r \gg L$, the lines connecting the ends of the dipole and the point $P$ may be considered as parallel so that

$$s_1 = r - \frac{L}{2} \cos\theta \qquad (10)$$

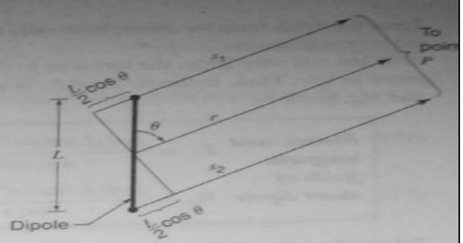



**Figure 6–3b** Relations for short dipole when $r \gg L$.

and

$$s_2 = r + \frac{L}{2} \cos\theta \qquad (11)$$

Substituting (10) and (11) into (9), it may be shown that the fields of a short electric dipole are:

**Electric fields of short dipole** — **General case**

$$E_r = \frac{I_0 L \cos\theta e^{j\omega[t-(r/c)]}}{2\pi\varepsilon_0}\left(\frac{1}{cr^2}+\frac{1}{j\omega r^3}\right) \quad (12)$$

$$E_\theta = \frac{I_0 L \sin\theta e^{j\omega[t-(r/c)]}}{4\pi\varepsilon_0}\left(\frac{j\omega}{c^2 r}+\frac{1}{cr^2}+\frac{1}{j\omega r^3}\right) \quad (13)$$

In obtaining (12) and (13) the relation was used that $\mu_0\varepsilon_0 = 1/c^2$, where $c$ = velocity of light. Turning our attention now to the ***magnetic field,*** this may be calculated from curl of $A$ as follows:

$$\nabla\times\mathbf{A} = \frac{\hat{\mathbf{r}}}{r\sin\theta}\left[\frac{\partial(\sin\theta)A_\phi}{\partial\theta}-\frac{\partial(A_\theta)}{\partial\phi}\right]+\frac{\hat{\theta}}{r\sin\theta}\left[\frac{\partial A_r}{\partial\phi}-\frac{\partial(r\sin\theta)A_\phi}{\partial r}\right] + \frac{\hat{\phi}}{r}\left[\frac{\partial(rA_\theta)}{\partial r}-\frac{\partial A_r}{\partial\theta}\right] \quad (14)$$

Since $A_\phi = 0$, the first and fourth terms of (14) are zero, since $A_r$ and $A_\theta$ are independent of $\phi$, so that the second and third terms of (14) are also zero. Thus, only the last two terms contribute, so that $\nabla\times\mathbf{A}$, and hence also $\mathbf{H}$, have only a $\phi$ component. Thus,

**Magnetic fields of short dipole** — **General case**

$$|\mathbf{H}| = H_\phi = \frac{I_0 L \sin\theta e^{j\omega[t-(r/c)]}}{4\pi}\left(\frac{j\omega}{cr}+\frac{1}{r^2}\right)$$

$$H_r = H_\theta = 0$$

Thus, the fields from the dipole have only three components $E_r$, $E_\theta$ and $H_\phi$. The components $E_\phi$, $H_r$ and $H_\theta$ are everywhere zero.

When $r$ is very large, the terms in $1/r^2$ and $1/r^3$ in (12), (13), and (15) can be neglected in favor of terms in $1/r$. Thus, in the *far field* $E_r$ is negligible, and we have effectively only two field components, $E_\theta$ and $H_\phi$, given by

**Electric and magnetic fields of short dipole** — **Far-field case**

$$E_\theta = \frac{j\omega I_0 L \sin\theta e^{j\omega[t-(r/c)]}}{4\pi\varepsilon_0 c^2 r} = j\frac{I_0\beta L}{4\pi\varepsilon_0 cr}\sin\theta e^{j\omega[t-(r/c)]}$$

$$H_\phi = \frac{j\omega I_0 L \sin\theta e^{j\omega[t-(r/c)]}}{4\pi cr} = j\frac{I_0\beta L}{4\pi r}\sin\theta e^{j\omega[t-(r/c)]}$$

Taking the ratio of $E_\theta$ to $H_\phi$ as given by (17) and (18), we obtain

$$\frac{E_\theta}{H_\phi} = \frac{1}{\varepsilon_0 c} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 376.7\ \Omega \qquad \textbf{Impedance of space}$$

## 6-4 Radiation Resistance of Short Electric Dipole

Let us now calculate the radiation resistance of the short dipole of Fig. 6–1*b*. This may be done as f… The Poynting vector of the far field is integrated over a large sphere to obtain the total power radiate… power is then equated to $I^2R$ where $I$ is the rms current on the dipole and $R$ is a resistance, called the ra… resistance of the dipole.

The *average* Poynting vector is given by

$$S = \tfrac{1}{2}\operatorname{Re}(E\times H^*)$$

The far-field components are $E_\theta$ and $H_\phi$ so that the radial component of the Poynting vector is

$$S_r = \tfrac{1}{2}\operatorname{Re} E_\theta H_\phi^*$$

where $E_\theta$ and $H_\phi^*$ are complex.

The far-field components are related by the intrinsic impedance of the medium. Hence,

$$E_\theta = H_\phi Z = H_\phi\sqrt{\frac{\mu}{\varepsilon}} \quad (3)$$

Thus, (2) becomes

$$S_r = \tfrac{1}{2}\operatorname{Re} Z H_\phi H_\phi^* = \tfrac{1}{2}|H_\phi|^2 \operatorname{Re} Z = \tfrac{1}{2}|H_\phi|^2\sqrt{\frac{\mu}{\varepsilon}} \quad (4)$$

The total power $P$ radiated is then

$$P = \iint S_r \, ds = \frac{1}{2}\sqrt{\frac{\mu}{\varepsilon}} \int_0^{2\pi} \int_0^{\pi} |H_\phi|^2 r^2 \sin\theta \, d\theta \, d\phi \tag{5}$$

where the angles are as shown in Fig. 6–2 and $|H_\phi|$ is the absolute value of the magnetic field, which from (6–3–18) is

$$|H_\phi| = \frac{\omega I_0 L \sin\theta}{4\pi c r} \tag{6}$$

Substituting this into (5), we have

$$P = \frac{1}{32}\sqrt{\frac{\mu}{\varepsilon}} \frac{\beta^2 I_0^2 L^2}{\pi^2} \int_0^{2\pi} \int_0^{\pi} \sin^3\theta \, d\theta \, d\phi \tag{7}$$

The double integral equals $8\pi/3$ and (7) becomes

$$P = \sqrt{\frac{\mu}{\varepsilon}} \frac{\beta^2 I_0^2 L^2}{12\pi} \tag{8}$$

This is the *average* power or rate at which energy is streaming out of a sphere surrounding the dipole. Hence, it is equal to the power radiated. Assuming no losses, it is also equal to the power delivered to the dipole.

Therefore, $P$ must be equal to the square of the rms current $I$ flowing on the dipole times a resistance called the *radiation resistance* of the dipole. Thus,

$$\sqrt{\frac{\mu}{\varepsilon}} \frac{\beta^2 I_0^2 L^2}{12\pi} = \left(\frac{I_0}{\sqrt{2}}\right)^2 R_r$$

Solving for $R_r$,

$$R_r = \sqrt{\frac{\mu}{\varepsilon}} \frac{\beta^2 L^2}{6\pi}$$

For air or vacuum $\sqrt{\mu/\varepsilon} = \sqrt{\mu_0/\varepsilon_0} = 377 = 120\pi \, \Omega$ so that (10) becomes[1]

| | | |
|---|---|---|
| ***Dipole with uniform current*** | $R_r = 80\pi^2 \left(\frac{L}{\lambda}\right)^2 = 80\pi^2 L_\lambda^2 = 790 L_\lambda^2 \quad (\Omega)$ | ***Radiation resistance*** |

6. Derive an expression for the far field components of a small loop antenna. [10] CO5 L3

7-2

section through the loop in the $yz$ plane is presented in Fig. 7–3. Since the individual small dipoles 2 and 4 are nondirectional in the $yz$ plane, the field pattern of the loop in this plane is the same as that for two isotropic point sources as treated in Sec. 5–9. Thus,

$$E_\phi = -E_{\phi 0} e^{j\psi/2} + E_{\phi 0} e^{-j\psi/2} \tag{2}$$

where $E_{\phi 0}$ = electric field from individual dipole and

$$\psi = \frac{2\pi d}{\lambda} \sin\theta = d_r \sin\theta \tag{3}$$

It follows that

$$E_\phi = -2jE_{\phi 0} \sin\left(\frac{d_r}{2}\sin\theta\right) \tag{4}$$

The factor $j$ in (4) indicates that the total field $E_\phi$ is in phase quadrature with the field $E_{\phi 0}$ of the individual dipole. This may be readily seen by a vector construction of the type of Fig. 5–15*b* of Chap. 5. Now if $d \ll \lambda$, (4) can be written

$$E_\phi = -jE_{\phi 0} d_r \sin\theta \tag{5}$$

The far field of the individual dipole was developed in Chap. 6, being given in Table 6–1. In developing the dipole formula, the dipole was in the $z$ direction, whereas in the present case it is in the $x$ direction (see Figs. 7–2 and 7–3). The angle $\theta$ in the dipole formula is measured from the dipole axis and is 90° in the present case. The angle $\theta$ in (5) is a different angle with respect to the dipole, being as shown in Figs. 7–2 and 7–3. Thus, we have for the far field $E_{\phi 0}$ of the individual dipole

$$E_{\phi 0} = \frac{j60\pi [I] L}{r\lambda} \tag{6}$$

where $[I]$ is the retarded current on the dipole and $r$ is the distance from the dipole. Substituting (6) in (5) then gives

$$E_\phi = \frac{60\pi [I] L d_r \sin\theta}{r\lambda} \tag{7}$$

However, the length $L$ of the short dipole is the same as $d$, that is, $L = d$. Noting also that $d_r = 2\pi d/\lambda$ and that the area $A$ of the loop is $d^2$, (7) becomes

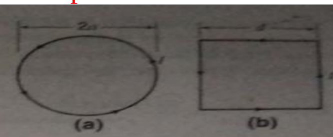



**Figure 7–1** Circular loop (*a*) and square loop (*b*) of equal area.

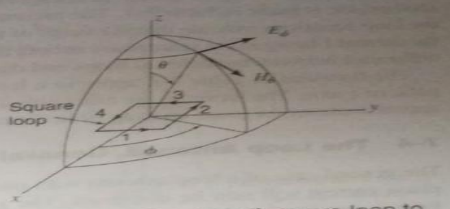



**Figure 7–2** Relation of square loop to coordinates.

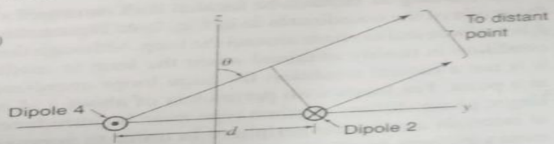
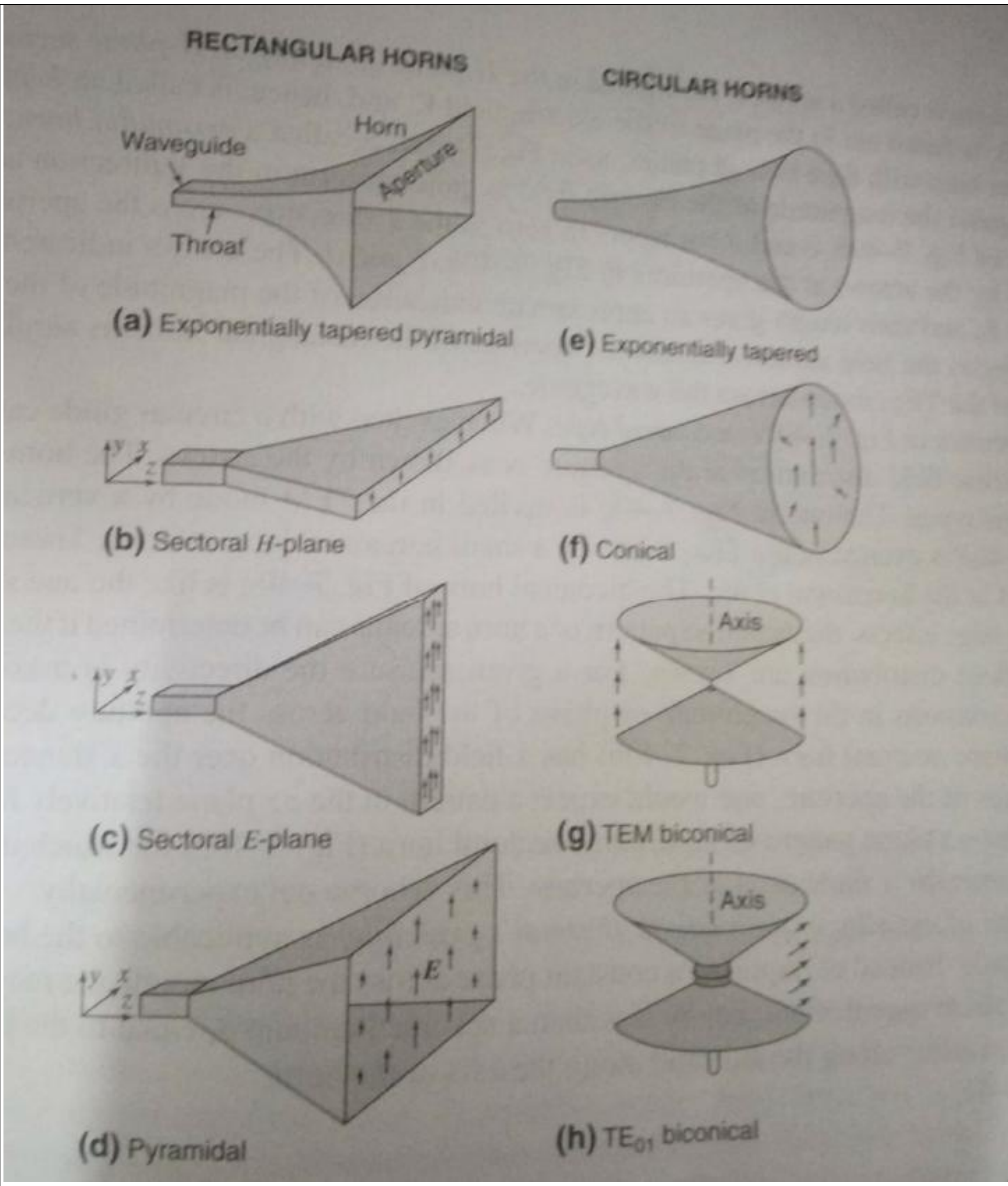



**Figure 7–3** Construction for finding far field of dipoles 2 and 4 of square loop.

| | | |
|---|---|---|
| ***Small loop*** | $E_\phi = \frac{120\pi^2 [I] \sin\theta}{r} \frac{A}{\lambda^2}$ | ***Far $E_\phi$ field*** |

(8)

| | | | | |
|---|---|---|---|---|
| | This is the instantaneous value of the $E_\phi$ component of the far field of a small loop of area $A$. The peak value of the field is obtained by replacing $[I]$ by $I_0$, where $I_0$ is the peak current in time on the loop. The other component of the far field of the loop is $H_\theta$, which is obtained from (8) by dividing by the intrinsic impedance of the medium, in this case, free space. Thus,<br>$H_\theta = \frac{E_\phi}{120\pi} = \frac{\pi[I]\sin\theta}{r}\frac{A}{\lambda^2}$ | | | |
| 7. (a) | Find the length, L, H-plane aperture and flare angles θE and θH of a pyramidal horn for which E-plane aperture is 10λ. Horn is fed by a rectangular waveguide with TE10 mode. Assume δ= 0.2λ in E-plane and 0.375λin H-plane. Also find E-plane, H-plane beam widths and directivity.<br>$L = \frac{a^2}{8\delta} = \frac{100\lambda}{8/5} = 62.5\lambda$<br>$\theta_E = 2\tan^{-1}\frac{a}{2L} = 2\tan^{-1}\frac{10}{125} = 9.1^\circ$<br>Taking $\delta = 3\lambda/8$ in the $H$ plane we have from (7–19–5) that the flare angle in the $H$ plane<br>$\theta_H = 2\cos^{-1}\frac{L}{L+\delta} = 2\cos^{-1}\frac{62.5}{62.5+0.375} = 12.52^\circ$<br>and from (7–19–5) that the $H$-plane aperture<br>$a_H = 2L\tan\frac{\theta_H}{2} = 2\times 62.5\lambda\tan 6.26^\circ = 13.7\lambda$<br>From Table 7–4,<br>$\text{HPBW } (E \text{ plane}) = \frac{56^\circ}{a_{E\lambda}} = \frac{56^\circ}{10} = 5.6^\circ$<br>$\text{HPBW } (H \text{ plane}) = \frac{67^\circ}{a_{H\lambda}} = \frac{67^\circ}{13.7} = 4.9^\circ$<br>From (3),<br>$D \simeq 10\log\left(\frac{7.5A_p}{\lambda^2}\right) = 10\log(7.5\times 10\times 13.7) = 30.1 \text{ dBi}$ | [06] | CO5 | L4 |
| (b) | Explain the different types of rectangular and circular horn antenna. For rectangular horn, write design equation for flare angle.<br>$\cos\frac{\theta}{2} = \frac{L}{L+\delta}$<br>$\sin\frac{\theta}{2} = \frac{a}{2(L+\delta)}$<br>$\tan\frac{\theta}{2} = \frac{a}{2L}$<br>where<br>$\theta$ = flare angle ($\theta_E$ for $E$ plane, $\theta_H$ for $H$ plane), deg<br>$a$ = aperture ($a_E$ for $E$ plane, $a_H$ for $H$ plane), m<br>$L$ = horn length, m<br>$\delta$ = path length difference, m<br>From the geometry we have also that<br>$L = \frac{a^2}{8\delta} \quad (\delta \ll L)$<br>and<br>$\theta = 2\tan^{-1}\frac{a}{2L} = 2\cos^{-1}\frac{L}{L+\delta}$ | [04] | CO5 | L2 |




| | | | | |
|---|---|---|---|---|
| 8. (a) | The radius of a circular loop antenna is 0.02λ. How many turns of the antenna will give a radiation resistance of 35Ω?. | [05] | CO5 | L4 |

Given,

$a = 0.02\lambda$ (radius of a circular loop)

Radiation resistance of a loop antenna, $R_r = 35\,\Omega$

$$R_r = 31,200\left(\frac{n \cdot A}{\lambda^2}\right)^2 \Omega$$

$A = \pi \cdot a^2$ (Area of a circular loop)

$$35 = 31,200\left[n \cdot \frac{\pi \cdot (0.02\lambda)^2}{\lambda^2}\right]^2$$

$$35 = 31,200\, n^2 \cdot \pi^2 \cdot \frac{(0.02\lambda)^4}{\lambda^4}$$

$$n^2 = \frac{35 \cdot \lambda^4}{31,200 \times \pi^2 \times (0.02\lambda)^4}$$

$n^2 = 710.384$

$n = 26.653$ turns

$n \cong 27$ turns (No. of turns of the circular loop antenna)

| | | | | |
|---|---|---|---|---|
| (b) | Calculate the maximum effective aperture of a thin loop antenna 0.1λ in diameter with a uniform in-phase current distribution.<br>Relation between Directivity and Effective Aperture<br>$D = \frac{4\pi}{\lambda^2}(A_e)$<br>$A_e = \frac{\lambda^2}{4\pi} D$<br>$D = \frac{3}{2}$ for small loop<br>$A_e = \frac{\lambda^2}{4\pi}\left(\frac{3}{2}\right)$<br>$A_e = 0.1193\lambda^2$ | [05] | CO5 | L3 |
| 3 (b) | Calculate the radiation resistance of a dipole of length =λ/5. Assume triangular current distribution.<br>Radiation resistance of a short dipole is<br>$Rr = 790\ (L/\lambda\text{^}2\ )$<br>Then taking L=λ/5<br>Rr=31.6 ohms | [6] | CO5 | L3 |