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Internal Assesment Test - I

Sub: Engineering Maths-III			Code: 15MAT31
Date: 18 / 09 /2017	Duration: 90 mins	Max Marks: 50	Sem: 3	Branch: IS A & B, TCE

NOTE: First question is compulsory. Answer any six questions from the rest.

		Marks	OBE CO	OBE RBT
1	From the data given below, find the number of students who obtained marks between 40 and 45.	[8]	CO301.6	L3

Marks	30-40	40-50	50-60	60-70	70-80
No. of students	31	42	51	35	31

		Marks	OBE CO	OBE RBT
2	Evaluate f(8) and f(15) using Newton's Divided difference formula from table below.	[7]	CO301.6	L3

x	4	5	7	10	11	13
y	48	100	294	900	1210	2028

		Marks	OBE CO	OBE RBT
3	Find the interpolating polynomial f(x) using Lagrange's formula and hence find f(3) for the following data.	[7]	CO301.6	L3

x	0	1	2	5
F(x)	2	3	12	147

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4 Evaluate $\int_0^6 \frac{dx}{1+x^2}$ dividing the interval into 6 equal parts using Weddle's Rule. Compare the answer with the exact value. [7] CO301.6 L3

5. (i)Use Regula-falsi method to obtain a root of the equation $2x - \log_{10} x = 7$ that lies between 3.5 and 4. Perform 2 iterations. (ii) Use Newton-Raphson method to obtain a root of $\cos x = xe^x$ correct to 3 decimals. [7] CO301.6 L3

6. Find a Fourier series of $f(x) = x - x^2$ from $x = -\pi$ to $x = \pi$. Hence deduce the sum of the series $\sum \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots$ [7] CO301.1 L3

7 Find the half range cosine series of $f(x) = (x-1)^2$ in $0 \le x \le 1$and hence deduce the sum of the series $\sum \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \ldots$. [7] CO301.1 L3

8 Obtain the Fourier expansion of y for the data given upto 2 harmonics [7] CO301.1 L3

x	0	1	2	3	4	5
y	9	18	24	28	26	20

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①

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
40	31				
		42			
			9		
50	73			-25	
		51			37
			-16		
60	124			12	
		35			
70	159		-4		
		31			
80	190				

3M

ⓐ To find $f(45)$ We use forward diff formula

$$y_r = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \cdots$$ 1M

$$r = \frac{x - x_0}{h} = \frac{45-40}{10} = 0.5$$ 1M

$$f(45) = 31 + (0.5)42 + \frac{(0.5)(0.5-1)}{2} 9 + \frac{(0.5)(0.5-1)(0.5-2)}{6}(-25)$$

$$+ \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)}{24}(37)$$ 1M

$f(45) = 47.86$

$\boxed{f(45) \sim 48}$ 1M

$\therefore$ Number of students obtaining less than 45 marks is 48.

& $f(40) = 31$ by data

$\therefore$ Number of students scoring marks between 40 & 45 is 17. 1M

— x —

2)

x	$f(x)$	Ist diff	IInd diff	III diff	IV Diff
4	48				
		52			
			15		
5	100			1	
		97			
					0
7	294		21		
				1	
		202			
10	900		27		0
		310		1	
11	1210		33		
					2M
		409			
13	2028				

$$f(x) = f(x_0) + (x-x_0) f(x_0, x_1) + (x-x_0)(x-x_1) f(x_0, x_1, x_2) + \ldots$$

$$+ (x-x_0)(x-x_1)(x-x_2) f(x_0, x_1, x_2, x_3) + \ldots$$ 1M

$$f(x) = 48 + (x-4)52 + (x-4)(x-5)15 + (x-4)(x-5)(x-7)1$$

$$f(x) = x^3 - x^2$$ 2M

$\therefore f(8) = 448$ 1M & $f(15) = 3150$. 1M

3) $x_0 = 0 \quad x_1 = 1 \quad x_2 = 2 \quad x_3 = 5$

$y_0 = 2 \quad y_1 = 3 \quad y_2 = 12 \quad y_3 = 147$

Formula $f(x) = \dfrac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \dfrac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$

$+ \dfrac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_1)(x_2-x_1)(x_2-x_3)} y_2 + \dfrac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$

2M

$= \frac{1}{5}(-x^3+8x^2-17x+0)+\frac{3}{4}(x^3-7x^2+10x)+(-2)(x^3-6x^2+5x)$

$+\frac{49}{20}(x^3-3x^2+2x)$

$= x^3+x^2-x+2$ 3 M

$\therefore$ Interpolating polynomial is $f(x) = x^3+x^2-x+2$

$x=3$; $f(3) = 35$ 2 M

— x —

4) $\displaystyle\int_0^6 \frac{dx}{1+x^2}$

Actual value $= (\tan^{-1} x)_0^6 = 1.4056$ 2M

By Weedler rule

$$\int_0^6 \frac{1}{1+x^2}\,dx = \frac{3h}{10}\left(y_0+5y_1+y_2+6y_3+y_4+5y_5+y_6\right)$$ 1M

$$\boxed{\int_0^6 \frac{1}{1+x^2}\,dx = 1.3735}$$ 2M

from the table $h=1$

x	0	1	2	3	4	5	6
$f(x)$	1	0.5	0.2	0.1	0.0588	0.0385	0.027.

2M

---

5) (a) $2x - \log_{10} x = 7$. by Regula falsi

root between 3.5 & 4

$f(3.5) = f(a) = -0.54406$

$f(4) = f(b) = 0.3979$ 1M

Ist iteration

$$x_1 = \frac{a\,f(b) - b\,f(a)}{f(b)-f(a)} = \frac{3.5(0.3979) - 4(-0.54406)}{0.3979-(-0.54406)}$$ 1M

$$\boxed{x_1 = 3.78768}$$ 1M

$f(x_1) = f(3.78768) = -0.003013$

$\therefore$ Root lies between 3.78768 & 4

IInd iteration

$a = 3.78768$ & $b = 4$

$\therefore x_2 = 3.7893$

$\therefore$ The root is 3.7893 1M

(b) $\cos x = x e^x$

$\therefore f(x) = \cos x - x e^x$

$f(0) = 1$ & $f(1) = -2.17798$

Root lies between 0 & 1 1M

I iteration: $a = 0$ $b = 1$ let us take $x_0 = 0.5$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 0.51803$$ 1M

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.51776$$

$\therefore$ Real root is 0.518 1M

(6) $f(x) = x - x^2$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$ 1M

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)\,dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx\,dx$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx\,dx$$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} (x - x^2)\,dx = \frac{1}{\pi} \left(\frac{x^2}{2} - \frac{x^3}{3} \right)_{-\pi}^{\pi}$$

$$\boxed{\frac{a_0}{2} = -\frac{\pi^2}{3}}$$ 1M

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} (x - x^2) \cos nx\,dx.$$

$$= \frac{1}{\pi} \left[(x - x^2)\left(\frac{\sin nx}{n}\right) - (1 - 2x)\left(\frac{-\cos nx}{n^2}\right) + (-2)\left(\frac{-\sin nx}{n^3}\right) \right]_{-\pi}^{\pi}$$

$$= \frac{-4}{n^2} \cos n\pi = \frac{-4}{n^2} (-1)^n.$$

$$\boxed{a_n = \frac{4(-1)^{n+1}}{n^2}}$$ 2M

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(x-x^2)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[(x-x^2)\left(\frac{-\cos nx}{n}\right)(1-2x)\left(\frac{-\sin nx}{n^2}\right) + (-2)\left(\frac{\cos nx}{n^3}\right)\right]_{-\pi}^{\pi}$$

$$b_n = \frac{-1}{\pi n}(2\pi\cos n\pi) = -\frac{2}{n}(-1)^n$$

$$\boxed{b_n = \frac{2}{n}(-1)^{n+1}}$$ 2M

$$x-x^2 = -\pi^2/3 + 4\sum_1^\infty \frac{(-1)^{n+1}}{n^2}\cos nx + 2\sum_1^\infty \frac{(-1)^{n+1}}{n}\sin nx$$

Put $x=0$

$$\therefore 0 = -\pi^2/3 + 4\sum_1^\infty \frac{(-1)^{n+1}}{n^2}\times 1 + 0$$

$$\pi^2/12 = 1/1^2 - 1/2^2 + 1/3^2 - 1/4^2 - \dots$$ 1M

(7) $f(x) = (x-1)^2 \quad 0 \le x \le 1$

comparing the interval with $[0,1]$

$l = 1$

$\therefore f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\pi x$ 1M

$a_0 = \frac{2}{1}\int_0^1 f(x)\,dx = 2\int_0^1 (x-1)^2\,dx$

$= 2\left(\frac{(x-1)^3}{3}\right)_0^1 = 2/3$

$$\boxed{\frac{a_0}{2} = 1/3}$$ 2M

$a_n = 2\int_0^1 (x-1)^2 \cos n\pi x\,dx$

$= 2\left[(x-1)^2 \frac{\sin n\pi x}{n\pi} - 2(x-1) \times \frac{(-\cos n\pi x)}{n^2\pi^2} + 2\left(\frac{-\sin n\pi x}{n^3\pi^3}\right)\right]_0^1$

$= \frac{4}{n^2\pi^2}\left((x-1)\cos n\pi x\right)_0^1$

$a_n = \frac{4}{n^2\pi^2}$ 2M

$\therefore f(x) = \frac{1}{3} + \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos n\pi x.$ 1M

put $x=0$, $1 = \frac{1}{3} + \frac{4}{\pi^2}\left[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots\right]$

1M

$\Rightarrow \frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \cdots$

—— X ——

(8)

x	$\theta = \frac{\pi x}{3}$	y	$\cos\theta$	$\sin\theta$	$y\cos\theta$	$y\sin\theta$
0	0	9	1	0	9	0
1	60	18	0.5	0.866	9	15.588
2	120	24	-0.5	0.866	-12	20.784
3	180	28	-1	0	-28	0
4	240	26	-0.5	-0.866	-13	-22.516
5	300	20	0.5	-0.866	10	-17.32
		125			-25	-3.464

3M

$\sum y\sin\theta = -3.464$

$a_0 = \frac{2}{N}\sum y = \frac{1}{3}(125) = 41.67$

$$\boxed{\frac{a_0}{2} = 20.835}$$

1M

$\sum y\cos 2\theta = -7$

$\sum y\sin 2\theta = -0.004$

$$a_1 = \frac{2}{N}\sum y\cos\theta = \frac{2}{6}(-25) = -8.333$$

$$b_1 = \frac{2}{N}\sum y\sin\theta = \frac{2}{6}(-3.464) = -1.155$$

2M

$$a_2 = \frac{2}{N}\sum y\cos 2\theta = \frac{1}{3}(-7) = -2.33$$

$$b_2 = \frac{2}{N}\sum y\sin 2\theta = -0.00133$$

$$= -\tfrac{1}{3}\times(0.004)$$

1M