CMR INSTITUTE OF TECHNOLOGY	USN	CMRIT

Internal Assesment Test-III

Sub:	Engineering Electromagnetics							Code:	17EC36
Date:	22/11 /2018	Duration:	90 mins	Max Marks:	50	Sem:	3rd	Branch:	ECE(A,B,C)

Answer FIVE FULL Questions

		Marks	OBE CO	OBE RBT
1.(a)	Using Faraday's law derive an expression for e.m.f induced in stationary conductor placed in a time varying magnetic field. Also explain motional e.m.f.	[06]	CO4	L2

– Two separate windings on an iron toroid and placed a galvanometer in one ckt. and battery on the other.

– Closing the battery ckt, momentary deflection of the galvanometer.

– A similar deflection in the opposite direction occurred when the battery was disconnected.

– either a moving mag. field or a moving coil could also produce a galvanometer deflection

– Time varying mag field produces an emf which may establish a current in a suitable closed ckt.

Faraday's law: $e \propto -\frac{d\phi}{dt} \Rightarrow e = -k\frac{d\phi}{dt}$ in S.I $k=1$

$$\text{emf} = -\frac{d\phi}{dt} \qquad \therefore e = -\frac{d\phi}{dt}$$

~~on the~~ we consider a closed path, conducting line or not, the mag. flux is that flux that passes through any and every surface whose perimeter is the closed path.

and $\frac{d\phi}{dt} \rightarrow$ time rate of change of flux

A non zero value of $\frac{d\phi}{dt}$ occurs when:

i) Time-changing flux linking a closed path

ii) Relative motion b/w a steady flux and a closed path.

iii) A combination of the two.

The minus sign is an indication that the emf is in such a direction as to produce a current whose flux, if added to the original flux, would reduce the magnitude of the emf.

Induced voltage acts to produce an opposing flux is Lenz's law.

– If the closed path is that taken N-turn filamentary conductor, it is sufficiently accurate to consider the turns as coincident and let,

$emf = -N \frac{d\phi}{dt}$

where, $\phi \rightarrow$ flux passing through any one of the coincident paths.

e.m.f is defined as,

$emf = \oint \vec{E} \cdot d\vec{l} \rightarrow$ voltage about a specific closed path

for electrostatic $\oint \vec{E} \cdot d\vec{l} = 0$

$e.m.f = \oint \vec{E} \cdot d\vec{l} = -\frac{d\phi}{dt} = -\frac{d}{dt}\int \vec{B} \cdot d\vec{s}$

fingers of our right hand indicate the direction of the closed path. thumb indicates the direction of $d\vec{s}$

– A flux density $\vec{B}$ in the direction of $d\vec{s}$ and increasing with time thus produces average value of $\vec{E}$ which is opposite to the +ve direction about the path

Stationary path.

magnetic flux $\rightarrow$ time varying quantity.

$\therefore emf = \oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$

Applying Stokes's theorem

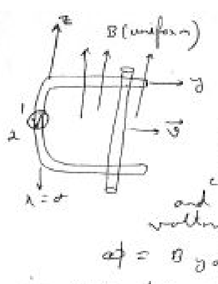
$\int_s (\vec{\nabla} \times \vec{E}) \cdot d\vec{s} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$

Surface integrals taken over identical general surfaces.

$\therefore (\vec{\nabla} \times \vec{E}) \cdot d\vec{s} = -\frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$

$\Rightarrow \boxed{(\vec{\nabla} \times \vec{E}) = -\frac{\partial \vec{B}}{\partial t}} \rightarrow$ differential or point form

$\oint \vec{E} \cdot d\vec{l} = -\int \frac{\partial B}{\partial t} \cdot d\vec{s} \rightarrow$ integral



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constant magnetic flux density B and moving path. The sliding bar moves to the right with a velocity v and the ckt. is completed through the two rails and an extremely small high-resistance voltmeter. The flux passing through the surface within the closed path.

$\phi = B y d$.

$\therefore$ Voltmeter reading: $emf = -\frac{d\phi}{dt} = -B \frac{dy}{dt} d$

$= -Bvd$.

Motional emf

(b)	List Maxwell's equations in point and integral forms for time varying field.	[04]	CO4	L1

<table>
<tr><td></td><td>Maxwell's equations in point form
$\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$
$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$
$\vec{\nabla} \cdot \vec{D} = \rho_v$
$\vec{\nabla} \cdot \vec{B} = 0$

(i) Faraday's law: $\oint \vec{E} \cdot d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}$
$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \Rightarrow \int (\vec{\nabla} \times \vec{E}) \cdot d\vec{s} = -$
$\therefore \boxed{\oint \vec{E} \cdot d\vec{l} = -\int_s \frac{\partial \vec{B}}{\partial t} \cdot d\vec{s}}$

(ii) Ampere's circuital law:
$\oint \vec{H} \cdot d\vec{L} = I + \int_s \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}$
$\int_s \vec{\nabla} \times \vec{H} = \int_s \vec{J} \cdot d\vec{s} + \int_s \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}$
$\Rightarrow \vec{\nabla} \times \vec{H} = I + \int_s \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}$

(iii) Gauss's law for electric field
$\oint (\vec{D} \cdot d\vec{s}) = \int_{vol} \rho_v \, dv$
$\vec{\nabla} \cdot \vec{D} = \rho_v$
$\int_{vol} \vec{\nabla} \cdot \vec{D} \, dv = \int_{vol} \rho_v \, dv$
$\Rightarrow \oint_s \vec{D} \cdot d\vec{s} = \int_{vol} \rho_v$

(iv) Gauss's law for mag. field
$\vec{\nabla} \cdot \vec{B} = 0$
$\Rightarrow \boxed{\oint_s \vec{B} \cdot d\vec{s} = 0}$</td><td></td><td></td><td></td></tr>
<tr><td>2.(a)</td><td>What is the inconsistency of Ampere's law with the equation of continuity? Derive modified form of Ampere's law.</td><td>[05]</td><td>CO4</td><td>L2</td></tr>
</table>

From Faraday's experimental law,

$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$.

If curl $\neq 0$ → the field has the special property of circulation.
Its line integral about a general closed path is not zero.

From Ampere's circuital law, $(\vec{\nabla} \times \vec{H}) = \vec{J}$. ①

∴ $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) \equiv 0 = \vec{\nabla} \cdot \vec{J}$
identically.

⇒ $\vec{\nabla} \cdot \vec{J} = 0$.

But from eqn. of continuity,

$\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho_v}{\partial t}$

∴ For time varying field we add an unknown term $\vec{G}$ to eqn. ①

$\vec{\nabla} \times \vec{H} = \vec{J} + \vec{G}$

⇒ $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{H}) = \vec{\nabla} \cdot \vec{J} + \vec{\nabla} \cdot \vec{G}$

⇒ $\vec{\nabla} \cdot \vec{J} = -\vec{\nabla} \cdot \vec{G}$

∴ $\vec{\nabla} \cdot \vec{G} = \frac{\partial \rho_v}{\partial t}$

∴ $\vec{\nabla} \cdot \vec{G} = \frac{\partial}{\partial t}(\vec{\nabla} \cdot \vec{D}) = \vec{\nabla} \cdot \frac{\partial \vec{D}}{\partial t}$

∴ $\vec{G} = \frac{\partial \vec{D}}{\partial t}$.

$\frac{\partial D}{\partial t} \longrightarrow A/m^2$

∴ Denote it by $\vec{J}_D$ → displacement current density.

$\vec{J}_D = \frac{\partial \vec{D}}{\partial t}$

Now, conduction current density, ($\vec{J} = \sigma \vec{E}$ is the motion of charge (usually electrons) in a region of zero net charge density) and (convection current density.

$\vec{J} = \rho_v \vec{v}$) motion of volume charge density.

In a non-conducting medium where no volume charge density is present,

$\vec{J} = 0$.

Then, $\vec{\nabla} \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$ (if $\vec{J} = 0$)

[analogous, $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$]

The total displacement current crossing any given surface is expressed by the surface integral.

$I_d = \int_s \vec{J}_d \cdot d\vec{s} = \int_s \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}$

∴ The Time-varying version of Ampere's circuital law:

$\int_s (\vec{\nabla} \times \vec{H}) \cdot d\vec{s} = \int_s \vec{J} \cdot d\vec{s} + \int_s \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}$

Applying Stoke's theorem,

$\boxed{\oint \vec{H} \cdot d\vec{L} = I + I_d = I + \int_s \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s}}$

(b)	Let $\mu = 3 \times 10^{-5}$ H/m, $\epsilon = 1.2 \times 10^{-10}$ F/m and $\sigma = 0$ everywhere. If $\boldsymbol{H} = 2\cos(10^{10}t - \beta x)\boldsymbol{a_z}$ A/m, use Maxwell's equations to obtain the expressions for **B**, **D**, **E.**	[05]	CO4	L3

	$\vec{B} = \mu\vec{H} = 2 \times 3\times10^{-5} \cos(10^{10}t - \beta x)\hat{a}_z$ $= 6\times10^{-5}\cos(10^{10}t - \beta x)\,\hat{a}_z \ Wb/m^2$ $\vec{J}_D = \vec{\nabla}\times\vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & 2\cos(10^{10}t - \beta x) \end{vmatrix}$ $= -\hat{a}_y \frac{\partial}{\partial x}\left[2\cos(10^{10}t - \beta x)\right]$ $= -\hat{a}_y\, 2\beta \sin(10^{10}t - \beta x)$ $\vec{D} = \int \vec{J}_D\, dt = -2\beta\hat{a}_y \int \sin(10^{10}t - \beta x)\,dt$ $= +2\beta\,\hat{a}_y \frac{\cos(10^{10}t - \beta x)}{10^{10}}$ $= \frac{2\beta}{10^{10}}\cos(10^{10}t - \beta x)\,\hat{a}_y \ C/m^2 + k$ $k = 0$, as no dc field $\vec{E} = \frac{\vec{D}}{\epsilon} = \frac{2\beta}{1.2\times10^{-10}\times10^{10}}\cos(10^{10}t - \beta x)\,\hat{a}_y$ $= 1.67\beta\cos(10^{10}t - \beta x)\,\hat{a}_y \ V/m$			
3.(a)	Discuss TEM wave propagation in a good conductor. Define skin depth. Propagation in good conductor (skin effect) For good conductor, $\frac{\epsilon''}{\epsilon'} >> 1$. $\epsilon'' = \frac{\sigma}{\omega}$ $\therefore \boxed{\frac{\sigma}{(\omega\epsilon')} >> 1}$ → criterion for good conductor The general expression for propagation constant $jk = j\omega\sqrt{\mu\epsilon'}\sqrt{1 - j\frac{\sigma}{\omega\epsilon'}}$ $= j\omega\sqrt{\mu\epsilon'}\sqrt{-j\frac{\sigma}{\omega\epsilon'}}$ $\left[\because \frac{\sigma}{\omega\epsilon'} >> 1\right]$ $\Rightarrow jk = j\sqrt{-j\frac{\omega^2\mu\epsilon'\sigma}{\omega\epsilon'}} = j\sqrt{-j\omega\mu\sigma}$ Sol, $-j = 1\angle -90^\circ$. and $\sqrt{1\angle-90^\circ} = 1\angle-45^\circ = \frac{1}{\sqrt{2}}(1-j)$ $\therefore jk = j(1-j)\sqrt{\frac{\omega\mu\sigma}{2}} = (j - j^2)\sqrt{\pi f\mu\sigma}$ $= (1+j)\sqrt{\pi f\mu\sigma} = \alpha + j\beta$	[07]	CO4	L2

	If only E_x component travelling in +z direction, then, $E_x = E_{x0} e^{-z\sqrt{\pi f \mu \sigma}} \cos(\omega t - z\sqrt{\pi f \mu \sigma})$ z > 0 good conductor z < 0 perfect dielectric ∴ At the boundary surface, z = 0. ∴ $E_x = E_{x0} \cos(\omega t)$ (z = 0) This is considered as the source field that establishes the fields within the conductor. $\vec{J} = \sigma \vec{E}$. [∵ displacement current density negligible] ∴ $J_x = \sigma E_x = \sigma E_{x0} e^{-z\sqrt{\pi f \mu \sigma}} \cos(\omega t - z\sqrt{\pi f \mu \sigma})$ • $e^{-z\sqrt{\pi f \mu \sigma}}$ → indicates a decrease in conduction current density ~~at any point~~ with penetration and electric field intensity into the conductor. This exponential factor = 1 at z $= e^{-1} = 0.368$ when $z = \frac{1}{\sqrt{\pi f \mu \sigma}}$. This distance is denoted by δ. Called depth of penetration or skin depth $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\alpha} = \frac{1}{\beta}$			
(b)	For a medium with σ = 4.0 S/m and $\varepsilon_r = 81$, evaluate skin depth at 1 MHz. The skin depth is $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{(\pi \times 10^6)(4\pi \times 10^{-7})(4)}} = 0.25 \text{ m} = 25 \text{ cm}$	[03]	CO4	L3

4.	State and explain Poynting's theorem.	[10]	CO5	L2

Poynting's theorem :-

Poynting's theorem states that the net power flowing out of a given volume v is equal to the time-rate of decrease in the energy stored within v minus the ohmic losses

If the medium is conductive, Maxwell's equation becomes,

$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ ... ①

Taking scalar product of both sides with $\vec{E}$,

$\vec{E} \cdot (\vec{\nabla} \times \vec{H}) = \vec{E} \cdot \vec{J} + \vec{E} \cdot \frac{\partial \vec{D}}{\partial t}$ ... ②

consider the vector identity

$\vec{\nabla} \cdot (\vec{E} \times \vec{H}) = -\vec{E} \cdot (\vec{\nabla} \times \vec{H}) + \vec{H} \cdot (\vec{\nabla} \times \vec{E})$

$\Rightarrow \vec{E} \cdot (\vec{\nabla} \times \vec{H}) = \vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{\nabla} \cdot (\vec{E} \times \vec{H})$ ③

Using this result in eqn. ②,

$\vec{H} \cdot (\vec{\nabla} \times \vec{E}) - \vec{\nabla} \cdot (\vec{E} \times \vec{H}) = \vec{E} \cdot \vec{J} + \vec{E} \cdot \frac{\partial \vec{D}}{\partial t}$

$\Rightarrow \vec{H} \cdot \left(-\frac{\partial \vec{B}}{\partial t}\right) - \vec{\nabla} \cdot (\vec{E} \times \vec{H}) = \vec{J} \cdot \vec{E} + \frac{\partial \vec{D}}{\partial t}$

$\Rightarrow -\vec{\nabla} \cdot (\vec{E} \times \vec{H}) = \vec{J} \cdot \vec{E} + \epsilon \vec{E} \frac{\partial \vec{E}}{\partial t} + \mu \vec{H} \left(\frac{\partial \vec{H}}{\partial t}\right)$ ④

Now, $\epsilon \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} = \frac{\partial}{\partial t} \frac{1}{2} (\vec{D} \cdot \vec{E})$

and $\mu \vec{H} \cdot \frac{\partial \vec{H}}{\partial t} = \frac{\partial}{\partial t} \left(\frac{1}{2} \vec{B} \cdot \vec{H}\right)$ ...

∴ Eqn. ④ becomes,

$$-\vec{\nabla} \cdot (\vec{E} \times \vec{H}) = \vec{J} \cdot \vec{E} + \frac{\partial}{\partial t} \left(\frac{1}{2} \vec{D} \cdot \vec{E}\right) + \frac{\partial}{\partial t} \left(\frac{1}{2} \vec{B} \cdot \vec{H}\right)$$

Integrating over a volume,

$$-\int_{vol} \vec{\nabla} \cdot (\vec{E} \times \vec{H})\, dv = \int_{vol} \vec{J} \cdot \vec{E}\, dv + \int_{vol} \frac{\partial}{\partial t} \left(\frac{1}{2} \vec{D} \cdot \vec{E}\right) dv + \int_{vol} \frac{\partial}{\partial t} \left(\frac{1}{2} \vec{B} \cdot \vec{H}\right) dv$$

$\Rightarrow -\oint_{area} (\vec{E} \times \vec{H}) \cdot d\vec{s} = R.H.S$ ~~⇒ Poynting's theorem~~

On R.H.S,

1st term = ohmic power dissipated within the volume

2nd term = total energy stored in the electric field

3rd " = " " " " magnetic field

R.H.S = total power flowing into the volume.

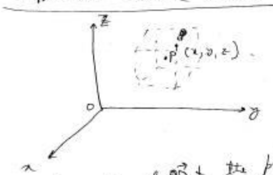
∴ The total power flowing out of the volume is

$\oint_{area} (\vec{E} \times \vec{H}) \cdot d\vec{s}$ W

The cross-product, $(\vec{E} \times \vec{H}) = \vec{S}$ W/m^2 → Poynting's vector.

5.	Derive point form of Maxwell's equation of electrostatics.	[10]	CO1	L2

Differential volume element



Let, the value of $\vec{D}$ at the pt. P be D_0, such that

$$\vec{D}_0 = D_{x0}\,\hat{a}_x + D_{y0}\,\hat{a}_y + D_{z0}\,\hat{a}_z .$$

According to Gauss's law,

$$\oint \vec{D}\cdot d\vec{s} = Q .$$

Around pt. P volume element.
consider cube of sides $\Delta x, \Delta y, \Delta z$.
The integral is broken into 6 integrals on the 6 sides.

i.e. $\oint \vec{D}\cdot d\vec{s} = \int_{front} + \int_{back} + \int_{left} + \int_{right} + \int_{top} + \int_{bottom}$

- $\vec{D}$ is constant over the entire area of the surface

$$\int_{front} = \vec{D}_{front}\cdot \vec{\Delta s}_{front}$$
$$= \vec{D}_{front}\cdot \Delta y\,\Delta z\,\hat{a}_x$$
$$= D_{x,front}\,\Delta y\,\Delta z$$

D_x at front surface:

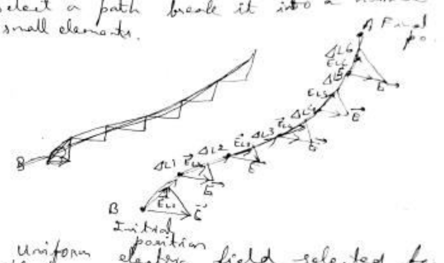
$$D_{x,front} = D_{x0} + \frac{\Delta x}{2} \times \text{rate of change of } D_x \text{ with } x$$
$$= D_{x0} + \frac{\Delta x}{2}\frac{\partial D_x}{\partial x}$$
$$\therefore \int_{front} = \left(D_{x0} + \frac{\Delta x}{2}\frac{\partial D_x}{\partial x}\right)\Delta y\,\Delta z .$$

Integral over back surface:

$$\int_{back} = \vec{D}_{back}\cdot \vec{\Delta s}_{back}$$
$$= \vec{D}_{back}\cdot\left(-\Delta y\,\Delta z\,\hat{a}_x\right)$$
$$= -D_{x,back}\,\Delta y\,\Delta z$$
$$D_{x,back} = D_{x0} - \frac{\Delta x}{2}\frac{\partial D_x}{\partial x} .$$
$$\therefore \int_{back} = \left(-D_{x0} + \frac{\Delta x}{2}\frac{\partial D_x}{\partial x}\right)\Delta y\,\Delta z$$
$$\therefore \int_{front} + \int_{back} = \frac{\partial D_x}{\partial x}\,\Delta x\,\Delta y\,\Delta z$$
$$\int_{left} + \int_{right} = \frac{\partial D_y}{\partial y}\,\Delta x\,\Delta y\,\Delta z$$
$$\int_{top} + \int_{bottom} = \frac{\partial D_z}{\partial z}\,\Delta x\,\Delta y\,\Delta z$$
$$\therefore \oint \vec{D}\cdot d\vec{s} = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right)\Delta x\,\Delta y\,\Delta z$$
$$= \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right)\Delta v = Q$$

	charge enclosed in a volume Δv $= \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) \times$ volume Δv. **Divergence** Let, $\Delta v \to 0$. $\left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) = \frac{\oint \vec{D}\cdot d\vec{s}}{\Delta v} = \frac{Q}{\Delta v}$ $\left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) = \lim_{\Delta v \to 0} \frac{\oint \vec{D}\cdot d\vec{s}}{\Delta v} = \lim_{\Delta v \to 0} \frac{Q}{\Delta v}$ $= \rho_v$. Two conclusions:- $\left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) = \lim_{\Delta v \to 0} \frac{\oint_s \vec{D}\cdot d\vec{s}}{\Delta v}$ ..(1) and $\left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right) = \rho_v$. .(2) Electric flux density can be generalised to any vector $\vec{A}$, i.e. $\left(\frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}\right) = \lim_{\Delta v \to 0} \frac{\oint_s \vec{A}\cdot d\vec{s}}{\Delta v}$ L.H.S = divergence of A = div A $= \lim_{\Delta v \to 0} \frac{\oint \vec{A}\cdot d\vec{s}}{\Delta v}$. **Divergence:** The divergence of the vector flux density $\vec{A}$ is the outflow of flux from a small closed surface per unit volume as the volume shrinks to zero. **Maxwell's First equation (electrostatics)** div $\vec{D} = \lim_{\Delta v \to 0} \frac{\oint \vec{D}\cdot d\vec{s}}{\Delta v}$ where, div $\vec{D} = \left(\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right)$ $\boxed{\text{div } \vec{D} = \rho_v}$ → Maxwell's first eqn. as applied to electrostatics and steady mag. field.			
6.	Find the total charge in a volume defined by the six planes for which $1 \leq x \leq 2$, $2 \leq y \leq 3$, $3 \leq z \leq 4$ if $\mathbf{D} = 4x\,\mathbf{a_x} + 3y^2\,\mathbf{a_y} + 2z^3\,\mathbf{a_z}$ C/m^2.	[10]	CO1	L3

	9) Find the total charge in the volume defined by the planes $1 \le x \le 2$, $2 \le y \le 3$, $3 \le z \le 4$ if $\vec{D} = 4x\vec{a_x} + 3y^2\vec{a_y} + 2z^3\vec{a_z}$ C/m² $\rho_v = \vec{\nabla}\cdot\vec{D}$ $Q_{enc} = \int_v (\vec{\nabla}\cdot\vec{D})\,dv$ $\vec{\nabla}\cdot\vec{D} = \frac{\partial}{\partial x}(4x) + \frac{\partial}{\partial y}(3y^2) + \frac{\partial}{\partial z}(2z^3)$ $\rho_v = 4 + 6y + 6z^2$ $Q_{enc} = \iiint (4+6y+6z^2)\,dx\,dy\,dz$ $= \iiint_v 4\,dx\,dy\,dz + \iiint_v 6y\,dx\,dy\,dz + \iiint 6z^2\,dx\,dy\,dz$ $= 4\times1\times1\times1 + \frac{6}{2}(1)(9-4)(1) + \frac{6}{3}(1)(1)(4^3-3^3)$ $= 4 + 15 + 74$ $\boxed{Q_{enc} = 93\,C}$ (ii) Surface integral: $\vec{D} = 4x\vec{a_x} + 3y^2\vec{a_y} + 2z^3\vec{a_z}$ C/m² $1 \le x \le 2,\ 2 \le y \le 3,\ 3 \le z \le 4.$ $\oint_S \vec{D}\cdot\vec{ds} = -\iint_S 4x\,dy\,dz\Big\vert_{x=1} + \iint_S 4x\,dy\,dz\Big\vert_{x=2} - \iint_S 3y^2\,dx\,dz\Big\vert_{y=2} + \iint_S 3y^2\,dx\,dz\Big\vert_{y=3}$ $- \iint_S 2z^3\,dx\,dy\Big\vert_{z=3} + \iint_S 2z^3\,dx\,dy\Big\vert_{z=4}$ $= -4[y]_2^3[z]_3^4 + 4\cdot 2[y]_2^3[z]_3^4 - 3(2)^2[x]_1^2[z]_3^4 + 3(3)^2[x]_1^2[z]_3^4$ $- 2(3)^3[x]_1^2[y]_2^3 + 2(4)^3[x]_1^2[y]_2^3$ $= -4 + 8 - 12 + 27 - 54 + 128$ $\boxed{\oint_S \vec{D}\cdot\vec{ds} = 93\,C}$			
7.(a)	Define current and current density. Derive the equation of continuity of current. Current through a closed surface, $I = \oint_S \vec{J}\cdot\vec{ds}$ Outward flow of +ve charge is balanced by a decrease of +ve charge within the closed surface. Let, Q_i be the charge inside the closed surface. $\therefore I = \oint_S \vec{J}\cdot\vec{ds} = -\frac{dQ_i}{dt}$ → reduction in charge, giving -ve sign Using divergence theorem, $\oint_S \vec{J}\cdot\vec{ds} = \int_{vol} (\vec{\nabla}\cdot\vec{J})\,dv$ Now, $Q = \int_{vol} \rho_v\,dv$ $\therefore \int_{vol} (\vec{\nabla}\cdot\vec{J})\,dv = -\frac{d}{dt}\int_{vol} \rho_v\,dv$ If the surface is constant derivative becomes a partial derivative $\int_{vol} (\vec{\nabla}\cdot\vec{J})\,dv = \int_{vol} -\frac{\partial \rho_v}{\partial t}\,dv$ This is true for any volume however small This is true for an incremental volume $\therefore (\vec{\nabla}\cdot\vec{J})\,dv = -\frac{\partial \rho_v}{\partial t}\,dv$ $\therefore$ Point form of continuity equation $\boxed{(\vec{\nabla}\cdot\vec{J}) = -\frac{\partial \rho_v}{\partial t}}$	[02+04]	CO2	L2
(b)	Let **D** = $5r^2$ $\mathbf{a_r}$ mC/m² in the region for r < 0.08m. Find ρ_v at r = 0.06m.	[04]	CO1	L3

	i) for $r < 0.08$ m, $\vec{D} = 5r^2 \hat{a}_r$ mC/m^2 $\therefore D_r = 5r^2$. $\therefore \nabla \cdot \vec{D} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 5r^2)$ $= \frac{1}{r^2}\frac{\partial}{\partial r}(5r^4) = \frac{1}{r^2} \cdot 20r^3 = 20r$ $\therefore \rho_v = (20 \times 0.06) = 1.20$ mC/m^3			
8.(a)	Derive an expression for the work done in moving a point charge Q in the presence of an electric field **E**. - If we attempt to move the test charge against the field we have to do equal and opposite force on the charge as exerted by the field. Consider a distance $\vec{dL}$ in an electric field $\vec{E}$ ∴ Force on Q due to electric field $\vec{E}$. $\vec{F}_E = Q\vec{E}$ force arises from the field The component of the force in the direction $\vec{dL}$ which we must overcome is $F_{EL} = \vec{F}\cdot\hat{a}_L = Q\vec{E}\cdot\hat{a}_L$. $\hat{a}_L \rightarrow$ unit vector along $\vec{dL}$. we apply equal and opposite force associated with the field $\therefore F_{applied} = -Q\vec{E}\cdot\hat{a}_L$ ∴ work done by external source in moving $dW = (-Q\vec{E}\cdot\hat{a}_L)\,dL = -Q\vec{E}\cdot\vec{dL}$ $\therefore dW = -Q\vec{E}\cdot\vec{dL}$ The work required to move the charge a finite distance, $W = -Q\int_{init}^{final} \vec{E}\cdot\vec{dL}$. If the charge is moved ⊥ to the electric field, W=0. The line integral select a path break it into a number of small elements.  Uniform electric field selected for simplicity. - six line segments $\Delta L1, \Delta L2, \ldots \Delta L6$ - The components of $\vec{E}$ along each segment are: $E_{L1}, E_{L2} \ldots E_{L6}$ ∴ The work involved in moving a charge Q from B to A is $W = -Q(E_{L1}\Delta L_1 + E_{L2}\Delta L2 + \ldots E_{L6}\Delta L6)$ $\vec{W} = -Q(\vec{E}_1\cdot\vec{\Delta L_1} + \vec{E}_2\cdot\vec{\Delta L_2} + \ldots \vec{E}_{L6}\cdot\vec{\Delta L_6})$ as uniform field, $\vec{E_1} = \vec{E_2} = \ldots = \vec{E_6}$ $\therefore \vec{W} = -Q\vec{E}\cdot(\vec{\Delta L1} + \vec{\Delta L2} + \ldots \vec{\Delta L6})$. $\therefore \vec{W} = -Q\vec{E}\cdot\vec{L}_{BA}$	[05]	CO2	L2

(b)	Non uniform field is given by **E**=y $\mathbf{a_x}$ +x $\mathbf{a_y}$ + 2 $\mathbf{a_z}$ V/m. Determine the work done in moving a charge of 2C from B(1, 0, 1) to A(0.8, 0.6, 1) along the path $x^2 + y^2 = 1$, z=1.	[05]	CO2	L3

Soln.

$$W = -Q \int_B^A \vec{E} \cdot d\vec{l}$$

$$= -2 \int_B^A \left(y\hat{a}_x + x\hat{a}_y + 2\hat{a}_z\right) \cdot \left(dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z\right)$$

$$= -2\int_1^{0.8} y\,dx - 2\int_0^{0.6} x\,dy - 4\underbrace{\int_1^1 dz}_{=0}$$

$x^2+y^2 = 1$

$\Rightarrow y^2 = 1 - x^2$

$\Rightarrow y = \sqrt{1-x^2}$.

(select sign based on the quadrant in which the ~~point~~ path is situated.

$$\therefore W = -2\int_1^{0.8} \sqrt{1-x^2}\,dx - 2\int_0^{0.6} \sqrt{1-y^2}\,dy - 0$$

$$= -\left[x\sqrt{1-x^2} + \sin^{-1}x\right]_1^{0.8} - \left[y\sqrt{1-y^2} + \sin^{-1}y\right]_0^{0.6}$$

$= -[-1.57 + 0.48 + 0.927] - [0.48 + 0.64]$ =-52.04-

$= -0.9571 = -0.96$ J.

9.	Starting from Maxwell's equations, derive wave equation for TEM wave.	[10]	CO4	L2

Sol. <u>Wave propagation in free-space:-</u>

- In free space the fields are not bounded by any confining structure.
- May assume any magnitude and direction, as initially determined by the device (antenna).
- Medium is sourceless. ($\rho_v = J = 0$)

Maxwell's equations are:

$$\left.\begin{aligned} \vec{\nabla} \times \vec{H} &= \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad \text{--(1)} \\ \vec{\nabla} \times \vec{E} &= -\mu_0 \frac{\partial \vec{H}}{\partial t} \quad \text{--(2)} \\ \vec{\nabla} \cdot \vec{E} &= 0 \quad \text{--(3)} \\ \vec{\nabla} \cdot \vec{H} &= 0 \quad \text{--(4)} \end{aligned}\right\} \text{writing in terms of } \vec{E} \text{ \& } \vec{H} \text{ only.}$$

From (1): If $\vec{E}$ is changing with time at some point $\vec{H}$ has curl at that point.
∴ $\vec{H}$ varies spatially in a direction normal to its orientation direction.

(2) → Time varying $\vec{H}$ generates $\vec{E}$ which varies spatially in the direction normal to its orientation.

Postulate: the existence of a uniform plane wave, in which both fields, $\vec{E}$ & $\vec{H}$ lie in the transverse plane.
→ plane for which normal is the direction of propagation.
- Such a wave is called Transverse electro magnetic (TEM) wave.

Then eqn. (1) reduces to,

$$\vec{\nabla} \times \vec{H} = -\frac{\partial H_y}{\partial z}\hat{a}_x = \epsilon_0 \frac{\partial \vec{E}}{\partial t} = \epsilon_0 \frac{\partial E_x}{\partial t}\hat{a}_x - (6$$

$$\vec{\nabla} \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & H_y & 0 \end{vmatrix}$$

$$= -\hat{a}_x \frac{\partial H_y}{\partial z} + \hat{a}_z \frac{\partial H}{\ }$$

neglect

Equation (5) & (6) can be written as,

$$\frac{\partial \vec{E}_x}{\partial z} = -\mu_0 \frac{\partial \vec{H}_y}{\partial t} \quad \cdots (7) \; (6)$$

$$\frac{\partial \vec{H}_y}{\partial z} = -\epsilon_0 \frac{\partial \vec{E}_x}{\partial t} \quad \cdots (8) \; (7)$$

Differentiate eqn (7) w.r.t z,

$$\frac{\partial^2 \vec{E}_x}{\partial z^2} = -\mu_0 \frac{\partial^2 \vec{H}_y}{\partial t \partial z} \quad \cdots (9)$$

Differentiating eqn. (8) w.r.t t we get,

$$\frac{\partial^2 \vec{H}_y}{\partial t \partial z} = -\epsilon_0 \frac{\partial^2 \vec{E}_x}{\partial t^2} \quad \cdots (10)$$

Substituting (10) into (9) we get,

$$\boxed{\frac{\partial^2 \vec{E}_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 \vec{E}_x}{\partial t^2}} \quad \cdots (11)$$

→ equation for x-polarized TEM electric field in free space.

The propagation velocity,

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}} = 3 \times 10^8 \text{ m/s} = c \quad \cdots (12)$$

(vel. of light in free space)

Now differentiate (7) w.r.t t,

$$\frac{\partial^2 E_x}{\partial t \partial z} = -\mu_0 \frac{\partial^2 H_y}{\partial t^2}$$

Differentiate (8) w.r.t z we get,

$$\frac{\partial^2 H_y}{\partial z^2} = -\epsilon_0 \frac{\partial^2 E_x}{\partial t \partial z}$$

$$\therefore \frac{\partial^2 H_y}{\partial z^2} = -\epsilon_0 \frac{\partial^2 E_x}{\partial t \partial z} = -\epsilon_0 \left(-\mu_0 \frac{\partial^2 H_y}{\partial t^2}\right) = \epsilon_0 \mu_0 \frac{\partial^2 H_y}{\partial t^2}$$

i.e. $$\boxed{\frac{\partial^2 H_y}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 H_y}{\partial t^2}} \quad \cdots (13)$$

— Wave equation for the magnetic field.