CMR INSTITUTE OF TECHNOLOGY

USN | | | | | | | | | |

Internal Assesment Test-III								
Sub:	Engineering Electromagnetics						Code:	17EC36
Date:	22/11 /2018	Duration:	90 mins	Max Marks:	50	Sem: 3rd	Branch:	ECE(A,B,C)
Answer FIVE FULL Questions								

		Marks	OBE CO	OBE RBT
1.(a)	Using Faraday's law derive an expression for e.m.f induced in stationary conductor placed in a time varying magnetic field. Also explain motional e.m.f.	[06]	CO4	L2
(b)	List Maxwell's equations in point and integral forms for time varying field.	[04]	CO4	L1
2.(a)	What is the inconsistency of Ampere's law with the equation of continuity? Derive modified form of Ampere's law.	[05]	CO4	L2
(b)	Let $\mu = 3 \times 10^{-5}$ H/m, $\epsilon = 1.2 \times 10^{-10}$ F/m and $\sigma = 0$ everywhere. If $\boldsymbol{H} = 2\,cos(10^{10}t - \beta x)\boldsymbol{a_z}$ A/m, use Maxwell's equations to obtain the expressions for **B**, **D**, **E.**	[05]	CO4	L3
3.(a)	Discuss TEM wave propagation in a good conductor. Define skin depth.	[07]	CO4	L2
(b)	For a medium with $\sigma = 4.0$ S/m and $\varepsilon_r = 81$, evaluate skin depth at 1 MHz.	[03]	CO4	L3
4.	State and explain Poynting's theorem.	[10]	CO5	L2
5.	Derive point form of Maxwell's equation of electrostatics.	[10]	CO1	L2
6.	Find the total charge in a volume defined by the six planes for which $1 \leq x \leq 2$, $2 \leq y \leq 3$, $3 \leq z \leq 4$ if $\mathbf{D} = 4x\,\mathbf{a_x} + 3y^2\,\mathbf{a_y} + 2z^3\,\mathbf{a_z}$ C/m^2.	[10]	CO1	L3
7.(a)	Define current and current density. Derive the equation of continuity of current.	[02+04]	CO2	L2
(b)	Let $\mathbf{D} = 5r^2\,\mathbf{a_r}$ mC/m^2 in the region for $r < 0.08$m. Find ρ_v at $r = 0.06$m.	[04]	CO1	L3
8.(a)	Derive an expression for the work done in moving a point charge Q in the presence of an electric field **E**.	[05]	CO2	L2
(b)	Non uniform field is given by $\mathbf{E} = y\,\mathbf{a_x} + x\,\mathbf{a_y} + 2\,\mathbf{a_z}$ V/m. Determine the work done in moving a charge of 2C from B(1, 0, 1) to A(0.8, 0.6, 1) along the path $x^2 + y^2 = 1, \quad z = 1$.	[05]	CO2	L3
9.	Starting from Maxwell's equations, derive wave equation for TEM wave.	[10]	CO4	L2

SCHEME OF EVALUATION

		Mark Split-Up
1.a)	Transformer e.m.f Motional e.m.f.	3 3
1.b)	Point form Integral form	2 2
2.a)	Derivation Final Expression	2 3
2.b)	**B**, **D**, **E**	1+3+1
3.a)	Wave propagation in a good conductor Skin depth	5 2
3.b)	Formula Approach & Answer	1 2
4.	Statement Derivation Final expression	2 7 1
5.	Derivation Final Expression	8 2
6.	Formula Approach & Answer	3 7
7.a)	Current and Current Density Continuity Equation	2 4
7.b)	Formula Approach & Answer	2 2
8.a)	Diagram Derivation Final Expression	1 3 1
8.b)	Formula Approach & Answer	2 3
9.	Derivation Final Expression	8 2

1a) Transformer emf:

(i) Stationary path, magnetic flux is time varying.

$$\boxed{emf = \oint_L \vec{E} \cdot d\vec{L} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}}$$

Faraday's law applied to a fixed path.

To obtain point form of this integral equation, apply stokes' theorem

$$\int_S (\nabla \times \vec{E}) \, d\vec{s} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}.$$

↓ surface integrals taken over identical surfaces.

$$(\nabla \times \vec{E}) \cdot d\vec{s} = -\frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$$

$$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$

→ One of Maxwell's four equations.

If $\vec{B}$ is not a function of time,

$$\boxed{\nabla \times \vec{E} = 0}$$

$$\boxed{\oint_L \vec{E} \cdot d\vec{L} = 0}$$

→ (electrostatics)

(ii) Motional emf: (Conductor moving in a uniform constant magnetic field)

Force on charge Q moving at a velocity $\vec{v}$ in the magnetic field $\vec{B}$.

$$\vec{F} = Q(\vec{v} \times \vec{B})$$

$$\boxed{\frac{\vec{F}}{Q} = \vec{v} \times \vec{B}}$$

Sliding bar → positive & negative charges experiencing this force.

Force per unit charge → motional electric field intensity (E_m)

$$\boxed{\vec{E}_m = \vec{v} \times \vec{B}}$$

$$\boxed{emf = \oint \vec{E}_m \cdot d\vec{L} = \oint \underbrace{(\vec{v} \times \vec{B}) \cdot d\vec{L}}_{\text{non zero only if } \vec{v} \text{ is non zero}}} \quad X.$$

$$emf = \oint_L (\vec{v} \times \vec{B}) \cdot d\vec{L} = \int_d^0 vB\, dx = -Bvd.$$

B is not a function of time

$$\boxed{emf = -Bvd}$$

1 b) Maxwell's equations in Point form: (for time varying fields)

1. $$\boxed{\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}}$$ ← from Faraday's law

2. $$\boxed{\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$ Including Displacement current density

Remaining two equations are unchanged from their non-time varying form.

3. $$\boxed{\nabla \cdot \vec{D} = \rho_v}$$ ⟹ (charge density is the source of electric flux lines)

From 1st equation, electric flux lines may form closed loops but every coulomb of charge have one coulomb of electric flux diverging from it)

4. $$\boxed{\nabla \cdot \vec{B} = 0}$$

⇓

Magnetic charges (or poles are not known to exist,

Magnetic flux is always found in closed loops & never diverges

These four equations form the basis of all electromagnetic theory. They are partial differential equations, relate electric & magnetic fields to each other & to their sources, charge & current density.

Auxiliary equations relating electric & magnetic field quantities are:

$$\vec{D} = \varepsilon \vec{E}$$

$$\vec{B} = \mu \vec{H}$$

Conduction current density $\vec{J} = \sigma \vec{E}$

Convection current density $\vec{J} = \rho_v \vec{v}$

Maxwell's equations in Integral form:

Experiments must treat physical macroscopic quantities and their results are expressed in terms of integral relationships.

① $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

Integrating over surface & apply Stokes' theorem

$$\boxed{\oint_L \vec{E} \cdot d\vec{L} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}}$$

② $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$

Integrating over surface and apply stokes theorem

$$\boxed{\oint_L \vec{H} \cdot d\vec{L} = I + \int_S \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}}$$

③ $\nabla \cdot \vec{D} = \rho_v$

Integrating over volume and apply Divergence theorem

$$\boxed{\oint_S \vec{D} \cdot d\vec{s} = \int_V \rho_v \, dv}$$

④ $\nabla \cdot \vec{B} = 0$

Integrating over volume & apply Divergence theorem

$$\boxed{\oint_S \vec{B} \cdot d\vec{s} = 0}$$

These four integral equations enable us to find boundary conditions on $\vec{B}, \vec{D}, \vec{H}$ & $\vec{E}$

The boundary conditions are in general unchanged from their forms for static or steady fields.

Displacement current :

2 a)

Point form of Ampere's circuital law for steady magnetic fields

$$\boxed{\nabla \times \vec{H} = \vec{J}} \quad (1)$$

← inadequate for time varying conditions.

Taking divergence,

$$\nabla \cdot \nabla \times \vec{H} = \nabla \cdot \vec{J} = 0.$$

But from continuity equation

$$\boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t} \neq 0}$$ for time varying conditions

Suppose we add unknown $\vec{G}$ to (1)

$$\Rightarrow \boxed{\nabla \times \vec{H} = \vec{J} + \vec{G}}$$

Taking divergence,

$$\nabla \cdot \nabla \times \vec{H} = \nabla \cdot \vec{J} + \nabla \cdot \vec{G}.$$

$$0 = \nabla \cdot \vec{J} + \nabla \cdot \vec{G}$$

$$\Rightarrow \boxed{\nabla \cdot \vec{G} = \frac{\partial \rho_v}{\partial t}}$$

w.k.t $\nabla \cdot \vec{D} = \rho_v$.

$$\nabla \cdot \vec{G} = \frac{\partial \, \nabla \cdot \vec{D}}{\partial t}$$

$$\nabla \cdot \vec{G} = \nabla \cdot \frac{\partial \vec{D}}{\partial t}$$

The solution for $\vec{G}$ is obtained as

$$\vec{G} = \frac{\partial \vec{D}}{\partial t}$$

$\therefore$ Ampere's circuital law in point form.

$$\boxed{\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}}$$

$\frac{\partial \vec{D}}{\partial t} \rightarrow$ has the dimensions of current density, A/m^2.

it results from time varying displacement flux density

Maxwell termed it as displacement current density

$$\boxed{\nabla \times \vec{H} = \vec{J} + \vec{J_d}}$$

$$\boxed{\vec{J_d} = \frac{\partial \vec{D}}{\partial t}}$$

We have already met other two current densities

Conduction current density (motion of charge in the region of zero net charge density) $\boxed{\vec{J} = \sigma \vec{E}}$

Convection current density. (motion of volume charge density) $\boxed{\vec{J} = \rho_v \vec{v}}$

In a non conducting medium, where $\rho_v = 0 \Rightarrow \vec{J} = 0$,

$$\boxed{\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}} \quad \text{if } \vec{J} = 0$$

Total displacement current crossing any given surface,

$$\boxed{I_d = \int_S \vec{J_d} \cdot \vec{ds} = \int_S \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}}$$

A.C.L. for Time varying conditions:

$$\int_S (\nabla \times \vec{H}) \cdot \vec{ds} = \int_S \vec{J} \cdot \vec{ds} + \int_S \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}$$

Apply stokes' theorem

$$\boxed{\oint_L \vec{H} \cdot \vec{dL} = I + I_d = I + \int_S \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}}$$

2 b)

$$\left.\begin{array}{l}\mu = 3\times10^{-5}\ H/m \\ \epsilon = 1.2\times10^{-10}\ F/m\end{array}\right\} \quad \sigma = 0 \qquad \vec{H} = 2\cos(10^{10}t - \beta x)\hat{a}_z$$

Calculate $\vec{B}$, $\vec{D}$, $\vec{E}$ and β.

$$\vec{B} = \mu\vec{H} = 6\times10^{-5}\cos(10^{10}t - \beta x)\,\hat{a}_z$$

$$\vec{\nabla}\times\vec{H} = \frac{\partial\vec{D}}{\partial t}$$

$$\vec{\nabla}\times\vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & 2\cos(10^{10}t - \beta x) \end{vmatrix}$$

$$= -\hat{a}_y\frac{\partial}{\partial x}\left[2\cos(10^{10}t - \beta x)\right]$$

$$= -\hat{a}_y\, 2\left(-\sin(10^{10}t - \beta x)\right)(-\beta)$$

$$= -2\sin(10^{10}t - \beta x)\beta\,\hat{a}_y$$

$$\therefore\ \vec{D} = -2\beta\int\sin(10^{10}t - \beta x)\,\hat{a}_y\,dt$$

$$= \frac{+2\beta\cos(10^{10}t - \beta x)}{10^{10}}\,\hat{a}_y$$

$$= \frac{2\beta\cos(10^{10}t - \beta x)}{10^{10}}\,\hat{a}_y\ C/m^2$$

Now, $\vec{D} = \epsilon \vec{E}$

$$\therefore \vec{E} = \frac{\vec{D}}{\epsilon} = \frac{2\beta \cos(10^{10}t - \beta x)}{1.2\times10^{-10}\times10^{10}}\,\hat{a}_y$$

$$= 1.66\,\beta \cos(10^{10}t - \beta x)\,\hat{a}_y \text{ V/m}$$

$$\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 1.66\beta\cos(10^{10}t - \beta x) & 0 \end{vmatrix}$$

$$= \hat{a}_z\left(-1.66\beta \sin(10^{10}t - \beta x)\right)(-\beta)$$

$$= +1.66\,\beta^2 \sin(10^{10}t - \beta x)\,\hat{a}_z$$

Now, $\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$

$$\therefore \vec{B} = -\int \vec{\nabla}\times\vec{E}\,dt = -\int 1.66\,\beta^2\left(\sin(10^{10}t - \beta x)\right)\hat{a}_z\,dt$$

$$= 1.66\,\beta^2\,\hat{a}_z\,\frac{\cos(10^{10}t - \beta x)}{10^{10}}$$

$$= 1.66\times10^{-10}\,\beta^2\cos(10^{10}t - \beta x)\,\hat{a}_z$$

$$\therefore 1.66\times10^{-10}\,\beta^2 = 6\times10^{-5}$$

$$\therefore \beta^2 = \frac{6\times10^{-5}}{1.66\times10^{-10}} = 3.61\times10^5 = 361000$$

$$\therefore \beta = \pm 600.$$

3 a) Wave equation: for Good Conductors and Skin effect

$$\boxed{\nabla^2 \vec{E_s} = -\gamma^2 \vec{E_s}} \rightarrow (1)$$

From Maxwell's equations,

$\nabla \times \vec{E_s} = -j\omega\mu \vec{H_s}$

$\nabla \times \vec{H_s} = \sigma \vec{E_s} + j\omega\epsilon \vec{E_s}$

$\nabla \cdot \vec{D_s} = \rho_v$

$\nabla \cdot \vec{B} = 0$

For dielectric medium (no free charges, $\rho_v = 0$)

$\nabla \cdot \vec{E_s} = 0$.

$$\nabla \times \nabla \times \vec{E_s} = -j\omega\mu \, \nabla \times \vec{H_s}$$

$$\nabla(\nabla \cdot \vec{E_s}) - \nabla^2 \vec{E_s} = -j\omega\mu \left[\sigma \vec{E_s} + j\omega\epsilon \vec{E_s}\right]$$

$$-\nabla^2 \vec{E_s} = -j\omega\mu\sigma \vec{E_s} + \omega^2\mu\epsilon \vec{E_s}$$

$$\boxed{\nabla^2 \vec{E_s} = -\left(-j\omega\mu\sigma \vec{E_s} + (\omega^2\mu\epsilon)\vec{E_s}\right)} \rightarrow (2)$$

$$\boxed{\gamma^2 = -j\omega\mu(\sigma + j\omega\epsilon)}$$

Comparing (1) & (2)

$$-\gamma^2 = -(-j\omega\mu\sigma + \omega^2\mu\epsilon) \Rightarrow -\gamma^2 = j\omega\mu\sigma - \omega^2\mu\epsilon$$

$$\sqrt{-\gamma^2} = \sqrt{j\omega\mu\sigma - \omega^2\mu\epsilon}$$

$$\pm j\gamma = \pm\sqrt{j\omega\mu\sigma - \omega^2\mu\epsilon} = \pm\sqrt{j\omega\mu\sigma + j^2\omega^2\mu\epsilon}$$

$$j\gamma = \sqrt{(j\omega)^2\mu\epsilon\left[\frac{\sigma}{\omega\epsilon j} + 1\right]}$$

$$\boxed{j\gamma = j\omega\sqrt{\mu\epsilon}\sqrt{\left[1 - j\frac{\sigma}{\omega\epsilon}\right]}}$$

(or

$$\boxed{j\gamma = j\omega\sqrt{\mu\epsilon}\left[\sqrt{1 - j\frac{\epsilon''}{\epsilon'}}\right]}$$

For good conductors,

$\sigma \simeq \infty$.

$\frac{\sigma}{\omega\epsilon} \gg 1$

$j\gamma = j\omega\sqrt{\mu\epsilon'}\sqrt{1 - j\frac{\sigma}{\omega\epsilon'}}$

$j\gamma = j\omega\sqrt{\mu\epsilon'}\sqrt{-j\frac{\sigma}{\omega\epsilon'}}$

$j\gamma = j\omega\sqrt{\mu\epsilon'}\sqrt{\frac{\sigma}{\omega\epsilon'}}\sqrt{-j}$

$\sqrt{-j} = \sqrt{1\angle -90^\circ}$

$= 1\angle -45^\circ$

$\sqrt{-j} = \frac{1-j}{\sqrt{2}}$

$j\gamma = j\sqrt{\omega\mu\sigma} \cdot \sqrt{-j}$

$= j\sqrt{\omega\mu\sigma}\,\frac{(1-j)}{\sqrt{2}}$

$= \frac{j\sqrt{\omega\mu\sigma}}{\sqrt{2}} + \frac{\sqrt{\omega\mu\sigma}}{\sqrt{2}}$

$j\gamma = \sqrt{\frac{\omega\mu\sigma}{2}} + j\sqrt{\frac{\omega\mu\sigma}{2}} = \alpha + j\beta$

$$\boxed{\alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\pi f\mu\sigma}}$$

Intrinsic impedance,

$\eta = \sqrt{\mu/\epsilon} = \sqrt{\frac{\mu}{\epsilon' - j\epsilon''}} = \sqrt{\frac{\mu}{\epsilon'(1 - j\frac{\epsilon''}{\epsilon'})}} = \sqrt{\frac{\mu}{\epsilon'(1 - j\frac{\sigma}{\omega\epsilon'})}}$

$= \sqrt{\frac{\mu}{\epsilon' - j\frac{\sigma}{\omega}}}$

$\eta = \sqrt{\frac{\mu}{\epsilon' + \frac{\sigma}{j\omega}}} = \sqrt{\frac{j\omega\mu}{\sigma + j\omega\epsilon'}}$

$\sigma >> \omega\epsilon$

$$\eta = \sqrt{\frac{j\omega\mu}{\sigma}}$$

$$\sqrt{j} = \sqrt{1 \angle 90^\circ} = 1 \angle 45^\circ$$

$$\sqrt{j} = \frac{1+j}{\sqrt{2}}$$

$$= \frac{(1+j)}{\sqrt{2}} \cdot \sqrt{\frac{\omega\mu}{\sigma}}$$

$$= \sqrt{\frac{\omega\mu}{2\sigma}} + j\sqrt{\frac{\omega\mu}{2\sigma}}$$

In polar form

$$\boxed{\eta = (1+j)\sqrt{\frac{\pi f \mu}{\sigma}}} \Rightarrow \boxed{\eta = \sqrt{\frac{2\pi f\mu}{\sigma}} \angle 45^\circ} \Rightarrow \vec{E} \text{ leads } \vec{H} \text{ by } 45^\circ.$$

Solution of Wave equation:

$$\vec{E} = \vec{E_s}\, e^{j\omega t}$$

$$\vec{E} = \Big(E_{x0}\, e^{-j\gamma z}\, e^{j\omega t} + \underbrace{E'_{x0}\, e^{j\gamma z}\, e^{j\omega t}}_{\text{Backward wave}}\Big)\, \vec{a_x}$$

Consider only forward wave

$$\vec{E} = E_{x0}\, e^{-\alpha z} \cdot e^{-j\beta z}\, e^{j\omega t}.$$

$$\boxed{\vec{E} = E_{x0}\, e^{-\alpha z} \cos(\omega t - \beta z)\, \vec{a_x}}$$

$$\vec{H} = \frac{E_{x0}}{\eta}\, e^{-\alpha z} \cos(\omega t - \beta z)\, \vec{a_y}$$

$$\vec{H} = \frac{E_{x0}}{|\eta| \angle \theta_\eta}\, e^{-\alpha z} \cos(\omega t - \beta z)\, \vec{a_y}.$$

$$\boxed{\vec{H} = \frac{E_{x0}}{\sqrt{\frac{\omega\mu}{\sigma}}}\, e^{-\alpha z} \cos(\omega t - \beta z - 45^\circ)\, \vec{a_y}}$$

As $\vec{E}$ (or $\vec{H}$) travels in a conducting medium, its amplitude is attenuated by a factor $e^{-\alpha z}$.

The distance through which the amplitude of wave decreases to a factor e^{-1} (about 37% of its original value) is called skin depth or depth of penetration of the medium.

$$E_0 e^{-\alpha z} = E_0 e^{-1}$$

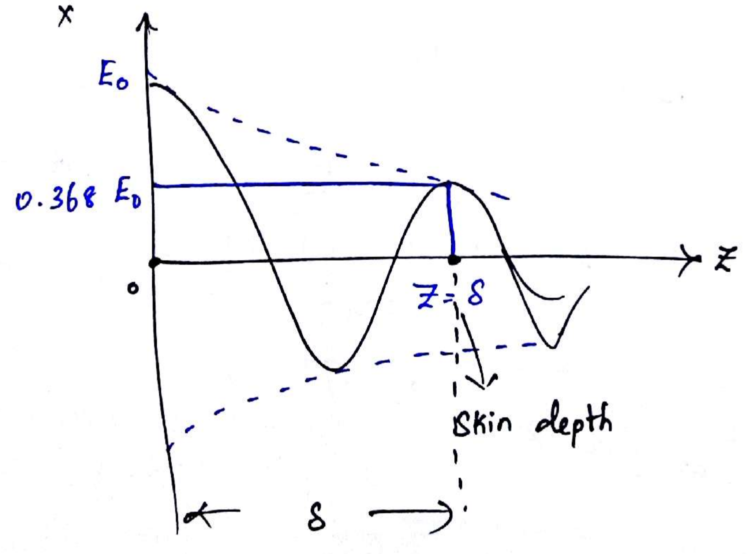
$z = \delta \leftarrow$ depth of penetration

$$e^{-\alpha \delta} = e^{-1}$$

$$\alpha \delta = 1$$

$$\boxed{\delta = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}}}$$

Skin depth is the measure of the depth to which an EM wave can penetrate the medium



$$\eta = (1+j)\sqrt{\frac{\omega\mu}{2\sigma}} \Rightarrow \eta = \sqrt{j}\sqrt{\frac{\omega\mu}{\sigma}}$$

$$= (1+j)\frac{\sqrt{\pi f \mu}}{\sqrt{\sigma}} \Rightarrow \eta = \sqrt{\frac{\omega\mu}{\sigma}}\angle 45^\circ$$

$$\eta = \sqrt{\frac{2\pi f\mu}{\sigma}}\angle 45^\circ \Rightarrow \boxed{\eta = \frac{\sqrt{2}}{\delta\sigma}\angle 45^\circ}$$

$$\boxed{\eta = \frac{(1+j)}{\delta\sigma}}$$

for good conductors,

$$\alpha = \beta = \frac{1}{\delta} = \sqrt{\pi f \mu \sigma}$$

$$\vec{E} = E_{x0}\, e^{-\alpha z} \cos(\omega t - \beta z)\, \vec{a_x}$$

$$\boxed{\vec{E} = E_{x0}\, e^{-z/\delta} \cos\left(\omega t - \frac{z}{\delta}\right) \vec{a_x}} \; V/m$$

$$\vec{H} = \frac{E_{x0}}{\eta}\, e^{-z/\delta} \cos\left(\omega t - \frac{z}{\delta}\right) \vec{a_y}$$

$$= \frac{E_{x0}}{\left(\frac{\sqrt{2}}{\delta\sigma}\right)\angle \pi/4}\, e^{-z/\delta} \cos\left(\omega t - \frac{z}{\delta}\right) \vec{a_y}$$

$$\boxed{\vec{H} = \frac{E_{x0}\,\delta\sigma}{\sqrt{2}}\, e^{-z/\delta} \cos\left(\omega t - \frac{z}{\delta} - \pi/4\right) \vec{a_y}} \; A/m$$

Solution. We first evaluate the loss tangent, using the given data:

3 b)

$$\frac{\sigma}{\omega\epsilon'} = \frac{4}{(2\pi \times 10^6)(81)(8.85 \times 10^{-12})} = 8.9 \times 10^2 \gg 1$$

Seawater is therefore a good conductor at 1 MHz (and at frequencies lower than this). The skin depth is

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{(\pi \times 10^6)(4\pi \times 10^{-7})(4)}} = 0.25 \text{ m} = 25 \text{ cm}$$

Now

$$\lambda = 2\pi\delta = 1.6 \text{ m}$$

and

$$v_p = \omega\delta = (2\pi \times 10^6)(0.25) = 1.6 \times 10^6 \text{ m/sec}$$

4)

Poynting's theorem & Wave power:

Poynting's theorem.

It states that the net power flowing out of a given volume is equal to the time rate of decrease in stored energy within the volume minus conduction losses.

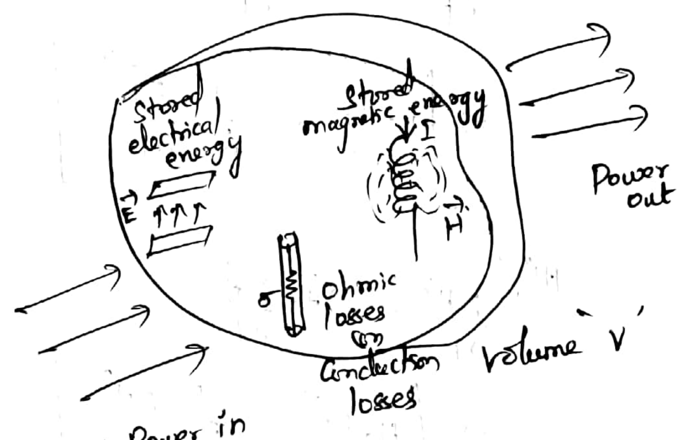


Fig. Illustration of power balance for EM fields.

Proof:

Maxwell's equations

$$\vec{\nabla}\cdot\vec{D} = \rho_V$$

$$\vec{\nabla}\times\vec{E} = -\mu\frac{\partial \vec{H}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{H} = \sigma\vec{E} + \varepsilon\frac{\partial \vec{E}}{\partial t}.$$

Vector identity:

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{H}\cdot\vec{\nabla}\times\vec{E} - \vec{E}\cdot\vec{\nabla}\times\vec{H}$$

Apply Maxwell's equations into this vector identity,

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\vec{H}\cdot\mu\frac{\partial \vec{H}}{\partial t} - \vec{E}\cdot\left[\sigma\vec{E} + \frac{\varepsilon\partial \vec{E}}{\partial t}\right]$$

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\mu\,\vec{H}\cdot\frac{\partial \vec{H}}{\partial t} - \sigma E^2 - \varepsilon\,\vec{E}\cdot\frac{\partial \vec{E}}{\partial t}.$$

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\sigma E^2 - \varepsilon\,\vec{E}\cdot\frac{\partial \vec{E}}{\partial t} - \mu\cdot\vec{H}\cdot\frac{\partial \vec{H}}{\partial t}$$

$\vec{E}\cdot\frac{\partial \vec{E}}{\partial t} = ?$

$\vec{H}\cdot\frac{\partial \vec{H}}{\partial t} = ?$

Let

$$\frac{\partial \vec{E}^2}{\partial t} = 2\vec{E}\cdot\frac{\partial \vec{E}}{\partial t}$$

$$\Rightarrow \boxed{\vec{E}\cdot\frac{\partial \vec{E}}{\partial t} = \frac{1}{2}\frac{\partial \vec{E}^2}{\partial t}}$$

III$^{\text{rly}}$

$$\frac{\partial H^2}{\partial t} = 2\vec{H}\cdot\frac{\partial \vec{H}}{\partial t}$$

$$\Rightarrow \boxed{\vec{H}\cdot\frac{\partial \vec{H}}{\partial t} = \frac{1}{2}\frac{\partial H^2}{\partial t}.}$$

Point form.

$$\boxed{\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = -\sigma E^2 - \frac{1}{2}\cdot\varepsilon\frac{\partial E^2}{\partial t} - \frac{1}{2}\mu\frac{\partial H^2}{\partial t}.}$$

Integrating over the given volume,

$$\underbrace{\iiint_V \vec{\nabla}\cdot(\vec{E}\times\vec{H})\,dv} = -\iiint_V \sigma E^2 dv - \frac{1}{2}\iiint_V \varepsilon\frac{\partial E^2}{\partial t}dv - \frac{1}{2}\iiint_V \mu\frac{\partial H^2}{\partial t}dv$$

Apply Divergence theorem

Divergence theorem,
$$\iiint_V (\vec{\nabla}\cdot\vec{A})\,dv = \oiint_S \vec{A}\cdot d\vec{s}$$

∴ The equation becomes — Poynting theorem.

$$\boxed{\underbrace{\oint\!\!\oint_S (\vec{E}\times\vec{H})\cdot\vec{ds}}_{\text{net power flowing out of the volume}} = \underbrace{-\sigma\iiint_V E^2\,dv}_{\text{Ohmic losses (or) Conduction losses}} \underbrace{-\frac{d}{dt}\left[\frac{1}{2}\iiint_V \varepsilon E^2\,dv\right]}_{\text{Rate of decrease in stored electric energy}} \underbrace{-\frac{d}{dt}\left[\frac{1}{2}\iiint_V \mu H^2\,dv\right]}_{\text{Rate of decrease in stored magnetic energy}}}$$

Hence proved.

w.k.t.

Electric potential energy, $\boxed{W_E = \frac{1}{2}\iiint_V \varepsilon E^2\,dv}$

and

Magnetic potential energy, $\boxed{W_H = \frac{1}{2}\iiint_V \mu H^2\,dv}$

Power flow of an electromagnetic wave

$$\boxed{P = \oint\!\!\oint_S (\vec{E}\times\vec{H})\cdot\vec{ds}} \quad W$$

where $\boxed{\vec{E}\times\vec{H} = \vec{P}}$ = Poynting vector = Power density vector

(or)

$\boxed{\vec{E}\times\vec{H} = \vec{S}}$ = Poynting vector (W/m^2)

5)

Application of Gauss' law:

Differential Volume element.

No symmetry.

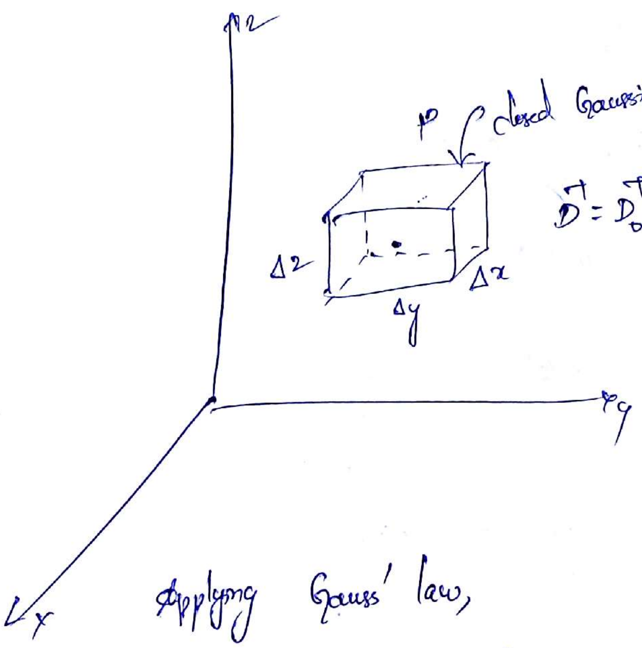
Choose a small gaussian surface → almost $\vec{D}$ is constant over that surface.

Result becomes correct only when volume $\Delta v \to 0$ (shrinks).

We will not obtain $\vec{D}$, obtain the valuable information about the way $\vec{D}$ varies in the region.

⇓

One of Maxwell's four equations (base to all electromagnetic theory).



$$\vec{D} = \vec{D_0} = D_{x0}\vec{a_x} + D_{y0}\vec{a_y} + D_{z0}\vec{a_z}.$$

Applying Gauss' law,

$$\oint_S \vec{D}\cdot\vec{ds} = Q$$

$$\int_{front} + \int_{back} + \int_{left} + \int_{Right} + \int_{top} + \int_{bottom} \vec{D}\cdot\vec{ds} = Q.$$

Surface element is very small → $\vec{D}$ is constant over the surface.

$$\int_{front} \vec{D}\cdot\vec{ds} = \vec{D}_{front}\cdot\vec{\Delta S}_{front}.$$

$$= \vec{D}_{front}\cdot \Delta y\,\Delta z\,\vec{a_x}.$$

$$= D_{x,front}\cdot \Delta y\,\Delta z$$

front face $\Rightarrow$ distance of $\frac{\Delta x}{2}$ from P.

$$\boxed{D_{x,\text{front}} = D_{x0} + \frac{\Delta x}{2} \cdot \frac{\partial D_x}{\partial x}}$$

Rate of change of D_x with x

$\because$ D_x varies with y & z.

value of D_x at P.

Expansion of Taylor's series.

$$\boxed{\int_{\text{front}} = \left(D_{x0} + \frac{\Delta x}{2} \frac{\partial D_x}{\partial x}\right) \Delta y \, \Delta z}$$

Integral over the back surface

$$\int_{\text{back}} = \vec{D}_{\text{back}} \cdot \Delta \vec{S}_{\text{back}}$$

$$= \vec{D}_{\text{back}} \cdot (-\Delta y \, \Delta z \, \vec{a}_x)$$

$$\int_{\text{back}} = -D_{x,\text{back}} \, \Delta y \, \Delta z.$$

$$\int_{\text{back}} = -\left[D_{x0} + \left(-\frac{\Delta x}{2}\right) \frac{\partial D_x}{\partial x}\right] \Delta y \, \Delta z$$

$$\boxed{\int_{\text{back}} = \left(-D_{x0} + \frac{\Delta x}{2} \frac{\partial D_x}{\partial x}\right) \Delta y \, \Delta z}$$

$$\boxed{\int_{\text{front}} + \int_{\text{back}} = \frac{\partial D_x}{\partial x} \Delta x \, \Delta y \, \Delta z.}$$

Exactly in the same way..

$$\int_{\text{right}} + \int_{\text{left}} = \frac{\partial D_y}{\partial y} \Delta x \, \Delta y \, \Delta z$$

&

$$\int_{\text{top}} + \int_{\text{bottom}} = \frac{\partial D_z}{\partial z} \Delta x \, \Delta y \, \Delta z.$$

Scanned by CamScanner

$$\therefore \oint_S \vec{D}\cdot d\vec{s} = \frac{\partial D_x}{\partial x}\Delta x \Delta y \Delta z + \frac{\partial D_y}{\partial y}\Delta x \Delta y \Delta z + \frac{\partial D_z}{\partial z}\Delta x \Delta y \Delta z$$

$$\boxed{\oint_S \vec{D}\cdot d\vec{s} = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right)\Delta x \Delta y \Delta z = Q}$$

It is an approximation which is better when Δv becomes smaller.

$\therefore \Delta v \to 0$. where $\Delta v = \Delta x \Delta y \Delta z$

$$\boxed{\text{Charge enclosed in volume } \Delta v = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right)\cdot \Delta v}$$

We now obtain the exact relationship by $\Delta v \to 0$.

$$\left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) = \frac{\oint_S \vec{D}\cdot d\vec{s}}{\Delta v} = \frac{Q}{\Delta v}.$$

$$\left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) = \lim_{\Delta v \to 0} \frac{\oint_S \vec{D}\cdot d\vec{s}}{\Delta v} = \left(\lim_{\Delta v \to 0} \frac{Q}{\Delta v}\right) \rightarrow \text{volume charge density}$$

$$\boxed{\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \lim_{\Delta v \to 0} \frac{\oint_S \vec{D}\cdot d\vec{s}}{\Delta v}} \rightarrow (1)$$

&

$$\boxed{\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho_v.} \rightarrow (2)$$

Divergence of $\vec{D}$ = div $\vec{D} = \lim_{\Delta v \to 0} \frac{\oint_S \vec{D} \cdot d\vec{s}}{\Delta v}$

Divergence of the vector flux density $\vec{A}$ is the outflow of flux from a small closed surface per unit volume as the volume shrinks to zero.

$$\boxed{\text{div } \vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}}$$ Rectangular

$$\boxed{\text{div } \vec{D} = \frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho D_\rho) + \frac{1}{\rho}\frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}}$$ cylindrical

$$\boxed{\text{div } \vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 D_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta \, D_\theta) + \frac{1}{r\sin\theta}\frac{\partial D_\phi}{\partial \phi}}$$ Spherical

$$\left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) = \frac{\oint_S \vec{D} \cdot d\vec{s}}{\Delta v} = \frac{Q}{\Delta v}.$$

$$\left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) = \lim_{\Delta v \to 0} \frac{\oint_S \vec{D} \cdot d\vec{s}}{\Delta v} = \left(\lim_{\Delta v \to 0} \frac{Q}{\Delta v}\right) \rightarrow \text{volume charge density}$$

$$\boxed{\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \lim_{\Delta v \to 0} \frac{\oint_S \vec{D} \cdot d\vec{s}}{\Delta v}} \rightarrow (1)$$

&

$$\boxed{\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho_v,} \rightarrow (2)$$

(x) MAXWELL's FIRST EQUATION OF ELECTROSTATICS:

1) $\operatorname{div} \vec{D} = \lim_{\Delta v \to 0} \dfrac{\oint_s \vec{D} \cdot d\vec{s}}{\Delta v}$ (Definition of divergence)

2) $\operatorname{div} \vec{D} = \dfrac{\partial D_x}{\partial x} + \dfrac{\partial D_y}{\partial y} + \dfrac{\partial D_z}{\partial z}$ (Result of applying the definition to a differential volume element (in Rectangular co-ordinates)

3) $\operatorname{div} \vec{D} = \rho_v$

Gauss's law

flux leaving any closed surface

$$\boxed{\oint_s \vec{D} \cdot d\vec{s} = Q}$$

charge enclosed

Gauss' law per unit volume,

$$\frac{\oint_s \vec{D} \cdot d\vec{s}}{\Delta v} = \frac{Q}{\Delta v}$$

As the volume shrinks to zero,

$$\lim_{\Delta v \to 0} \frac{\oint_S \vec{D} \cdot d\vec{s}}{\Delta v} = \lim_{\Delta v \to 0} \frac{Q}{\Delta v}$$

Divergence

volume charge density

$$\boxed{\operatorname{div} \vec{D} = \rho_v.}$$

(X)

First of Maxwell's four equations

Point form of Gauss's law (or) Maxwell's first equation

Statement:

The electric flux per unit volume leaving a vanishingly small volume unit is exactly equal to the volume charge density there.

Gauss' law →

$$\boxed{\oint_S \vec{D} \cdot d\vec{s} = Q = \int_V \rho_v \, dv.}$$

(XI) Integral form of Maxwell's first equation

6) 9)

Find the total charge in the volume defined by six planes

$1 \le x \le 2$
$2 \le y \le 3$ if $\vec{D} = 4x\vec{a_x} + 3y^2\vec{a_y} + 2z^3\vec{a_z}$ C/m^2
$3 \le z \le 4$

$\vec{\nabla}\cdot\vec{D} = \frac{\partial}{\partial x}(4x) + \frac{\partial}{\partial y}(3y^2) + \frac{\partial}{\partial z}(2z^3)$

$\rho_v = \vec{\nabla}\cdot\vec{D}$.

$Q_{enc} = \int_v (\vec{\nabla}\cdot\vec{D})\,dv$.

$\rho_v = 4 + 6y + 6z^2$ C/cu.m.

$$Q_{enc} = \iiint (4+6y+6z^2)\,dx\,dy\,dz$$

$$= \iiint_v 4\,dx\,dy\,dz + \iiint_v 6y\,dx\,dy\,dz + \iiint 6z^2\,dx\,dy\,dz$$

$$= 4\times1\times1\times1 + \frac{6}{2}(1)(9-4)(1) + \frac{6}{3}(1)(1)(4^3-3^3)$$

$$= 4 + 15 + 74$$

$$\boxed{Q_{enc} = 93\,C}$$

18) Surface integral: $\vec{D} = 4x\vec{a_x} + 3y^2\vec{a_y} + 2z^3\vec{a_z}$ C/m^2

$1 \le x \le 2,\ 2 \le y \le 3,\ 3 \le z \le 4$.

$$\oint_S \vec{D}\cdot\vec{ds} = -\iint_S 4x\,dy\,dz\Big|_{x=1} + \iint_S 4x\,dy\,dz\Big|_{x=2} - \iint_S 3y^2\,dx\,dz\Big|_{y=2} + \iint_S 3y^2\,dx\,dz\Big|_{y=3}$$

$$- \iint_S 2z^3\,dx\,dy\Big|_{z=3} + \iint_S 2z^3\,dx\,dy\Big|_{z=4}$$

$$= -4[y]_2^3[z]_3^4 + 4[2][y]_2^3[z]_3^4 - 3(2)^2[x]_1^2[z]_3^4 + 3(3)^2[x]_1^2[z]_3^4$$

$$- 2(3)^3[x]_1^2[y]_2^3 + 2(4)^3[x]_1^2[y]_2^3$$

$$= -4 + 8 - 12 + 27 - 54 + 128$$

$$\boxed{\oint_S \vec{D}\cdot\vec{ds} = 93\ C}$$

7 a) Continuity of Current:

principle of conservation of charge and continuity equation follow the principle

Charges can neither be created nor be destroyed, although equal amounts of positive and negative charge may be simultaneously created, obtained by separation, destroyed or lost by recombination

Consider any region bounded by a closed surface. Current through the closed surface,

$$\boxed{I = \oint_S \vec{J}\cdot d\vec{s}}$$

This outward flow of positive charge must be balanced by a decrease of positive charge within that closed surface.

charge inside closed surface $\rightarrow Q_i \Rightarrow$ rate of decrease of $Q_i = -\frac{dQ_i}{dt}$.

Principle of conservation of charge

$$\boxed{I = \oint_S \vec{J}\cdot d\vec{s} = -\frac{dQ_i}{dt}}$$ Integral form of Continuity equation

Differential form:

Divergence theorem

$$\oint_S \vec{J}\cdot d\vec{s} = \int_V (\nabla\cdot\vec{J})\, dv.$$

& $Q_i = \int_V \rho_v\, dv.$

$$\frac{dQ_i}{dt} = \frac{d}{dt}\int_V \rho_v\, dv.$$

closed surface is constant $\Rightarrow \frac{dQ_i}{dt} = \int_V \frac{\partial \rho_v}{\partial t}\, dv.$

$$\therefore \boxed{I = \int_V (\nabla\cdot\vec{J})\, dv = -\int_V \frac{\partial \rho_v}{\partial t}\, dv.}$$

Since the expression is true for any volume, however small, it is true for an incremental volume Δv.

$$\therefore \boxed{(\nabla\cdot\vec{J})\,\Delta v = -\frac{\partial \rho_v}{\partial t}\cdot\Delta v}$$

$$\boxed{\nabla \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}}$$ Point form of Continuity equation.

$\Downarrow$

Current or charge per second diverging from a small volume per unit volume is equal to the time rate of decrease of charge per unit volume at every point.

7 b)

(i)

$\rho_v = \vec{\nabla} \cdot \vec{D} = \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 D_r) + 0 + 0$

$\underline{\underline{r = 0.06}}$

$\vec{D} = 5r^2 \vec{a_r}$

$= \frac{1}{r^2} \frac{\partial}{\partial r} (5r^4) \, m$

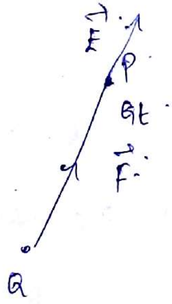
$= \frac{1}{r^2} \cdot 20r^3 \, m$

$\rho_v = 20r \quad m$

$\boxed{\rho_v \Big|_{r=0.06} = 1.2 \; m} \; e/m^3$

8 a)

Energy expended in moving a point charge in an electric field :



$\vec{E}$ ← force per unit test charge at P.

to Move the test charge against $\vec{E}$, we require force equal & opposite to the field.

↓

requires expend energy (or) do work.

If we wish to Move a charge Q, a distance $\vec{dL}$ in an electric field $\vec{E}$.

Force on Q due to $\vec{E}$, $\boxed{\vec{F_E} = Q\vec{E}}$.

force arises from electric field.

The component of this force in the direction $\vec{dL}$,

$F_{EL} = \vec{F_E} \cdot \vec{a_L}$ ← unit vector in the direction of $\vec{dL}$

$$\boxed{F_{EL} = Q\vec{E} \cdot \vec{a_L}}$$

The force we must apply is equal and opposite to F_{EL}.

$$\boxed{F_{appl} = - Q\vec{E} \cdot \vec{a_L}}$$

Energy expended = force x distance

Differential work done by an external source moving charge Q is

$$dW = - Q\vec{E} \cdot \vec{a_L} \cdot dL$$

$$\boxed{dW = - Q\vec{E} \cdot \vec{dL}}$$

Work required to move a charge a finite distance,

$$\boxed{W = - Q \int_{initial}^{final} \vec{E} \cdot \vec{dL}}$$

Path must be specified

Charge is assumed at rest at both its initial & final position

8 b)

EXAMPLE 4.1

We are given the nonuniform field

$$\mathbf{E} = y\mathbf{a}_x + x\mathbf{a}_y + 2\mathbf{a}_z$$

and we are asked to determine the work expended in carrying 2C from $B(1, 0, 1)$ to $A(0.8, 0.6, 1)$ along the shorter arc of the circle

$$x^2 + y^2 = 1 \quad z = 1$$

Solution. We use $W = -Q \int_B^A \mathbf{E} \cdot d\mathbf{L}$, where $\mathbf{E}$ is not necessarily constant. Working in rectangular coordinates, the differential path $d\mathbf{L}$ is $dx\mathbf{a}_x + dy\mathbf{a}_y + dz\mathbf{a}_z$, and the integral becomes

$$\begin{aligned} W &= -Q \int_B^A \mathbf{E} \cdot d\mathbf{L} \\ &= -2 \int_B^A (y\mathbf{a}_x + x\mathbf{a}_y + 2\mathbf{a}_z) \cdot (dx\,\mathbf{a}_x + dy\,\mathbf{a}_y + dz\,\mathbf{a}_z) \\ &= -2 \int_1^{0.8} y\,dx - 2 \int_0^{0.6} x\,dy - 4 \int_1^1 dz \end{aligned}$$

where the limits on the integrals have been chosen to agree with the initial and final values of the appropriate variable of integration. Using the equation of the circular path (and selecting the sign of the radical which is correct for the quadrant involved), we have

$$\begin{aligned} W &= -2 \int_1^{0.8} \sqrt{1 - x^2}\,dx - 2 \int_0^{0.6} \sqrt{1 - y^2}\,dy - 0 \\ &= -\left[x\sqrt{1 - x^2} + \sin^{-1} x\right]_1^{0.8} - \left[y\sqrt{1 - y^2} + \sin^{-1} y\right]_0^{0.6} \\ &= -(0.48 + 0.927 - 0 - 1.571) - (0.48 + 0.644 - 0 - 0) \\ &= -0.96\,\text{J} \end{aligned}$$

9)

General Wave equation (for the media considered) of UPW

Maxwell's equations:

$$\vec{\nabla}\cdot\vec{D} = 0 \Longrightarrow \vec{\nabla}\cdot\vec{E} = 0 \rightarrow (1)$$

$$\vec{\nabla}\times\vec{E} = -\mu\frac{\partial \vec{H}}{\partial t} \Longrightarrow \vec{\nabla}\times\vec{E} = -\mu\frac{\partial \vec{H}}{\partial t} \rightarrow (2)$$

$$\vec{\nabla}\cdot\vec{B} = 0 \Longrightarrow \vec{\nabla}\cdot\vec{H} = 0 \rightarrow (3)$$

$$\vec{\nabla}\times\vec{H} = \sigma\vec{E} + \varepsilon\frac{\partial \vec{E}}{\partial t} \Longrightarrow \vec{\nabla}\times\vec{H} = \sigma\vec{E} + \varepsilon\frac{\partial \vec{E}}{\partial t} \rightarrow (4)$$

Take curl on both sides of (2)

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\vec{\nabla}\times\left(\mu\frac{\partial \vec{H}}{\partial t}\right).$$

Vector identity $\rightarrow$ $\vec{\nabla}\times\vec{\nabla}\times\vec{E} = \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) - \nabla^2\vec{E}$

$$\vec{\nabla}(\cancel{\vec{\nabla}\cdot\vec{E}}) - \nabla^2\vec{E} = -\mu\frac{\partial}{\partial t}(\vec{\nabla}\times\vec{H}).$$

Apply equation (1)   Apply equation (4)

$$-\nabla^2\vec{E} = -\mu\frac{\partial}{\partial t}\left[\sigma\vec{E} + \varepsilon\frac{\partial \vec{E}}{\partial t}\right]$$

$$-\nabla^2\vec{E} = -\mu\sigma\frac{\partial \vec{E}}{\partial t} - \mu\varepsilon\frac{\partial^2 \vec{E}}{\partial t^2}$$

$$\nabla^2\vec{E} = \mu\sigma\frac{\partial \vec{E}}{\partial t} + \mu\varepsilon\frac{\partial^2 \vec{E}}{\partial t^2}$$

For UPW $\vec{E}(z,t)$ and $\vec{H}(z,t)$

$$\nabla^2\vec{E} = \frac{\partial^2\vec{E}}{\partial z^2}$$

$$\boxed{\frac{\partial^2\vec{E}}{\partial z^2} = \mu\sigma\frac{\partial \vec{E}}{\partial t} + \mu\varepsilon\frac{\partial^2 \vec{E}}{\partial t^2}} \rightarrow (5)$$

General wave equation for $\vec{E}$ field of UPW.

Similarly,

Take curl on both sides of (4)

$$\vec{\nabla}\times\vec{\nabla}\times\vec{H} = \vec{\nabla}\times\left(\sigma\vec{E} + \varepsilon\frac{\partial\vec{E}}{\partial t}\right)$$

$$\vec{\nabla}(\vec{\nabla}\cdot\vec{H}) - \nabla^2\vec{H} = \sigma(\vec{\nabla}\times\vec{E}) + \varepsilon\frac{\partial(\vec{\nabla}\times\vec{E})}{\partial t}$$

Applying equation (3) applying equation (2)

$$-\nabla^2\vec{H} = -\sigma\cdot\mu\frac{\partial\vec{H}}{\partial t} - \varepsilon\mu\frac{\partial^2\vec{H}}{\partial t^2}$$

$$\boxed{\nabla^2\vec{H} = \mu\sigma\frac{\partial\vec{H}}{\partial t} + \mu\varepsilon\frac{\partial^2\vec{H}}{\partial t^2}}$$

and $\vec{H}(z,t)$

$$\nabla^2\vec{H} = \frac{\partial^2\vec{H}}{\partial z^2}$$

$$\boxed{\frac{\partial^2\vec{H}}{\partial z^2} = \mu\sigma\frac{\partial\vec{H}}{\partial t} + \mu\varepsilon\frac{\partial^2\vec{H}}{\partial t^2}} \quad (6)$$

$\Rightarrow$ General wave equation for $\vec{H}$ field of UPW.

For free space equations (5) and (6) becomes.

$\sigma = 0$

$\varepsilon = \varepsilon_0$

$\mu = \mu_0$

wave equation for free space for $\vec{E}$ field

$$(5) \Rightarrow \boxed{\frac{\partial^2\vec{E}}{\partial z^2} = \mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}} \quad \text{(or)} \quad \boxed{\frac{\partial^2\vec{E}}{\partial t^2} = \frac{1}{\mu_0\varepsilon_0}\frac{\partial^2\vec{E}}{\partial z^2}}$$

Wave equation for free space for $\vec{H}$-field.

(6) $\Rightarrow$ $$\boxed{\frac{\partial^2 \vec{H}}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{H}}{\partial t^2}} \quad \text{(or)} \quad \boxed{\frac{\partial^2 \vec{H}}{\partial t^2} = \frac{1}{\mu_0 \epsilon_0} \frac{\partial^2 \vec{H}}{\partial z^2}}$$

where $v_p = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 = c$ = velocity of light (or) EM waves in free space.

$$\frac{1}{\sqrt{\mu_0 \epsilon_0}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times \frac{10^{-9}}{36\pi}}} = \frac{1}{\sqrt{\frac{10^{-16}}{9}}} = \sqrt{9 \times 10^{+16}} = 3 \times 10^8 \text{ m/s.}$$

Wave equations for free space can be written as

$$\boxed{\frac{\partial^2 \vec{E}}{\partial z^2} = \frac{1}{v_p^2} \frac{\partial^2 \vec{E}}{\partial t^2}} \quad \text{(or)} \quad \boxed{\frac{\partial^2 \vec{E}}{\partial t^2} = v_p^2 \frac{\partial^2 \vec{E}}{\partial z^2}}$$

and

$$\boxed{\frac{\partial^2 \vec{H}}{\partial z^2} = \frac{1}{v_p^2} \frac{\partial^2 \vec{H}}{\partial t^2}} \quad \text{(or)} \quad \boxed{\frac{\partial^2 \vec{H}}{\partial t^2} = v_p^2 \frac{\partial^2 \vec{H}}{\partial z^2}}$$