USN | | | | | | | | | |

CMRIT — CMR Institute of Technology, Bengaluru. Accredited with A+ Grade by NAAC. Celebrating 25 Years

Internal Assessment Test 1 – Sept. 2017

Sub:	Engineering Physics					Sub Code:	15PHY12	Branch:	All
Date:	21/09/2017	Duration:	90 mins	Max Marks:	50	Sem / Sec:	I / I,J,K,L,M,N,O		OBE

Answer any **FIVE FULL** Questions

Note: Value of Constants: $h = 6.625\times10^{-34}$Js $k = 1.38\times10^{-23}$ J/K $m = 9.11\times10^{-31}$ kg. $e = 1.6 \times 10^{-19}$C, $c = 3\times10^{8}$ m/s

Q.	Question	MARKS	CO	RBT
1 (a)	State and explain Planck's law of black body radiation. Show that it reduces to Wien's law and Rayleigh-Jeans law under suitable wavelength conditions.	[07]	CO1	L3
(b)	Calculate wavelength of the maximum intensity (black body radiation) that could be emitted by the normal human body, assuming temperature of the body as 38 ºC (given that Wien's constant is 2.898×10^{-3} m-K).	[03]	CO1	L2
2 (a)	Define phase velocity and group velocity. Derive an expression for group velocity on the basis of superposition of two travelling waves.	[06]	CO1	L2
(b)	If the kinetic energy of the electron is 3 eV, calculate de-Broglie wavelength and phase velocity of the de-Broglie wave associated with the electron.	[04]	CO1	L2
3 (a)	Using Heisenberg's uncertainty principle, show that an electron cannot exist inside the nucleus.	[07]	CO1	L3
(b)	An electron has a speed of 500 m/s measured with an accuracy of 0.003%. With what accuracy one can locate the position of the electron? .	[03]	CO1	L2

Q.	Question	MARKS	CO	RBT
4 (a)	Derive the time independent Schrodinger wave equation for a free particle in one dimension.	[06]	CO1	L3
(b)	An electron is bound in a one dimensional potential well of width 0.15 nm. Find the energy value (in eV) and the de-Broglie wavelength of the electron in the second excited state.	[04]	CO1	L2
5 (a)	Derive the expression for energy Eigen value and Eigen function for a particle in a one dimensional potential well of infinite height.	[07]	CO1	L3
(b)	Calculate the wavelength of the X-ray photon scattered by an electron at an angle 90º, if the incident photon has a wavelength of 1Å.	[03]	CO1	L2
6 (a)	Explain the failures of classical free electron theory.	[06]	CO2	L2
(b)	Discuss the dependence of Fermi factor on temperature at T = 0 K and T > 0 K.	[04]	CO2	L1
7 (a)	Obtain an expression for the electrical conductivity of a metal from quantum mechanical consideration.	[06]	CO2	L3
(b)	Explain the following terms briefly. a) Drift velocity, b) Fermi energy and c) Fermi Temperature	[04]	CO2	L2

SCHEME OF EVALUATION
2017-18 ODDSEMESTER
IAT-1

1.a.(7)

Max Planck developed a structural model for black body radiation that leads to a theoretical equation for the wavelength distribution that is in complete agreement with the experimental results at all wavelengths.

According to his theory

1. a black body is imagined to be consisting of large number of electrical oscillators.
2. an oscillator emits or absorbs energy in discrete units. It can emit or absorb energy by making a transition from one quantum state to another in the form of discrete energy packets known as photons whose energy is an integral multiple of hv where h is the planks constant and v is the frequency.
3. Energy emitted per unit volume per unit energy range is given by the product of number of modes of vibration in the given energy range and the energy per mode.The Energy density per unit wavelength range per unit volume is given by

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\left[\frac{1}{e^{\left[\frac{h\gamma}{kT}\right]}-1}\right]d\lambda \qquad \textbf{(3)}$$

Where h is Planck's constant, c is velocity of light, T is absolute temperature, λ is the wavelength and k is Boltzmann constant

Deduction of Weins law: **(2)**

It is applicable at smaller wavelengths.

For smaller wavelengths $e^{\frac{h\gamma}{kT}} >> 1$

$$\therefore e^{\frac{h\gamma}{kT}} >> 1 = e^{\frac{h\gamma}{kT}}$$

So Planck's radiation law becomes

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5}\left[\frac{1}{e^{\left[\frac{h\gamma}{kT}\right]}}\right]$$

Deduction of Rayleigh Jeans Law: **(2)**

It is applicable at longer wavelengths.

For longer wavelengths $\frac{h\gamma}{kT} << 1$

$$\therefore e^{\frac{h\gamma}{kT}} = 1 + \frac{h\gamma}{kT} + \left(\frac{h\gamma}{kT}\right)^2 \frac{1}{2|} + ... = 1 + \frac{h\gamma}{kT}$$

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{1+\frac{h\gamma}{kT}-1} d\lambda = \frac{8\pi kT}{\lambda^4} d\lambda$$

1. b. (3)

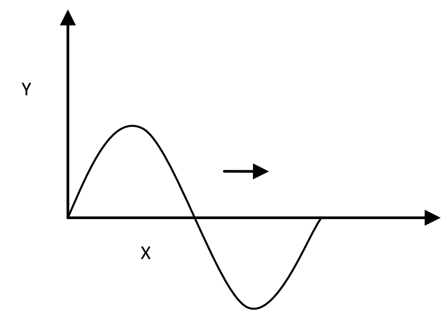
From Weins displacement law

$$\lambda_{max}.T = 2.89x10^{-3}\,mK$$

Put T=311K, $\lambda_{max}. = 9318.3nm$

2.a. (6)

Phase velocity(V_p): It is the speed with which an isolated pulse / constant phase propagates in a medium. **(1)**



A single pulse is shown in this diagram .It is represented as

Y = A sin [wt – kx]

where Y is the displacement of a particle at a distance 'x' from the origin at a time 't', A is the amplitude , w is the angular velocity and k is the wave number.

Imagine two points A and B at X_1 and X_2 at same phase on the wave. Then

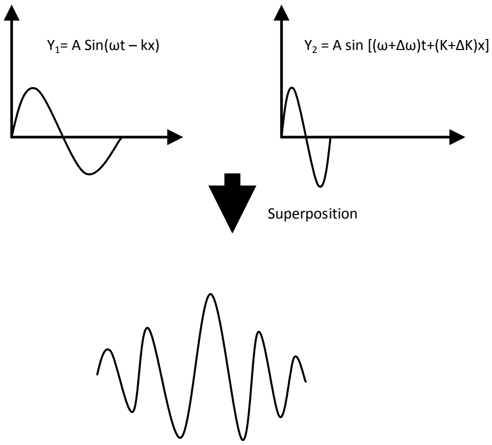
$(wt_1\text{-}kX_1) = (wt_2 - kX_2)$

$$\frac{w}{k} = \frac{x_1 - x_2}{t_1 - t_2}$$

$$\therefore V_p = \frac{w}{k}$$

Group Velocity(V_g): It is the velocity with which the resultant envelope of varying amplitude formed by the superposition of two or more waves propagates . **(1)**

$$V_g = \underset{dk\to 0}{Lt} \frac{dw}{dk}$$



Resultant Envelope with varying amplitude

Let the waves be represented as

Y_1 = A sin (wt – kx) ...... (1

Y_2 = A sin [(w+Δw) t – (k + Δk) x]

The susultant superposition is Y= Y_1 +Y_2

$= 2A\cos[(\frac{\Delta w}{2})t-(\frac{\Delta k}{2})x]\sin(\frac{2w+\Delta w}{2})t-(\frac{2k+\Delta k}{2})$ .... (2)

But Δw and Δk are small

2w + Δw ≈ 2w, 2k+ Δk ≈ 2k

$\therefore$ Y = 2A cos [($\frac{\Delta w}{2}$) t – ($\frac{\Delta k}{2}$) x] sin (wt-kx) .....(2)

Comparing equations (1) and (2), the coefficient of sin (wt-kx) in equation (2) can be considered as the amplitude of the wave.

Amplitude of the resultant wave =2A cos [(Δw/2) t – (Δk/2) x]

This amplitude varies as a wave .The velocity with which the variation in amplitude is propagated is referred as group velocity

V_g = (Δw/2) / (Δk/2)

$V_g = \Delta w / \Delta k = \underset{dk\to 0}{Lt}\frac{dw}{dk}$ **(4)**

2.b. (4)

Debroglie wavelength $\lambda = \frac{h}{\sqrt{2mE}}$

E=3eV = 3x1.6x10^{-19}J

$\lambda = 7x10^{-10}$ m particle velocity = 1.02x10^{6}m/s

Phase velocity = c^2/v_g =8.76x10^{10}m/s

3.a. (7)

The position and momentum of a particle cannot be determined accurately and simultaneously. The product of uncertainty in the measurement of position (Δx) and momentum (Δp) is always greater than or equal to $\frac{h}{2\Pi}$. **(2)**

$$(\Delta x).(\Delta p) \geq \frac{h}{4\Pi}$$

This uncertainty is not due to discrepancy with the apparatus or with the method of measurement, but because of the very wave nature of the object. This uncertainty persists as long as matter possesses wave nature.

Different forms of Heisenberg's Principle:

$$(\Delta x).(\Delta p) \geq \frac{h}{4\Pi}$$

$$(\Delta L).(\Delta \theta) \geq \frac{h}{4\pi}$$

$$(\Delta E).(\Delta t) \geq \frac{h}{4\pi}$$

Here ΔL is the uncertainty in angular momentum

Δθ is the uncertainty in angular displacement

ΔE is the uncertainty in the energy

Δt is the uncertainty in the time interval during which the particle exists in the state E

To Show that electron does not exist inside the nucleus:

We know that the diameter of the nucleus is of the order of 10^{-15}m.If the electron is to exist inside the nucleus, then the uncertainty in its position Δx cannot exceed the size of the nucleus

$$\Delta x \leq 10^{-14} m$$

Now the uncertainty in momentum is

$$\Delta p \geq \frac{h}{4\pi.\Delta x}$$

$$\Delta p \geq \frac{6.62\times 10^{-34}}{4\pi \times 10^{-15}}$$

$$\Delta p \geq 0.5\times 10^{-20} Ns$$

Then the momentum of the electron can at least be equal to the uncertainty in momentum.

$$p \geq 0.5\times 10^{-20} Ns$$

Now the energy of the electron with this momentum supposed to be present in the nucleus is given by - for small velocities (non-relativistic) case-

For high velocities,

$$E = \left(\frac{1}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}}\right)m_o c^2 \quad ....(1)$$

$$E^2 = \left(\frac{1}{1-\left(\frac{v^2}{c^2}\right)}\right)m^2{}_o c^4$$

Momentum $p = \left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\right)m_o v$

$$p^2 = \left(\frac{1}{1-\left(\frac{v^2}{c^2}\right)}\right) m_o^2 v^2$$

$$p^2c^2 = \left(\frac{1}{1-\left(\frac{v^2}{c^2}\right)}\right) m_o^2 v^2 c^2 \text{ ....(2)}$$

From (1) and (2) $E^2 = p^2c^2 + m_o^2 c^4$ **(5)**

On substitution

$$E = (9.1x10^{-31})^2(3x10^8)^2 + (0.5x10^{-19})^2(3x10^8)^4 \quad = 9.4MeV$$

3.b. **(3)**

Given v=500m/s accurate to 0.003%

$$\Delta v = 500x\frac{0.003}{100} = 0.015 m/s$$

From **Heisenberg's Principle,**

$$\Delta x.\Delta p \geq \frac{h}{4\pi}$$

$$\Delta x.m\Delta v = \frac{h}{4\pi}$$

$$\Delta x = \frac{h}{4\pi.\Delta v.m} = 3.86x10^{-3} m$$

4.a. **(6)**

Time independent Schrödinger equation

A matter wave can be represented in complex form as

$$\Psi = A\sin kx(\cos wt + i\sin wt)$$

$$\Psi = A\sin kxe^{iwt}$$

Differentiating wrt x

$$\frac{d\Psi}{dx} = kA\cos kxe^{iwt}$$

$$\frac{d^2\Psi}{dx^2} = -k^2 A\sin kxe^{iwt} = -k^2\Psi \text{ .... (1)}$$

From debroglie's relation **(2)**

$$\frac{1}{\lambda} = \frac{h}{mv} = \frac{h}{p}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\Pi p}{h}$$

$$k^2 = 4\Pi^2\frac{p^2}{h^2} \text{ .... (2)}$$

Total energy of a particle E = Kinetic energy + Potential Energy

$$E = \frac{1}{2} m v^2 + V$$

$$E = \frac{p^2}{2m} + V$$

(2)

$$p^2 = (E - V)2m$$

Substituting in (2)

$$k^2 = \frac{4\Pi^2(E-V)2m}{h^2}$$

$\therefore$ From (1)

$$\boxed{\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 m(E-V)\Psi}{h^2} = 0}$$

(For one dimension) **(2)**

4.b. **(4)**

Energy of a particle in an infinite potential well $E = \frac{n^2h^2}{8ma^2}$

Second excited state corresponds to n= 3

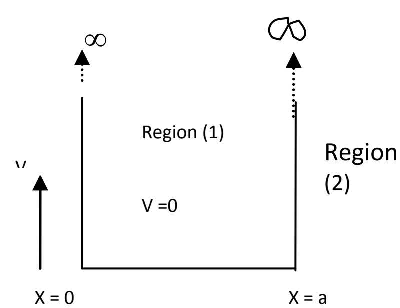
$$E = \frac{3^2h^2}{8.(9.1x10^{-31}).(0.15x10^{-9})^2} = 2.42x10^{-17} J = 151eV$$

$$\lambda = \frac{h}{\sqrt{2mE}} = 1x10^{-10} m$$

5.a. **(7)**

Particle in an infinite potential well problem:

Consider a particle of mass m moving along X-axis in the region from X=0 to X=a in a one dimensional potential well as shown in the diagram. The potential energy is zero inside the region and infinite outside the region.



Applying, Schrodingers equation for region (1) as particle is supposed to be present in region (1) **(2)**

$$\frac{d^2\Psi}{dx^2} + \frac{8\Pi^2 mE\psi}{h^2} = 0 \qquad \because V = 0$$

But $k^2 = \frac{8\Pi^2 mE}{h^2}$

$$\therefore \frac{d^2\Psi}{dx^2} + k^2\Psi = 0$$

The general solution to this expression is given by

$$\Psi = C\cos kx + D\sin kx$$

At x=0, $\Psi = 0 \quad \therefore C = 0$

At x=a, $\Psi = 0$ D sin ka = 0 $\Rightarrow$ ka = nΠ where n = 1, 2 3$\therefore$

$$\Psi = D\sin\left(n\frac{\Pi}{a}\right)x$$

E = $\frac{n^2h^2}{8ma^2}$ **(2)**

To evaluate the constant D:

Normalisation : For one dimension

$$\int_0^a \Psi^2 dx = 1$$

$$\int_0^a D^2 \sin^2(\frac{n\Pi}{a})x dx = 1$$

But $\cos 2\theta = 1 - 2\sin^2\theta$

$$\int_0^a D^2 \frac{1}{2}(1 - \cos 2(\frac{n\Pi}{a})x)dx = 1$$

$$\int_0^a \frac{D^2}{2}dx - \int_0^a \frac{1}{2}\cos 2(\frac{n\Pi}{a})x)dx = 1$$

$$\frac{D^2 a}{2} - [\sin 2\left(\frac{n\Pi}{a}\right)\frac{x}{2}]_0^a = 1$$

$$D^2\frac{a}{2} - 0 = 1$$

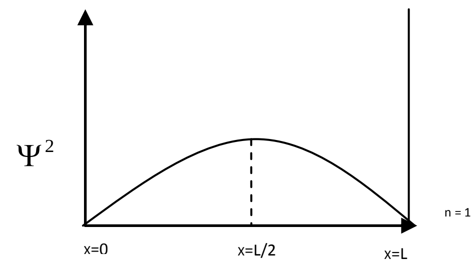
D = $\sqrt{\frac{2}{a}}$ **(2)**

$$\therefore \Psi_n = \sqrt{\frac{2}{a}} \sin\left(n\frac{\Pi}{a}\right)x$$

For n = 1, First state

$$\therefore \Psi_1 = \sqrt{\frac{2}{a}} \sin\left(1.\frac{\Pi}{a}\right)x$$

The graph of Ψ^2 $versus$ x is shown below.

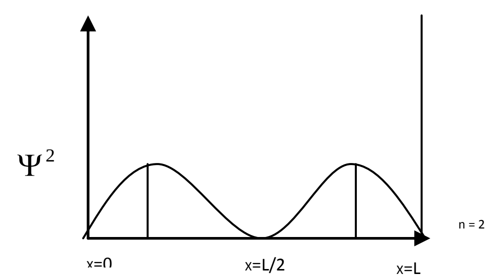


It is seen from the graph that probability density is maximum at the centre for the particle in the first state.

For n = 2, Second state

$$\therefore \Psi_2 = \sqrt{\frac{2}{a}} \sin\left(2.\frac{\Pi}{a}\right)x \quad \textbf{(1)}$$

The graph of Ψ^2 $versus$ x is shown below.



It is seen from the graph that probability density is maximum at x = L / 4 and x = 3L/4 for the particle in the second state.

5.b. **(3)**

Change in wavelength of x ray after Compton scattering is given by

$$d\lambda = \lambda^1 - \lambda = \frac{h}{m_0 c}(1 - \cos\theta)$$

Wavelength of Incident photon = $\lambda = 1\text{x}10^{-10}\text{m}$

Here , when θ=90^0,

Wavelength of scattered photon

$$\lambda^1 = \lambda + \frac{h}{m_0 c}(1 - \cos 90) = 1\text{x}10^{-10} + 0.0242\text{x}10^{-10} = 1.0242\text{x}10^{-10}\text{m}$$

6.a. **(6)**

Failures of Classical free electron theory:

1. Prediction of low specific heats for metals: **(2)**

Classical free electron theory assumes that conduction electrons are classical particles similar to gas molecules. Hence, they are free to absorb energy in a continuous manner. Hence metals possessing more electrons must have higher heat content. This resulted in high specific heat given by the expression

$C_V = C_v = \frac{3}{2}R$.

This was contradicted by experimental results which showed low specific heat for metals given by $C_V = 10^{-4}R$.

2. Temperature dependence of electrical conductivity: **(2)**

From the assumption of kinetic theory of gases

$$\frac{3}{2}kT = \frac{1}{2}mv^2$$
$$\therefore v \propto \sqrt{T}$$

Also mean collision time τ is inversely proportional to velocity,

$$\tau \propto \frac{1}{v}$$
$$\tau \propto \frac{1}{\sqrt{T}}$$
$$\therefore \sigma = \frac{ne^2\tau}{m} \Rightarrow \sigma \propto \frac{1}{\sqrt{T}}$$

However experimental studies show that $\sigma \propto \frac{1}{T}$

3. Dependence of electrical conductivity on electron concentration: **(2)**

As per free electron theory, $\sigma \propto n$

The electrical conductivity of Zinc and Cadmium are 1.09 x 10^7 /ohm m and .15 x 10^7 /ohm m respectively which are very much less than that for Copper and Silver for which the values are 5.88x10^7 /ohm m and 6.2 x 10^7 /ohm m. On the contrary, the electron concentration for zinc and cadmium are 13.1x10^{28} /m^3 and 9.28 x10^{28} /m^3 which are much higher than that for Copper and Silver which are 8.45x10^{28} /m^3 and 5.85 x10^{28} /m^3.

These examples indicate that $\sigma \propto n$ does not hold good.

4. Mean free path, mean collision time found from classical theory are incorrect

6.b. (4)

Case 1:

For E < E_F, at T = 0,

$$f(E) = \frac{1}{e^{(\frac{E-E_F}{kT})} + 1} = 1$$

This shows that energy levels below Fermi energy are completely occupied:

Case 2:

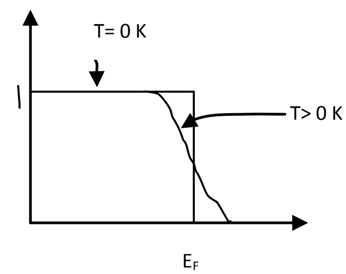
For E > E_F , at T=0

$$f(E) = \frac{1}{e^{(\frac{E-E_F}{kT})} + 1} = 0$$

This shows that energy levels above Fermi energy are empty:

At ordinary temperatures, for E = E_F,

$$f(E) = \frac{1}{2}$$



7.a. (6)

Expression for Electrical conductivity:

Imagine a conductor across which an electric field E is applied. The equation of motion for an electron moving under the influence of external field is given by

F= dp/dt = eE

Let the wave number change from k_1 to k_2 in time interval τ_F in the presence of electric field.

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{h/p} = \frac{2\pi p}{h}$$
$$p = \frac{hk}{2\pi}$$
$$\frac{dp}{dt} = \frac{h}{2\pi}\left(\frac{dk}{dt}\right) \quad \textbf{(3)}$$
$$dk = \frac{2\pi}{h}eEdt$$

On integration $k_2 - k_1 = \Delta k = \frac{2\pi.eE.\tau_F}{h}$ ....(1)

From quantum theory, conductivity J = $\Delta k.ne.\frac{h}{2\pi.m}$ ....(2)

Substituting (1) in (2)

We get J = $\frac{ne^2\tau_F}{m^*}E$ ...(3) **(1)**

Since from Ohm's, J = σE, conductivity σ can be written as

$$\sigma = \frac{ne^2}{m^*}\frac{\lambda}{v_F} \quad \textbf{where } \tau = \frac{\lambda}{v_F} \quad \textbf{(2)}$$

7.b. (4)

Fermi energy (E_F):

It is the highest energy possessed by an electron at zero Kelvin.

$$E_F = \frac{h^2}{8m}\left(\frac{3n}{\Pi}\right)^{\frac{2}{3}} \quad \textbf{(1)}$$

<u>Fermi Temperature:</u> It is the temperature to which a metal is to be heated such that the free electrons acquire energy equal to Fermi energy.

$$T_F = \frac{E_F}{k} \quad \textbf{(1)}$$

Drift velocity:

The net displacement in the position of electrons per unit time caused by the application of electric field is known as drift velocity.

$$v_d = \frac{eE\tau}{m} \quad \textbf{(2)}$$