CMR INSTITUTE OF TECHNOLOGY									CMRIT
Sub:	Elements of Civil Engineering and Engineering Mechanics								
Date:	21.03.2017	Duration:	90 mins	Max Marks:	50	Sem:	I		

1(a) **Explain the basic idealisations made in mechanics.**

4.4 Basic Idealisations for Solving Mechanics Problems

When applying the laws and principles of mechanics to practical problems, a number of ideal conditions are assumed to exist. In the absence of such assumptions idealisations - it may not be feasible to find solutions. Such idealisations will not vary the accuracy of results of analysis obtained below the required optimum level. Following are some of the assumptions made to study the practical problem.

4.4.1 Particle

[02]

A particle is defined as an object that has no size but has mass and is assumed to a single point in space. The examples of practical problems involving the distances considerably larger when compared to size of body. The movement of earth in the celestial sphere, earth is treated as a particle. Also an aeroplane can be considered as a particle for the description of its flight path.

4.4.2 Continuum

[02]

A body consists of several particles and each particle at microscopic level can be visualized as molecules, atoms and electrons. The real picture of molecules and atoms is too complex to deal with. However, the physical quantities obtained by averaging the effects of molecules and atoms at macroscopic level, the body is assumed to consist of a continuous distribution of matter. Such a hypothetical continuous distribution of matter in the body is treated as a *Continuum*. This concept of Cotniuum of body enables us to treat the bodies as rigid bodies in solving the mechanics problems.

4.4.3 Rigid Body

[02]

A body which does not undergo deformation on the application of force is called a *rigid body*. The body will retain its shape and size or the distance between any two points of the body does not change, under the action of applied force, neglecting small relative deformations.

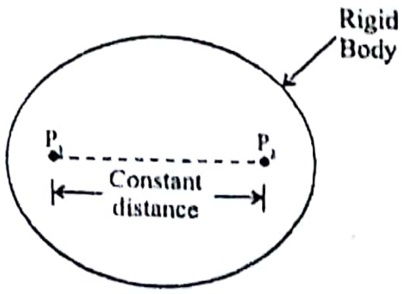

Fig. 4.1

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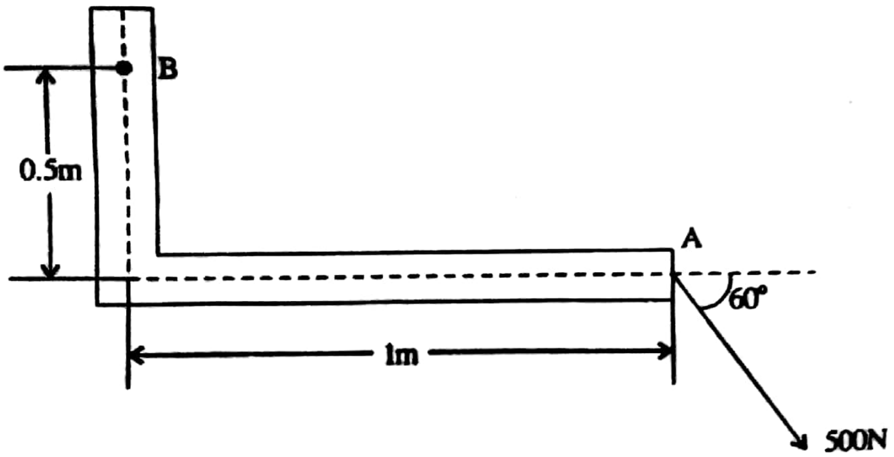
4.4.4 Point Force

When a force is transmitted from one body to another that are in contact, the bodies will generally undergo localised deformations, with the increase in contact area. Then the force acts actually on a finite area rather than at a point. Since the bodies dealt with are rigid bodies not much accuracy is lost by treating it as a point force, which helps to simplify the problem.

(b) A 500N force is applied at point A of a L shaped plate. Find the equivalent force couple system at B



Fig, 1.b

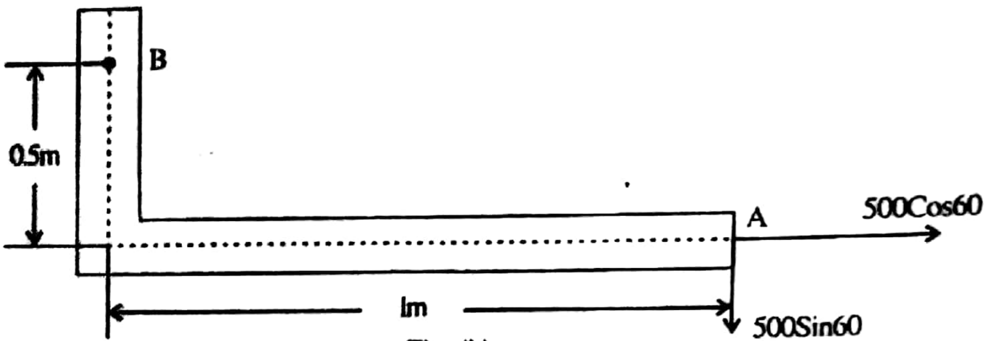


Fig. (b)

The resultant will be 500N acting at an angle 60°, since there is only one force at A.

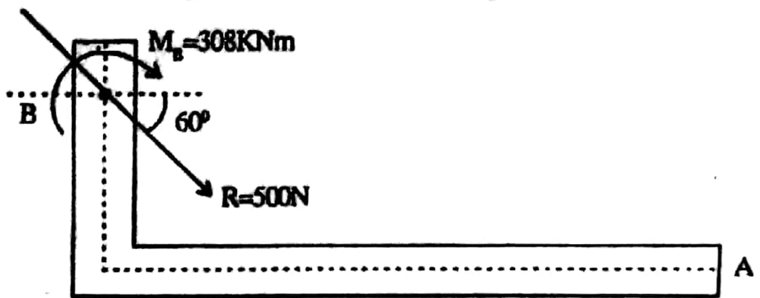
Taking Moment about B. 01

$$\Sigma M_B = (500\sin 60) \times 1 - (500\cos 60) \times 0.5$$

$$= 308\text{KNm (Clockwise)}.$$

02

The Equivalent Force-Couple at B is shown in fig. (c).



01

Equivalent Force Couple at B

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2 (a) **Determine the magnitude and direction of the resultant force for the force system shown in Fig.2a**

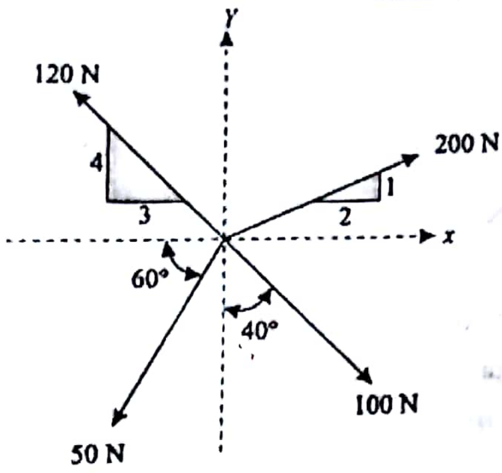

Fig. 2a

Solution. If θ_1 is the inclination of 200 N force to x-axis, then

$$\tan\theta_1 = \frac{1}{2} \quad \therefore \theta_1 = 26.565°$$

Similarly inclination of 120 N force to x-axis is given by

$$\tan\theta_2 = \frac{4}{3} \quad \text{i.e., } \theta_2 = 53.13°.$$

$$\sum F_x = 200\cos 26.565 - 120\cos 53.13 - 50\cos 60 + 100\sin 40 = 146.2 \text{ N} \quad \boxed{02}$$

$$\sum F_y = 200\sin 26.565 + 120\sin 53.13 - 50\sin 60 - 100\cos 40 = 65.5 \text{ N} \quad \boxed{02}$$

$$\therefore \quad R = \sqrt{146.2^2 + 65.5^2} = 160.2 \text{ N} \quad \boxed{01}$$

$$\alpha = \tan^{-1}\frac{65.5}{146.2} = 24.1° \text{ as shown in Fig. 5.10(b).} \quad \boxed{01}$$

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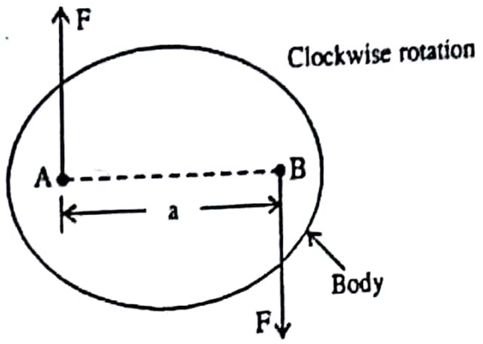
(b) Define couple. Explain the characteristics of a couple.

When two equal and opposite parallel forces act on a body, at some distance apart, then these two forces constitute a COUPLE, which has a tendency to rotate a body. The perpendicular distance between the parallel forces is known as *arm of the couple* or *lever arm* or moment arm.

The simple examples of couple are the forces applied to the key of a clock in winding it up. Forces applied while Locking or Unlocking of the lock. The couple can produce clockwise rotation or anticlockwise rotation.



02 marks

4.15 Characteristics of Couple

A couple, whether clockwise or anti-clockwise has the following characteristics :

1) The Algebraic sum of forces, constituting the couple is zero i.e., the resultant of the forces constituting couple is zero. From figure 4.12, -F+F=0.

2) The Algebraic sum of the moments of the forces, constituting the couple, about any point is equal to the moment of the couple itself.

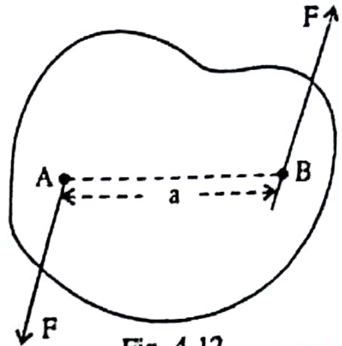


Fig. 4.12

02 marks

In figure 4.13 taking moment of forces about the point O.

$$M_O = Fd_2 - Fd_1$$

$$= F(d_2 - d_1) \quad \text{..........(1)}$$

Moment for couple = Fa ....(2)

From (1) and (2) it is proved.

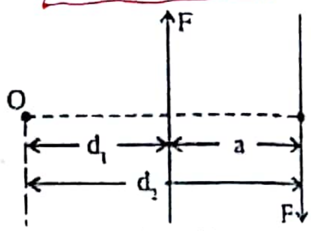


Fig. 4.13

3) A couple can be balanced only by an equal and opposite couple in the same plane.

4) Any number of coplanar couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of the moments of all the couples.

5) The moment of couple is constant for any point choosen in the plane of the couple.

6) Any two couples whose moments are equal and of same sign are equivalent.

3(a) State and Prove Parallelogram law of forces.

If two forces acting simultaneously on a body at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection of the two sides

01 marks

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representing the forces.

Consider the two forces F_1 and F_2 acting on a particle as shown in Fig. 5.3(a). Let the angle between the two forces be θ. If parallelogram $ABCD$ is constructed as shown in Fig. 5.3(b), with AB representing F_1 and AD representing F_2 to some scale, according to parallelogram law of forces AC represents the resultant R. Drop perpendicular CE to AB.

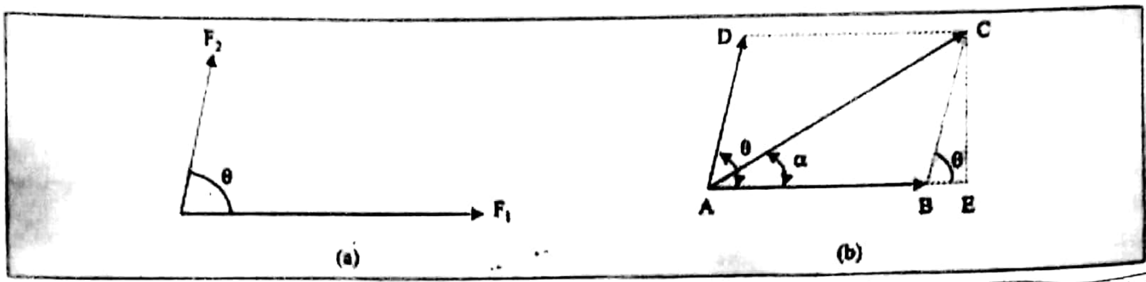

Fig. 5.3

[01 marks]

Now the resultant R of F_1 and F_2 is given by

$$R = AC$$

$$= \sqrt{AE^2 + CE^2}$$

$$= \sqrt{(AB + BE)^2 + CE^2}$$

But $\quad AB = F_1$

$$BE = BC \cos\theta = F_2 \cos\theta$$

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and $$CE = BC \sin\theta = F_2 \sin\theta$$

$$\therefore \quad R = \sqrt{(F_1 + F_2\cos\theta)^2 + (F_2\sin\theta)^2}$$

$$= \sqrt{F_1^2 + 2F_1F_2\cos\theta + F_2^2\cos^2\theta + F_2^2\sin^2\theta}$$

$$= \sqrt{F_1^2 + 2F_1F_2\cos\theta + F_2^2} \quad \boxed{\text{03 marks}} \qquad \text{Eqn. (5.1)}$$

The inclination of the resultant to force F_1 is given by α, where

$$\tan\alpha = \frac{CE}{AE} = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

Thus $$\alpha = \tan^{-1}\frac{F_2\sin\theta}{F_1 + F_2\cos\theta} \quad \boxed{\text{01 marks}} \qquad \text{Eqn. (5.2)}$$

Particular cases:

(i) When $\theta = 90°$ [Ref. Fig. 5.4(a)] $\quad R = \sqrt{F_1^2 + F_2^2}$

(ii) When $\theta = 0°$ [Ref. Fig. 5.4(b)] $\quad R = \sqrt{F_1^2 + 2F_1F_2 + F_2^2} = F_1 + F_2$

(iii) When $\theta = 180°$ [Ref. Fig. 5.4(c)] $\quad R = \sqrt{F_1^2 - 2F_1F_2 + F_2^2} = F_1 - F_2$

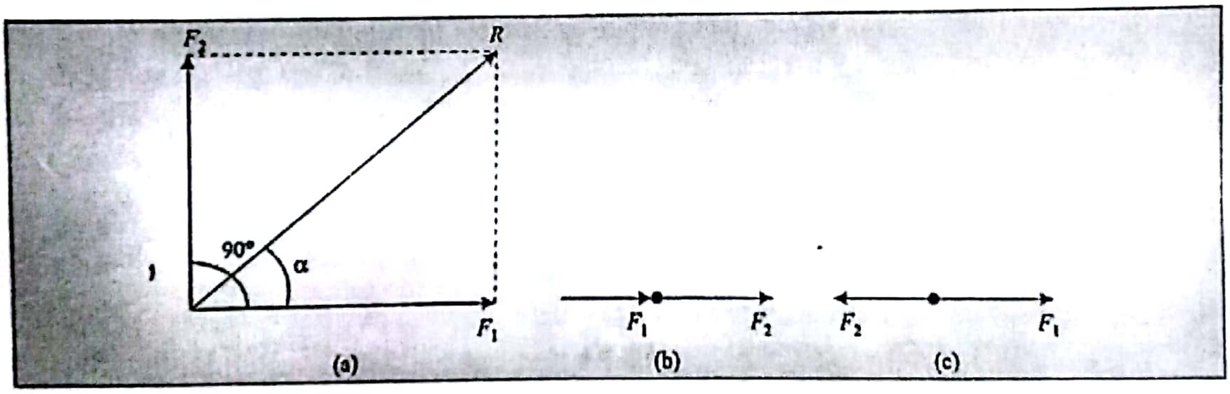

Fig. 5.4

From cases (ii) and (iii) it is clear that when the forces acting on a body are collinear, their resultant is equal to the algebraic sum of the forces.

(b) **Three forces are acting at point O as shown in Fig.3b. Determine two additional forces along OA and OB such that resultant of five forces is zero.**

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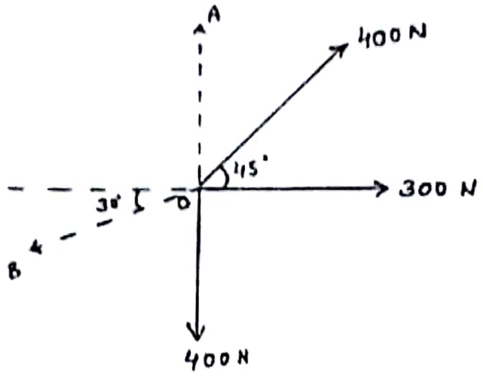
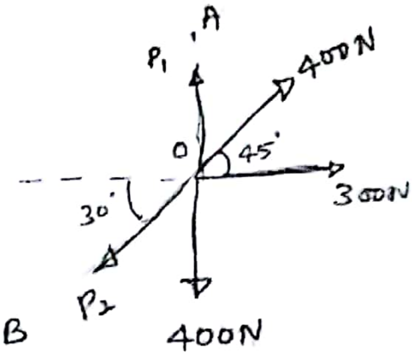


Fig.3.b

Soln:

$\Sigma F_x = 0$ & $\Sigma F_y = 0$

let P_1 & P_2 be two forces acting along OA & OB



$\Sigma F_x = 0$

$300 + 400\cos 45 - P_2 \cos 30^\circ = 0$

$P_2 = 673\ N$ $\boxed{02 \text{ marks}}$

$\Sigma F_y = 0$

$P_1 + 400 \sin 45 - 673 \sin 30 - 400 = 0$

$P_1 = 453.657\ N$ $\boxed{02 \text{ marks}}$

4 (a) **Two smooth cylinders each of weight 100 N and radius 15cm are connected at their centers by a string AB of length 40cm and rest upon horizontal plane as shown in Fig.4.(a). The cylinder above them has a weight 200N and radius of 15cm. Find the force in the string AB and the reaction developed at the point of contacts D and E.**

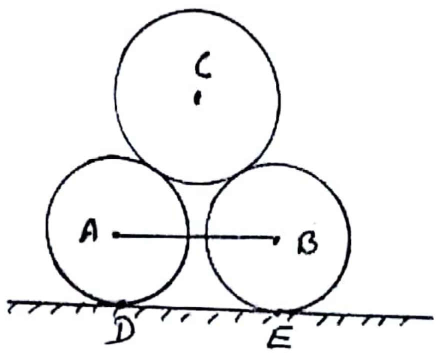


Fig.4a

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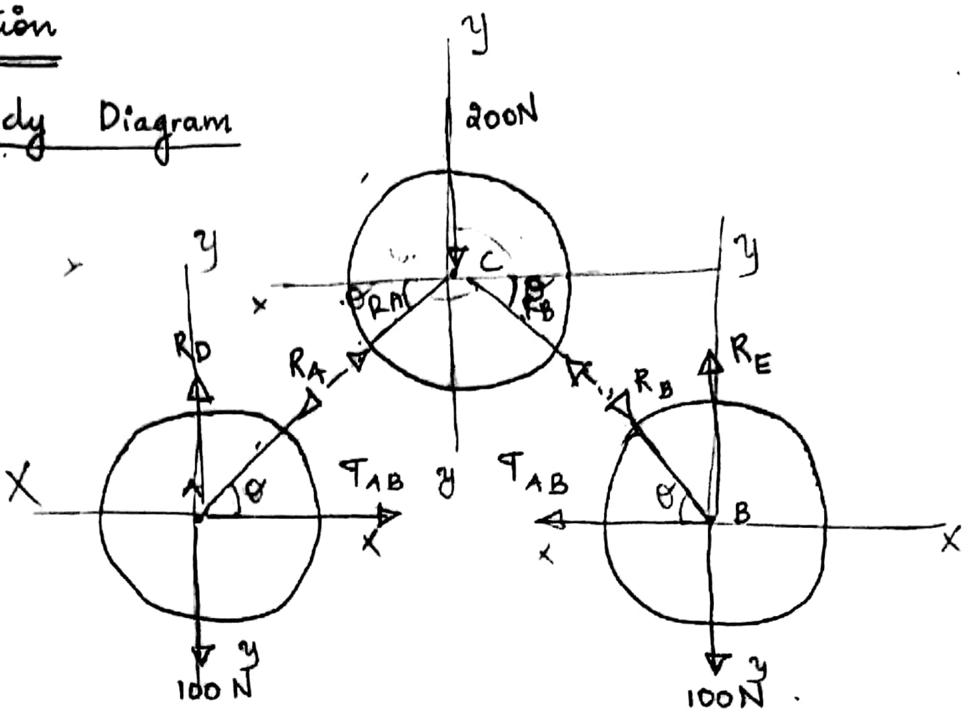
Solution

Freebody Diagram



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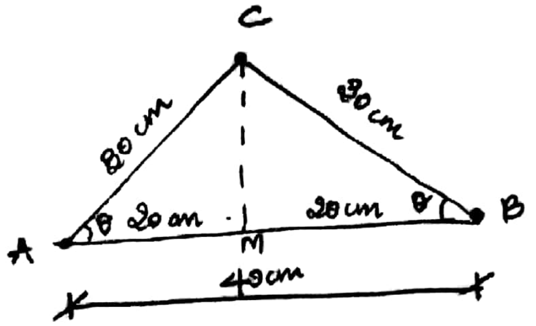
where

T_{AB} tension in the string

R_D, R_E · Reaction from ground

R_A R_B · Reaction at the point of contact of the balls

We need to find the angle of inclination of R_A and R_B to the horizontal.



In Δ BCM

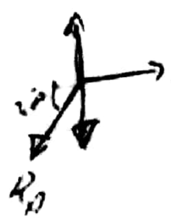
Applying Pythagoras theorm

$BC^2 = CM^2 + BM^2$

$80^2 = CM^2 + 20^2$

$CM = \sqrt{80^2 - 20^2}$

$= 22.36$



$\therefore \tan\theta = \frac{opp}{adjacent} = \frac{22.36}{20}$

$\theta = 48.2°$

Resolving forces at A :-

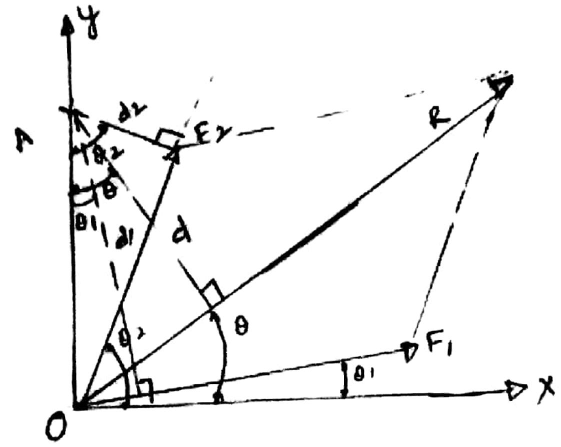
For equilibrium at A

5(a)	State and Prove Varignon's theorem of moments.

) Varignon's theorem :-

Varignon's theorem states that the algebraic sum of moments due to all forces acting on an object about any point is equal to the moment of their resultant about the same point.

01 mark



01 mark

$\cos\theta_1 = \frac{d_1}{OA}$ $\quad d_1 = OA\cos\theta_1$

$d_2 = OA\cos\theta_2$

$d = OA\cos\theta$

The moment of 'R' about A

✓

- $M_R = R.d$

$= R(OA\cos\theta)$

$\therefore \quad M_R = OA\,(R\cos\theta)$

The x- component of resultant is

$R_x = R\cos\theta$

$\therefore \quad M_R = OA.\,R_x$

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The moment of F_1 about 'A' is

$$M_1 = F_1 d_1$$

$$= F_1 (OA \cos\theta_1)$$

$$M_1 = OA (F_1 \cos\theta_1)$$

But $F_1 \cos\theta_1 = F_{1x}$

$$M_1 = OA \; F_{1x}$$

Moment of F_2 about 'A' is

$$M_2 = F_2 \times d_2$$

$$= F_2 \times (OA \cos\theta_2)$$

$$= OA \; F_2 \cos\theta_2$$

$$M_2 = OA \; F_{2x}$$

$$M_1 + M_2 = OA \; F_{1x} + OA \; F_{2x}$$

$$M_1 + M_2 = OA (F_{1x} + F_{2x})$$

$$F_{1x} + F_{2x} = R_x$$

$$M_1 + M_2 = OA \; R_x$$

$$M_1 + M_2 = M_R.$$

$\boxed{\text{03 marks}}$

Thus the algebraic sum of moments due to F_1 & F_2 about 'A' is equal to the moment of their resultant about 'A'.

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(b) **Find the resultant magnitude, direction and its point of application from A for the square plate subjected to forces shown in Fig.5b**

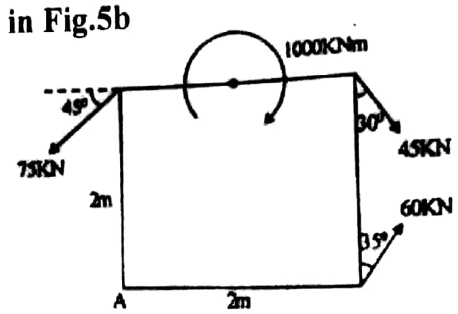

Fig.5b

Solution : Resolved components of forces shown in figure b.

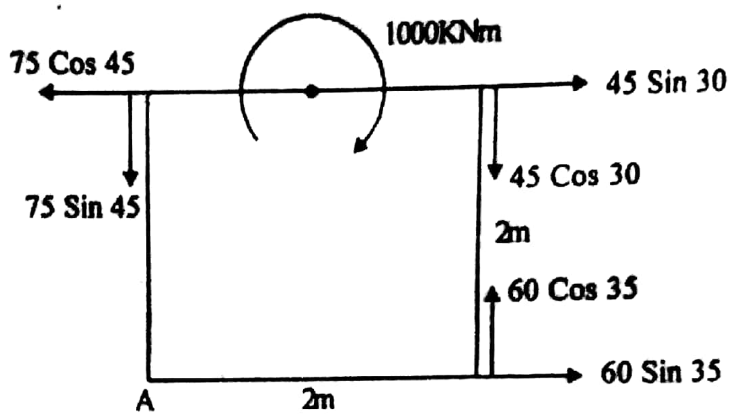

Fig. (b)

Resolving force Horizontally,

$$\Sigma H = -75Cos45 + 45Sin30 + 60Sin35$$
$$= 3.88KN$$

Resolving forces Vertically,

$$\Sigma V = -75Sin45 - 45Cos30 + 60Cos35$$
$$= -42.86KN$$

Resultant $R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$

$$R = \sqrt{(3.88)^2 + (-42.86)^2}$$
$$R = 43.03\ KN$$

02 marks

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Let 'θ' be angle made by resultant,

$$\tan\theta = \frac{\Sigma V}{\Sigma H} = \frac{-42.86}{3.88} = -11.046$$

$$\theta = -84.83^{0}$$

Since ΣH is +ve and ΣV is -ve, resultant lies in 4[th] quadrant figure c.

Let x be the ⊥[ar] distance of resultant from A from figure (b) and figure (d).

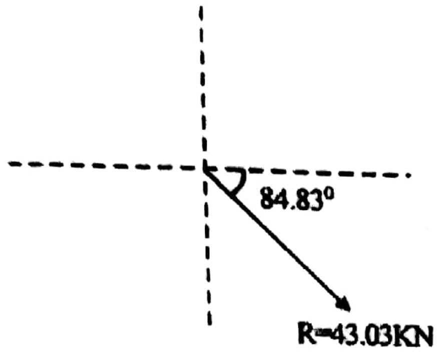


Fig. (c)

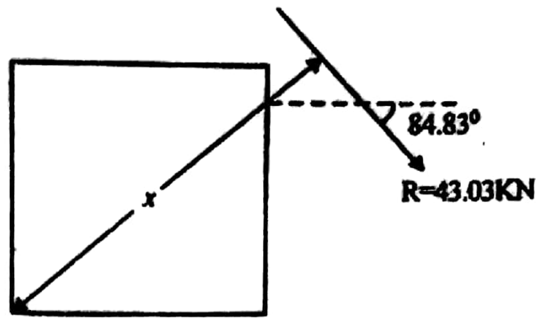


Fig. (d)

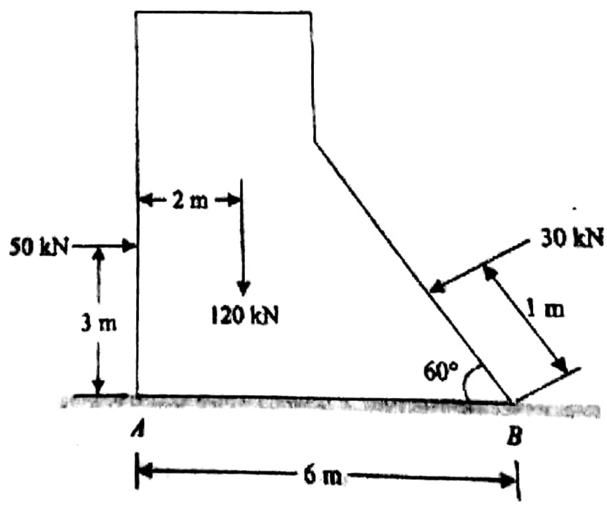
Taking moment about A and applying Varignon's principle,

$$43.03x = (-70\cos45)2 + (45\sin 30)2 + (45\cos30)2 - (60\cos35)2 + 1000$$

$$43.03x = 925.65$$

$$x = 21.51\text{m from A on RHS}$$ 03 marks

6 (a) **Check the stability of the dam carrying the forces as shown in Fig.Q.6.(a). The dam is said to be stable if the resultant lies in the middle 1/3 of the base AB.**



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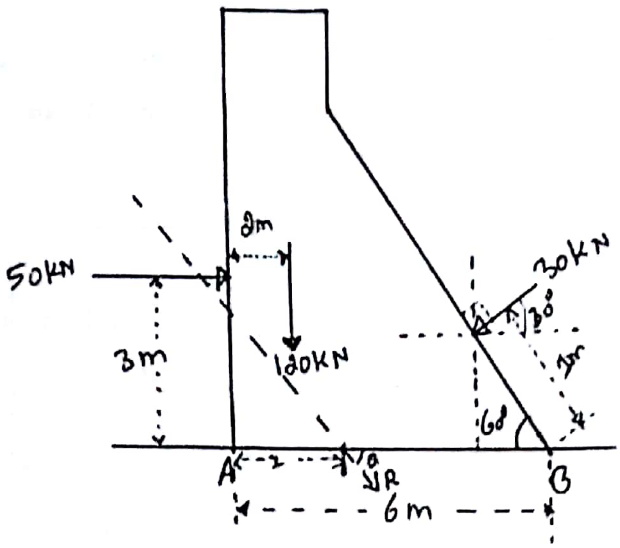

To determine magnitude & direction of Resultant

$$\Sigma F_x = 50 - 30\cos 30$$

$$\boxed{\Sigma F_x = 24.02}\ kN$$

$$\Sigma F_y = -120 - 30\sin 30$$

$$\boxed{\Sigma F_y = -135\ kN}$$

Magnitude,

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(24.02)^2 + (-135)^2}$$

$$\boxed{R = 137.12\ kN}$$

02 marks

Direction

$$\theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

$$= \tan^{-1}\left(\frac{135}{24.02}\right)$$

$$\boxed{\theta = 79.91^\circ}$$

01 marks

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To find the Position [x intercept] of resultant w.r.t A

By applying Varignon's theorem of moments about A.

$$M_{R \text{ about } A} = \Sigma M_{F \text{ about } A}$$

$$137.12 \sin 79.91 \cdot x = 50 \times 3 + 120 \times 2 + 30 \sin 30 (6 - 1 \cos 60) - 30 \cos 30 \times 1 \sin 60$$

$$x = \frac{150 + 240 + 82.5 - 112.5}{134.99}$$

$$\boxed{x = 2.66 \text{ m.}}$$

03 marks

where $\frac{1}{3}$ of base AB $= \frac{1}{3} \times 6^{2} = 2m \neq 2.66m.$

01 marks

$\therefore$ dam is not stable.

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(b) State and explain Lami's theorem

Lami's theorem states that *if a body is in equilibrium under the action of only three forces, each force is proportional to the sine of the angle between the other two forces.* Thus for the system of forces shown in Fig. 8.1(a),

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma} \qquad \text{Eqn. (8.3)}$$

{01 marks}

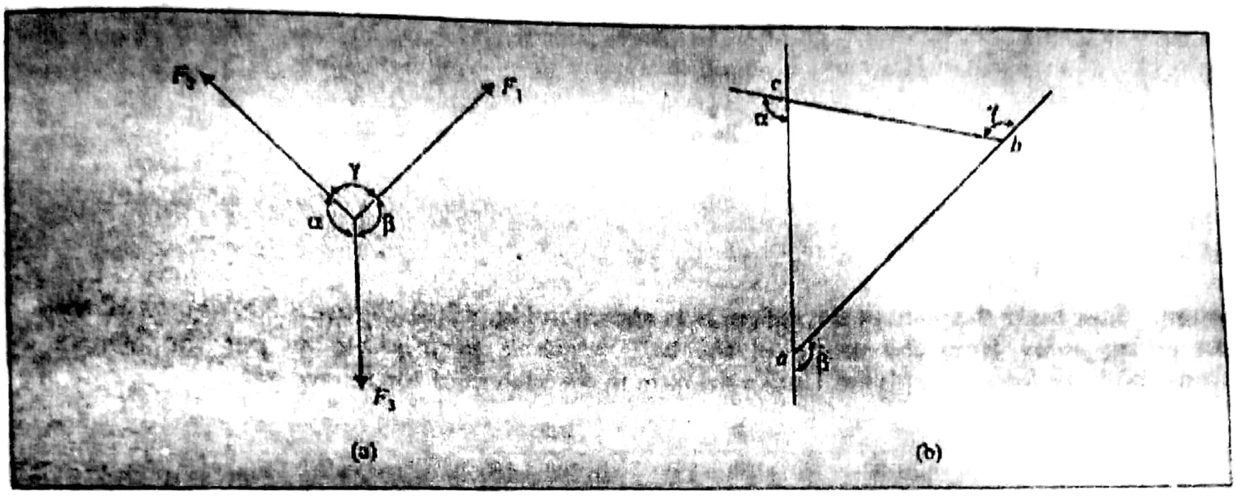

Fig. 8.1

Proof. Draw the three forces F_1, F_2 and F_3 one after the other in direction and magnitude starting from point '*a*'. Since the body is in equilibrium the resultant should be zero, which means the last point of force diagram should coincide with '*a*'. Thus, it results in a triangle of forces *abc* as shown in Fig. 8.1(b). Now the external angles at *a*, *b* and *c* are equal to β, γ and α, since *ab*, *bc* and *ca* are parallel to F_1, F_2 and F_3 respectively. In the triangle of forces *abc*,

$$ab = F_1$$

$$bc = F_2$$

and

$$ca = F_3$$

Applying sine rule for the triangle ***abc*,**

$$\frac{ab}{\sin(180-\alpha)} = \frac{bc}{\sin(180-\beta)} = \frac{ca}{\sin(180-\gamma)}$$

i.e.,

$$\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}$$

{02 marks}

CI CCI HOD

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