CMR INSTITUTE OF TECHNOLOGY

USN

Internal Assessment Test II

Sub:	**ENGINEERING MATHEMATICS I**					Code:	15MAT11	
Date:	08/ 11 / 2017	Duration:	90 mins	Max Marks:	50	Sem:	**I**	Sec: **B, D,F**

Answer Q1.a or Q1.b and any 7 questions from the rest.

		Marks	OBE CO	OBE RBT
1.a)	Diagonalise the matrix $\begin{bmatrix} 2 & -2 & -1 \\ 0 & 1 & 0 \\ 2 & -4 & -1 \end{bmatrix}$.	[08]	CO7	L3
	OR			
b)	Reduce the quadratic form $2xy + 2xz + 2yz$ to canonical form.	[08]	CO7	L3
2.	If $u = tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.	[06]	CO3	L3
3.	If $v(x,y) = (1 - 2xy + y^2)^{-\frac{1}{2}}$ and $x\frac{\partial v}{\partial x} - y\frac{\partial v}{\partial y} = y^2v^k$, find the value of k.	[06]	CO3	L3

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4. If $x = rsin\theta cos\varphi, y = r\ sin\theta sin\varphi\ and\ z = rcos\ \theta$, find $J\left(\frac{x,y,z}{r,\theta,\varphi}\right)$.	[06]	CO3	L3
5. A particle moves along a curve whose parametric equations are $x = e^{-t}$, $y = 2\cos 3t, z = 2\sin 3t$, where t is the time. Find the component of velocity and acceleration in the direction of $\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$, at $t = 0$.	[06]	CO4	L3
6. Find $div\ \vec{F}$ and $curl\ \vec{F}$, if $\vec{F} = grad(x^3 + y^3 + z^3 - 3xyz)$.	[06]	CO4	L3
7. Verify that $\vec{f} = 2xyz^3\hat{\imath} + x^2z^3\hat{\jmath} + 3x^2yz^2\hat{k}$ is irrotational, and find a scalar function ϕ such that $\vec{f} = \nabla\phi$.	[06]	CO4	L3
8. Find the angle between the directions of the normals to the surface $xy = z^2$ at the points $(4,1,2)$ and $(3,3,-3)$.	[06]	CO4	L3
9. For any scalar field φ and vector field $\vec{F}$, prove that $div\ (\varphi\vec{F}) = \varphi\ div\ \vec{F} + grad\ \varphi.\vec{F}$.	[06]	CO4	L3

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Solution

1 a) $A = \begin{pmatrix} 2 & -2 & -1 \\ 0 & 1 & 0 \\ 2 & -4 & -1 \end{pmatrix}$

the characteristic equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -2 & -1 \\ 0 & 1-\lambda & 0 \\ 2 & -4 & -1-\lambda \end{vmatrix} = 0.$$

$$\lambda^3 - 2\lambda^2 + (-1+0+2)\lambda - (-2+2) = 0$$

$$\lambda^3 - 2\lambda^2 + \lambda = 0 \qquad \text{— 1 mk}$$

$$\lambda(\lambda^2 - 2\lambda + 1) = 0$$

$$\lambda(\lambda-1)^2 = 0 \Rightarrow \lambda = 0, 1, 1 \qquad \text{— 1 mk}$$

$$[A - \lambda I]X = 0$$

$\underline{\lambda = 0}$ $\quad 2x - 2y - z = 0$

$y = 0$

$2x - 4y - z = 0$

$y = 0$, $2x - z = 0 \Rightarrow z = 2x$

Let $x = 1$. Then $z = 2$

$X_1 = (1, 0, 2)^T$ — 1 mk.

$\underline{\lambda = 1}$ $\quad \left.\begin{array}{l} x - 2y - z = 0 \\ 2x - 4y - 2z = 0 \end{array}\right\} \quad x = 2y + z$

Let $y = 1$, $z = 0$ Then $x = 2$

$X_2 = (2, 1, 0)^T$.

Let $y = 0$, $z = 1$. Then $x = 1$

$X_3 = (1, 0, 1)^T$ — 2 mk

$$P = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$ — 1 mk

$$P^{-1} = \frac{\operatorname{adj} P}{|P|} = \frac{1}{1-2} \begin{bmatrix} 1 & -2 & -1 \\ 0 & -1 & 0 \\ -2 & 4 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & 0 \\ 2 & -4 & -1 \end{bmatrix}$$

$$P^{-1}AP = \begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & 0 \\ 2 & -4 & -1 \end{bmatrix} \begin{bmatrix} 2 & -2 & -1 \\ 0 & 1 & 0 \\ 2 & -4 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2 & 1 \\ 0 & 1 & 0 \\ 2 & -4 & -1 \end{bmatrix} \begin{bmatrix} 0 & 2 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} =$$

$$= \operatorname{diag}(\lambda_1, \lambda_2, \lambda_3) = D$$ — 2 mk.

2b) $2xy + 2yz + 2zx$.

The symmetric matrix of the Q.F is

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

The ch. eqn is $|A - \lambda I| = 0$

$$\begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$$

$$\lambda^3 - 0 \cdot \lambda^2 + (-1 + -1 + -1)\lambda - (1 + 1) = 0$$

$$\lambda^3 - 3\lambda - 2 = 0$$

$$(\lambda - 2)(\lambda + 1)^2 = 0$$

$$\lambda = 2, -1, -1$$ — 2 mk

$$[A - \lambda I]X = 0.$$

2. $u = \tan^{-1}\left(\dfrac{x^3+y^3}{x+y}\right)$

$$\tan u = \frac{x^3+y^3}{x+y} = \frac{x^3\left(1+y^3/x^3\right)}{x\left(1+y/x\right)} = x^2 f(y/x)$$

$\tan u$ is homogeneous of degree 2. — 2 mk

$\therefore$ By Euler's theorem, $x\dfrac{\partial}{\partial x}(\tan u) + y\dfrac{\partial}{\partial y}(\tan u) = n \cancel{\tan} \tan u =$ — 2 mk

$$x \sec^2 u \frac{\partial u}{\partial x} + y \sec^2 u \frac{\partial u}{\partial y} = 2\tan u$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{\sec^2 u}\cdot 2\tan u$$

$$= \cos^2 u \cdot 2\frac{\sin u}{\cos u} = 2\sin u\cos u$$

$$= \underline{\underline{\sin 2u}}$$ — 2 mk

3. $v(x,y) = (1-2xy+y^2)^{-1/2}$

$$\frac{\partial v}{\partial x} = -\frac{1}{2}\left(1-2xy+y^2\right)^{-3/2}(-2y)$$

$$= y\left(1-2xy+y^2\right)^{-3/2}$$

$$\frac{\partial v}{\partial y} = -\frac{1}{2}\left(1-2xy+y^2\right)^{-3/2}(-2x+2y)$$

$$= (x-y)\left(1-2xy+y^2\right)^{-3/2}$$ — 3 mk

$$x\frac{\partial v}{\partial x} - y\frac{\partial v}{\partial y} = xy\left(1-2xy+y^2\right)^{-3/2} - y(x-y)\left(1-2xy+y^2\right)^{-3/2}$$

$$= \left(1-2xy+y^2\right)^{-3/2}\left[xy - xy + y^2\right]$$

$$= y^2\left[\left(1-2xy+y^2\right)^{-1/2}\right]^3$$

$$= y^2 v^k$$ — 2 mk

$\Rightarrow$ $\underline{\underline{k=3}}$ — 1 mk

$\underline{\lambda = 2}$ $\quad -2x + y + z = 0$

$x - 2y + z = 0$

$x + y - 2z = 0$

$$\begin{matrix} -2 & 1 & 1 \\ 1 & -2 & 1 \end{matrix}$$

$$\frac{x}{1+2} = \frac{-y}{-2-1} = \frac{z}{4-1}$$

Evector: $(3, +3, 3)^T$ or $(1, +1, 1)^T$

$X_1 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)^T$ — 1mk

$\underline{\lambda = -1}$ $\quad x + y + z = 0$

$x = -(y + z)$

Let $y = k_1$, $z = k_2$. Then $x = -(k_1 + k_2)$

Taking $k_1 = 1$, $k_2 = 0$, we get the eigenvector

$(-1, 1, 0)$

A vector $(-(k_1 + k_2), k_1, k_2)$ that is orthogonal to $(-1, 1, 0)$ satisfies $k_1 + k_2 + k_1 = 0$

ie $k_2 = -2k_1$.

Let $k_1 = 1$. then $k_2 = -2$

$\therefore$ An orthogonal evector is $(1, 1, -2)$ — 3mk

Normalising, we get

$X_2 = \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right)$ and $X_3 = \left(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}\right)$ — 3mk

$\therefore P = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & 0 & \frac{-2}{\sqrt{6}} \end{pmatrix}$ is the orthogonal modal matrix.

The transformation $X = PY$. ie, $x = \frac{1}{\sqrt{3}}x' - \frac{1}{\sqrt{2}}y' + \frac{1}{\sqrt{6}}z'$, $y = \frac{1}{\sqrt{3}}x' + \frac{1}{\sqrt{2}}y' + \frac{1}{\sqrt{6}}z'$, $z = \frac{1}{\sqrt{3}}x' - \frac{2}{\sqrt{6}}z'$ gives the canonical form $\underline{2x'^2 - y'^2 - z'^2}$ — 2mk

2. $u = \tan^{-1}\left(\frac{x^3+y^3}{x+y}\right)$

$\tan u = \frac{x^3+y^3}{x+y} = \frac{x^3\left(1+y^3/x^3\right)}{x\left(1+y/x\right)} = x^2 f(y/x)$

$\tan u$ is homogeneous of degree 2. — 2 mk

$\therefore$ By Euler's theorem, $x\frac{\partial}{\partial x}(\tan u) + y\frac{\partial}{\partial y}(\tan u) = n \tan u = 2\tan u$

— 2 mk

$x \sec^2 u \frac{\partial u}{\partial x} + y \sec^2 u \frac{\partial u}{\partial y} = 2\tan u$.

$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{\sec^2 u}\cdot 2\tan u$

$= \cos^2 u \cdot 2\frac{\sin u}{\cos u} = 2\sin u\cos u$

$= \underline{\underline{\sin 2u}}$ — 2 mk

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$\frac{\partial v}{\partial x} = -\frac{1}{2}(1-2xy+y^2)^{-3/2}(-2y)$

$= y(1-2xy+y^2)^{-3/2}$ — 2 mk

$\frac{\partial v}{\partial y} = -\frac{1}{2}(1-2xy+y^2)^{-3/2}(-2x+2y)$

$= (x-y)(1-2xy+y^2)^{-3/2}$ — 3 mk

$x\frac{\partial v}{\partial x} - y\frac{\partial v}{\partial y} = xy(1-2xy+y^2)^{-3/2} - y(x-y)(1-2xy+y^2)^{-3/2}$

$= (1-2xy+y^2)^{-3/2}\left[xy - xy + y^2\right]$

$= y^2\left[(1-2xy+y^2)^{-1/2}\right]^3$

$= y^2 v^k$ — 2 mk

$\Rightarrow$ $\underline{\underline{k = 3}}$ — 1 mk

4. $x = r\sin\theta\cos\varphi$

$y = r\sin\theta\sin\varphi$

$z = r\cos\theta$

$$J\left(\frac{x,y,z}{r,\theta,\varphi}\right) = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial \varphi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial \varphi} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial \varphi} \end{vmatrix} \quad \text{— 1mk}$$

$$= \begin{vmatrix} \sin\theta\cos\varphi & r\cos\theta\cos\varphi & -r\sin\theta\sin\varphi \\ \sin\theta\sin\varphi & r\cos\theta\sin\varphi & r\sin\theta\cos\varphi \\ \cos\theta & -r\sin\theta & 0 \end{vmatrix} \quad \text{— 2mk}$$

$$= r \cdot r\sin\theta \begin{vmatrix} \sin\theta\cos\varphi & \cancel{r}\cos\theta\cos\varphi & -\sin\varphi \\ \sin\theta\sin\varphi & \cos\theta\sin\varphi & \cos\varphi \\ \cos\theta & -\sin\theta & 0 \end{vmatrix}$$

Expanding along last row,

$$J\left(\frac{x,y,z}{r,\theta,\varphi}\right) = r^2\sin\theta\left[\cos\theta\left(\cos\theta\cos^2\varphi + \cos\theta\sin^2\varphi\right) + \sin\theta\left(\sin\theta\cos^2\varphi + \sin\theta\sin^2\varphi\right)\right]$$

$$= r^2\sin\theta\left[\cos^2\theta\left(\cos^2\varphi + \sin^2\varphi\right) + \sin^2\theta\left(\cos^2\varphi + \sin^2\varphi\right)\right]$$

$$= r^2\sin\theta\left[\cos^2\theta + \sin^2\theta\right] = \underline{\underline{r^2\sin\theta}} \quad \text{— 3mk}$$

5. $\vec{r}(t) = e^{-t}\,\hat{i} + 2\cos 3t\,\hat{j} + 2\sin 3t\,\hat{k}$

$\vec{v} = \frac{d\vec{r}}{dt} = -e^{-t}\,\hat{i} - 6\sin 3t\,\hat{j} + 6\cos 3t\,\hat{k}.$

At $t=0$, $\vec{v} = -\hat{i} + 6\hat{k}$ — 1mk

$\vec{a} = \frac{d\vec{v}}{dt} = e^{-t}\,\hat{i} - 18\cos 3t\,\hat{j} - 18\sin 3t\,\hat{k}$

At $t=0$, $\vec{a} = \hat{i} - 18\hat{j}$ — 1mk

$\vec{d} = \hat{i} - 3\hat{j} + 2\hat{k}$

$$\hat{d} = \frac{1}{\sqrt{1+9+4}}\left(\hat{i} - 3\hat{j} + 2\hat{k}\right) = \frac{1}{\sqrt{14}}\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)$$

Component of velocity along $\vec{d} = \vec{v} \cdot \hat{d}$

$$= \left(-\hat{i} + 6\hat{k}\right) \cdot \frac{\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)}{\sqrt{14}}$$

$$= \frac{-1+12}{\sqrt{14}} = \underline{\underline{\frac{11}{\sqrt{14}}}} \quad \text{--- 2 mk}$$

Component of acceleration along $\vec{d}$

$$= \left(\hat{i} - 18\hat{j}\right) \cdot \frac{\left(\hat{i} - 3\hat{j} + 2\hat{k}\right)}{\sqrt{14}}$$

$$= \frac{1+54}{\sqrt{14}} = \underline{\underline{\frac{+55}{\sqrt{14}}}} \quad \text{--- 2 mk.}$$

6. $$\vec{F} = \nabla\left(x^3 + y^3 + z^3 - 3xyz\right) = \nabla\phi$$

$$= \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}$$

$$= \left(3x^2 - 3yz\right)\hat{i} + \left(3y^2 - 3xz\right)\hat{j} + \left(3z^2 - 3xy\right)\hat{k}$$

$$= f_1\hat{i} + f_2\hat{j} + f_3\hat{k} \quad \text{--- 2 mk}$$

$$\text{div } \vec{F} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

$$= \underline{6x + 6y} + 6z = 6(x+y+z) \quad \text{--- 2 mk}$$

$$\text{Curl } \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3x^2 - 3yz & 3y^2 - 3zx & 3z^2 - 3xy \end{vmatrix}$$

$$= \hat{i}\left(-3x + 3x\right) + \hat{j}\left(-3y + 3y\right) + \hat{k}\left(-3z + 3z\right)$$

$$= \underline{\underline{0}} \quad \text{--- 2 mk}$$

7. $\vec{f} = 2xyz^3\,\hat{i} + x^2z^3\,\hat{j} + 3x^2yz^2\,\hat{k}$

$$\text{Curl } \vec{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2xyz^3 & x^2z^3 & 3x^2yz^2 \end{vmatrix}$$

$$= \hat{i}\left(3x^2z^2 - 3x^2z^2\right) + \hat{j}\left(6xyz^2 - 6xyz^2\right) + \hat{k}\left(2xz^3 - 2xz^3\right)$$

$$= 0 \qquad \text{—— } \underline{2\text{ mk}}$$

$$\vec{f} = \nabla\varphi = \frac{\partial\varphi}{\partial x}\hat{i} + \frac{\partial\varphi}{\partial y}\hat{j} + \frac{\partial\varphi}{\partial z}\hat{k}$$

$$\frac{\partial\varphi}{\partial x} = 2xyz^3 \;;\quad \frac{\partial\varphi}{\partial y} = x^2z^3 \quad \frac{\partial\varphi}{\partial z} = 3x^2yz^2$$

Integrating w.r.t x, y & z resp,

$$\varphi = x^2yz^3 + f_1(y, z)$$
$$\varphi = x^2yz^3 + f_2(x, z)$$
$$\varphi = x^2yz^3 + f_3(x, y)$$

$$\therefore \underline{\underline{\varphi = x^2yz^3}} \qquad \text{—— } \underline{4\text{ mk}}$$

8. $\varphi : xy - z^2$

$$\nabla\varphi = \frac{\partial\varphi}{\partial x}\hat{i} + \frac{\partial\varphi}{\partial y}\hat{j} + \frac{\partial\varphi}{\partial z}\hat{k}$$
$$= y\hat{i} + x\hat{j} - 2z\hat{k}$$

At $(4, 1, 2)$, $\nabla\varphi = \hat{i} + 4\hat{j} - 4\hat{k}$

$$|\nabla\varphi| = \sqrt{1+16+16} = \sqrt{33}$$

$$n_1 = \frac{1}{\sqrt{33}}(\hat{i} + 4\hat{j} - 4\hat{k}) \qquad \text{—— } \underline{2\text{ mk}}$$

At $(3, 3, -3)$, $\nabla\varphi = 3\hat{i} + 3\hat{j} + 6\hat{k} = 3(\hat{i} + \hat{j} + 2\hat{k})$

$$|\nabla\varphi| = 3\sqrt{1+1+4} = 3\sqrt{6}. \qquad \text{—— } \underline{2\text{ mk}}$$

$$n_2 = \frac{1}{3\sqrt{6}} \cdot 3(\hat{i} + \hat{j} + 2\hat{k}) = \frac{1}{\sqrt{6}}(\hat{i} + \hat{j} + 2\hat{k})$$

$$\cos\theta = \hat{n}_1 \cdot \hat{n}_2 = \frac{1}{\sqrt{6}\sqrt{33}}(1+4-8) = \frac{-3}{\sqrt{6\times 33}} = \frac{-3}{3\sqrt{22}}$$

$$\theta = \cos^{-1}\left(\frac{1}{\sqrt{22}}\right) \quad \text{— 2 mk}$$

9. $\text{div}(\varphi \vec{F}) = \varphi \,\text{div}\, \vec{F} + \text{grad}\,\varphi \cdot \vec{F}$

Proof Let $\vec{F} = f_1\hat{i} + f_2\hat{j} + f_3\hat{k}$

$$\text{div}\,\varphi\vec{F} = \nabla\cdot(\varphi f_1\hat{i} + \varphi f_2\hat{j} + \varphi f_3\hat{k})$$

$$= \frac{\partial}{\partial x}(\varphi f_1) + \frac{\partial}{\partial y}(\varphi f_2) + \frac{\partial}{\partial z}(\varphi f_3)$$

$$= \varphi\frac{\partial f_1}{\partial x} + f_1\frac{\partial \varphi}{\partial x} + \varphi\frac{\partial f_2}{\partial y} + f_2\frac{\partial \varphi}{\partial y} + \varphi\frac{\partial f_3}{\partial z} + f_3\frac{\partial \varphi}{\partial z} \quad \text{— 2 m}$$

$$= \varphi\left(\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}\right) + \left(f_1\frac{\partial \varphi}{\partial x} + f_2\frac{\partial \varphi}{\partial y} + f_3\frac{\partial \varphi}{\partial z}\right)$$

$$= \varphi\left(\frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}\right) + (f_1\hat{i} + f_2\hat{j} + f_3\hat{k})\cdot\left(\frac{\partial \varphi}{\partial x}\hat{i} + \frac{\partial \varphi}{\partial y}\hat{j} + \frac{\partial \varphi}{\partial z}\hat{k}\right)$$

$$= \varphi(\nabla\cdot\vec{F}) + \vec{F}\cdot\nabla\varphi$$

$$= \varphi\,\text{div}\,\vec{F} + \nabla\varphi\cdot\vec{F} \quad \text{— 4 m}$$

$$= \varphi\,\text{div}\,\vec{F} + \text{grad}\,\varphi\cdot\vec{F}$$