CMRIT
CMR INSTITUTE OF TECHNOLOGY, BENGALURU.
ACCREDITED WITH A+ GRADE BY NAAC

USN ☐☐☐☐☐☐☐☐☐☐

Sub	ENGINEERING MATHEMATICS III			Code	15MAT31
Date 17 / 11 / 2017	Duration 90 mins	Max Marks 50	Sem III	Branch	EE A,B EC A,B,C, CS A,B

Question 1 is compulsory. Answer any SIX questions from the rest.

Improvement Test

		Marks	CO	RBT
1.	Verify Green's theorem in plane for $\vec{f} = (2x + y^2)\hat{i} + (3y - 4x)\hat{j}$ where C is taken around triangle ABC in xy plane with A(0,0), B(2,0),C(2,1).	08	C301.5	L3
2.	If the velocity V (km/hr) and resistance R (kg/tonne) are related by a relation of the form $R = a + bV^2$ find **a** and **b** by the method of least squares with the use of the following table.	07	C301.6	L3
3.	A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is a Catenary.	07	C301.3	L3

Table for question 2:

V	10	20	30	40	50
R	8	10	15	21	30

CMRIT
CMR INSTITUTE OF TECHNOLOGY, BENGALURU
ACCREDITED WITH A+ GRADE BY NAAC

USN ☐☐☐☐☐☐☐☐☐☐

Sub ENGINEERING MATHEMATICS III Code 15MAT31
Date 17 / 11 / 2017 Duration 90 mins Max Marks 50 Sem III Branch EE A,B EC A,B,C, CS A,B

Question 1 is compulsory. Answer any SIX questions from the rest. Marks OBE CO RBT

Improvement Test

1. Verify Green's theorem in plane for $\vec{f} = (2x + y^2)\hat{i} + (3y - 4x)\hat{j}$ where C is taken around triangle ABC in xy plane with A(0,0), B(2,0),C(2,1). 08 C301.5 L3

2. If the velocity V (km/hr) and resistance R (kg/tonne) are related by a relation of the form $R = a + bV^2$ find a and b by the method of least squares with the use of the following table. V 10 20 30 40 50; R 8 10 15 21 30. 07 C301.6 L3

3. A heavy cable hangs freely under gravity between two fixed points. Show that the shape of the cable is a Catenary. 07 C301.3 L3

4. Find the path in which a particle, in the absence of friction, will slide from one point to another in the shortest time under the action of gravity. **07** **C301.3** L3

5. If $\vec{f} = (x-y)\,\hat{i} + (x+y)\,\hat{j}$ evaluate $\oint_C \vec{f}\,.\,\vec{dr}$ around the curve C consisting of the parabolas $y^2 = x$ and $x^2 = y$. **07** **C301.5** L3

6. Verify Gauss Divergence theorem for $\vec{f} = 4xz\,\hat{i} - y^2\,\hat{j} + yz\,\hat{k}$ taken over the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0, z = 1. **07** **C301.5** L3

7. Test for an extremum of the functional **07** **C301.3** L3

$$I(y(x)) = \int_0^1 (xy + y^2 - 2y^2 y')\,dx,\; y(0) = 1,\; y(1) = 2.$$

8. Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n,\ given\ y_0 = y_1 = 0$ using Z transform . **07** **C301.2** L3

9. Fit a straight line **y = a+bx** to the following data **07** **C301.6** L3

Year	1975	1985	1995	2005	2015
Production	8	10	12	10	16

Find the expected production in the year 2020.

4. Find the path in which a particle, in the absence of friction, will slide from one point to another in the shortest time under the action of gravity. **07** **C301.3** L3

5. If $\vec{f} = (x-y)\,\hat{i} + (x+y)\,\hat{j}$ evaluate $\oint_C \vec{f}\,.\,\vec{dr}$ around the curve C consisting of the parabolas $y^2 = x$ and $x^2 = y$. **07** **C301.5** L3

6. Verify Gauss Divergence theorem for $\vec{f} = 4xz\,\hat{i} - y^2\,\hat{j} + yz\,\hat{k}$ taken over the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0, z = 1. **07** **C301.5** L3

7. Test for an extremum of the functional **07** **C301.3** L3

$$I(y(x)) = \int_0^1 (xy + y^2 - 2y^2 y')\,dx,\; y(0) = 1,\; y(1) = 2.$$

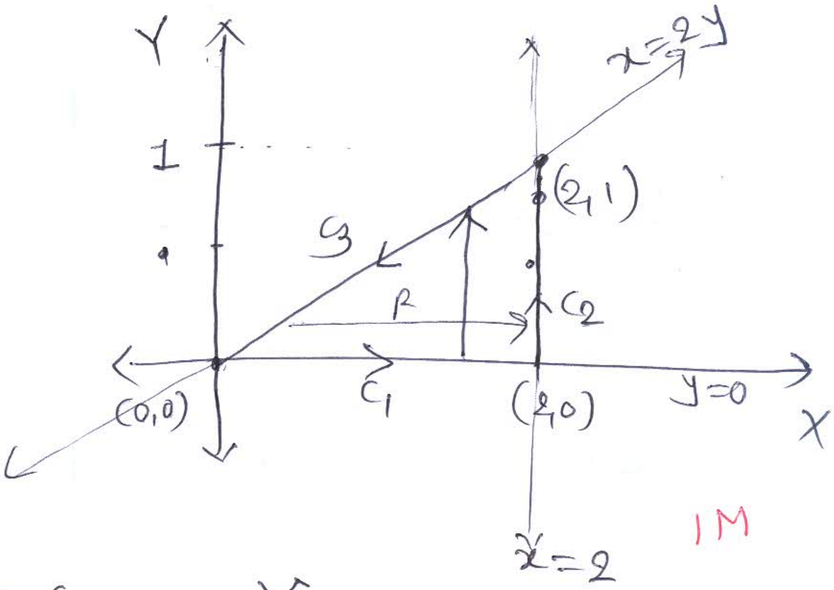
8. Solve the difference equation $y_{n+2} + 6y_{n+1} + 9y_n = 2^n,\ given\ y_0 = y_1 = 0$ using Z transform . **07** **C301.2** L3

9. Fit a straight line **y = a+bx** to the following data **07** **C301.6** L3

Year	1975	1985	1995	2005	2015
Production	8	10	12	10	16

Find the expected production in the year 2020.

Q.1 verify Green's Theorem in plane for $\vec{F} = (2x+y^2)\hat{i} + (3y-4x)\hat{j}$ where c is taken around triangle ABC in XY-plane with A(0,0), B(2,0), C(2,1).



1M

$\rightarrow \vec{F} = (2x+y^2)\hat{i} + (3y-4x)\hat{j}$

$\vec{r} = x\hat{i} + y\hat{j}$

$\therefore d\vec{r} = dx\,\hat{i} + dy\,\hat{j}$

$\vec{F} \cdot d\vec{r} = (2x+y^2)\,dx + (3y-4x)\,dy$

Line integral of vector function $\vec{F}$ along the curve c is

$$\int_C \vec{F} \cdot d\vec{r} = \int_C (2x+y^2)\,dx + (3y-4x)\,dy$$

by Green's Theorem

$$\int_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy \quad \text{└ ①.}$$

1M

$$\int_C (M\,dx + N\,dy) = \int_C (2x + y^2)\,dx + (3y - 4x)\,dy$$

line integral over the curve

→ C_1 : $y = 0 \Rightarrow dy = 0$

x varies from 0 to 2

$$\therefore \int_{C_1} (2x + y^2)\,dx + (3y - 4x)\,dy = \int_0^2 2x\,dx$$

$$= (x^2)_0^2 = \boxed{4}$$

→ C_2 : $x = 2 \Rightarrow dx = 0$

y varies from 0 to 1.

$$\therefore \int_{C_2} (2x + y^2)\,dx + (3y - 4x)\,dy = \int_0^1 (3y - 8)\,dy$$

$$= \left(\frac{3y^2}{2} - 8y\right)_0^1$$

$$= \frac{3}{2} - 8 = \boxed{\frac{-13}{2}}$$

→ C_3: $x = 2y \Rightarrow dx = 2dy$

y veries from 1 to 0.

$$\int_{C_3} (2x+y^2)\,dx + (3y-4x)\,dy = \int_1^0 \left[(4y+y^2)\,2dy + (3y-8y)\,dy\right]$$

$$= \int_1^0 (2y^2+3y)\,dy$$

$$= \left[\frac{2y^3}{3} + \frac{3y^2}{2}\right]_1^0$$

$$= \left[0 - \left(\frac{2}{3}+\frac{3}{2}\right)\right]$$

$$\boxed{= -\frac{13}{6}}$$

$$\therefore \int_C (M\,dx+N\,dy) = \int_{C_1} (M\,dx+N\,dy) + \int_{C_2} (M\,dx+N\,dy) + \int_{C_3} (M\,dx+N\,dy)$$

$$= 4 - \frac{13}{2} - \frac{13}{6}$$

$$= \frac{24-39-\overset{13}{\cancel{18}}}{6} = \frac{-14}{3} \quad \text{—} ②$$

3M

$N = 3y - 4x$ | $M = 2x + y^2$

$\frac{\partial N}{\partial x} = -4$ | $\frac{\partial M}{\partial y} = 2y$

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy = \iint_R (-4 - 2y)\,dx\,dy$$

y veries from 0 to $x/2$

x veries from 0 to 2

$$= \int_0^2 \int_0^{x/2} (-4 - 2y)\,dy\,dx$$

$$= \int_0^2 \left[-4y - \frac{\cancel{2}y^2}{\cancel{2}}\right]_0^{x/2} dx$$

$$= \int_0^2 \left[-\frac{\overset{2}{\cancel{4}}x}{\cancel{2}} - \frac{x^2}{4}\right] dx$$

$$= \left[-\cancel{2}\frac{x^2}{\cancel{2}} - \frac{x^3}{12}\right]_0^2$$

$$= -4 - \frac{\overset{2}{\cancel{8}}}{\underset{3}{\cancel{12}}}$$

$$= -4 - 2/3 = -\frac{14}{3} \quad ——③$$

3M

from eqⁿ ② and ③

Green's Theorem is varified.

Q.2 If the velocity V (km/hr) and resistance R (kg/tonne) are related by a relation of the form $R = a + bV^2$. find a and b by the method of least squares with the use of tabular values.

Soln.

$V = x$	$R = y$	x^2	$x^2 y$	x^4
10	8	100	800	10,000
20	10	400	4000	16,0000
30	15	900	13500	81,0000
40	21	1600	33600	2560000
50	30	2500	75000	6250000
150	84	5500	126900	9790000

4M

$$R = a + bV^2$$

Consider $R = y$, $V = x$

$$\therefore y = a + bx^2$$

Normal eqns : $\Sigma y = na + b\Sigma x^2$

$$\Sigma x^2 y = a\Sigma x^2 + b\Sigma x^4$$

$$\therefore 84 = 5a + (5500)b$$

$$126900 = (5500)a + (9790000)b$$

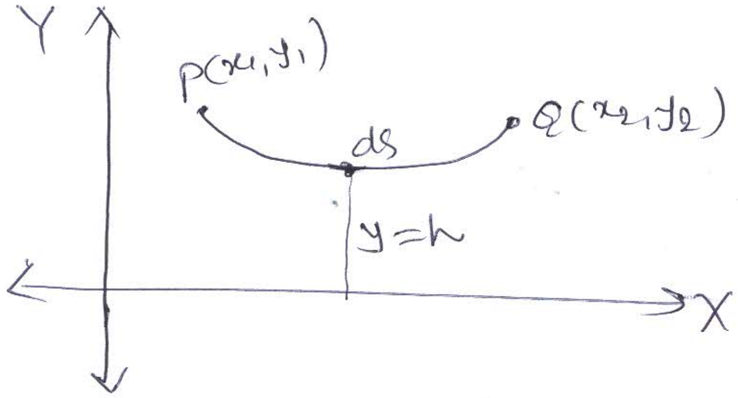
$a = 6.6525$ $b = 0.0092$ 2M

$$\therefore y = (6.6525) + (0.0092)x^2$$

$$\boxed{R = (6.6525) + (0.0092)V^2}$$ 1M

Q.3 A heavy cable hangs freely under gravity between two fixed points. show that the shape of the cable is a catenary.

Sol^n:-



(1M)

→ Let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be the two fixed points of the hanging cable.

→ consider an elementry arc length ds of the cable.

→ There are no external forces on the chain in the horizontal direction.

→ we have to minimise the potential Energy.

(1M)

$$P.E. = mgh \qquad (h = y, \quad m = \rho . ds)$$

$$= (\rho \, ds) \, gh$$

$$I = \int_{x_1}^{x_2} \rho \, ds \, gh = \int_{x_1}^{x_2} (\rho . g) y \sqrt{1+y'^2} \, dx$$

$$= (\rho g) \int_{x_1}^{x_2} y \sqrt{1+y'^2} \, dx$$

$\therefore$ $f(x, y, y') = y\sqrt{1+y'^2}$ independent of x

$$f - y'\left(\frac{\partial f}{\partial y'}\right) = K$$

$$y\sqrt{1+y'^2} - y'\left[\frac{yy'}{\sqrt{1+y'^2}}\right] = K$$

$$\frac{y(1+y'^2) - y'^2 y}{\sqrt{1+y'^2}} = K$$

$$\frac{y}{\sqrt{1+y'^2}} = K$$

$$\therefore \quad \frac{y^2}{(1+y'^2)} = K^2 \Rightarrow 1 + y'^2 = \frac{y^2}{K^2}$$

$$\Rightarrow y'^{\cancel{2}} = \frac{\sqrt{y^2 - K^2}}{K}$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{y^2-K^2}}{K}$$

$$\Rightarrow \int \frac{dy}{\sqrt{y^2-K^2}} = \int \frac{1}{K}\, dx$$

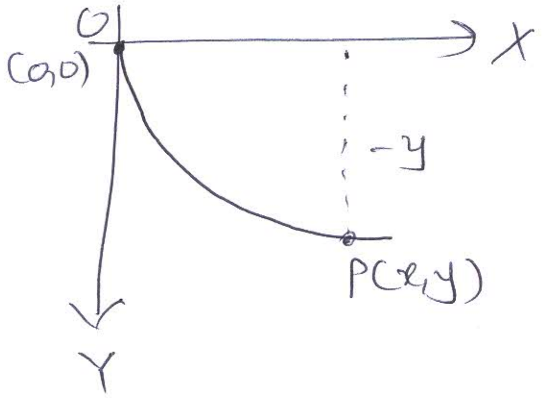
(4M)

$$\Rightarrow \cosh^{-1}(y/K) = \frac{1}{K}(x+c)$$

This eqn represents the shape catenary $\Rightarrow$ $\boxed{y = K\cosh\left(\frac{x+c}{K}\right)}$ (1M)

Q.4 Find the Path along which a particle, in the absence of friction will slide from one point to another in the shortest time under the action of gravity.

Soln:



(1M)

Let the particle start sliding from 'o' on the curve with velocity zero. At time t, the particle is at P(x,y).

→ ~~because~~ By Law of Conservation of energy,

$KE + P.E. = 0$ (∵ friction is zero)

$$K.E. \text{ at } P = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{ds}{dt}\right)^2$$

$$P.E. \text{ at } P = mgh = mg(-y) = -mgy$$

$$K.E. + P.E. = 0$$

$$\frac{1}{2}m\left(\frac{ds}{dt}\right)^2 + (-mgy) = 0$$

$$\frac{1}{2}m\left(\frac{ds}{dt}\right)^2 = mgy$$

$$\left(\frac{ds}{dt}\right)^2 = 2gy \Rightarrow \boxed{dt = \frac{ds}{\sqrt{2gy}}}$$

To find the path when T is minimum,

$$I = \int_0^T dt = \int_0^x \frac{ds}{\sqrt{2gy}}$$

$$I = \frac{1}{\sqrt{2g}} \int_0^x \frac{\sqrt{1+y'^2}}{\sqrt{y}}\, dx$$

∴ $f(x, y, y') = \frac{\sqrt{1+y'^2}}{\sqrt{y}}$ is independent of x

Euler's eqn $f - y'\left(\frac{\partial f}{\partial y'}\right) = K$

$$\frac{\sqrt{1+y'^2}}{\sqrt{y}} - \frac{y'}{\sqrt{y}} \frac{\cancel{2}\, y'}{\cancel{2}\sqrt{1+y'^2}} = K$$

$$\frac{1+y'^2 - y'^2}{\sqrt{y(1+y'^2)}} = K$$

$$\therefore \sqrt{y(1+y'^2)} = \frac{1}{K}$$

$\Rightarrow \boxed{y(1+y'^2) = a}$ (take $\frac{1}{K^2} = a$)

$1+y'^2 = \frac{a}{y} \Rightarrow y'^2 = \frac{a}{y} - 1$

$$\Rightarrow y' = \sqrt{\frac{a-y}{y}}$$

$$\frac{dy}{dx} = \sqrt{\frac{a-y}{y}}$$

$$\int \frac{\sqrt{y}}{\sqrt{a-y}}\, dy = \int dx$$

Put $y = a\sin^2(\theta/2)$

$$dy = \cancel{2}\, a \sin(\theta/2) \,.\, \cos(\theta/2)\ \frac{1}{\cancel{2}}\, d\theta$$

$$\therefore\ x = \int \sqrt{\frac{\cancel{a}\sin^2(\theta/2)}{\cancel{a}(1-\sin^2\theta/2)}}\ a \sin(\theta/2)\cos(\theta/2)\, d\theta$$

$$= a\int \frac{\sin(\theta/2)}{\cancel{\cos(\theta/2)}}\ \sin(\theta/2)\ \cancel{\cos(\theta/2)}\ d\theta$$

$$= a\int \sin^2(\theta/2)\, d\theta$$

$$= a\int \frac{1-\cos(\theta)}{2}\, d\theta$$

$$\boxed{x = \frac{a}{2}(\theta - \sin\theta) + b} \quad \text{—— ①}$$

5M

When $y=0 \Rightarrow \theta = 0$

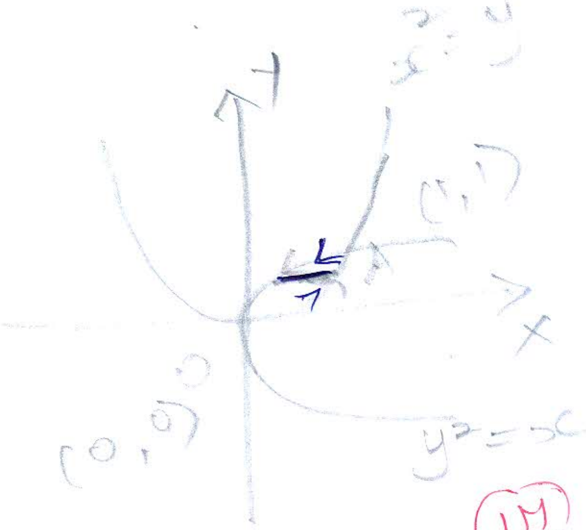
$\therefore$ by eqⁿ ① $x=0$, $b=0$

$$\therefore \quad \boxed{x = \frac{a}{2}(\theta - \sin\theta)}$$

$$\boxed{y = \frac{a}{2}(1 - \cos\theta)}$$

1M

5.



(1M)

$$\oint_C \vec{f}\cdot d\vec{r} = \iint_R (N_x - M_y)\,dx\,dy$$ (1M)

$\vec{r} = x\hat{i} + y\hat{j}$

$\vec{A} = (x-y)\hat{i} + (x+y)\hat{j}$ $\quad d\vec{r} = \hat{i}\,dx + \hat{j}\,dy$

$\vec{A}\cdot d\vec{r} = (x-y)\,dx + (x+y)\,dy$

Along OA $\quad y = x^2$

$dy = 2x\,dx$

$x = 0$ to $x = 1$

$$\int_{x=0}^{1} \left\{(x - x^2)\,dx + (x+x^2)\,2x\,dx\right\}$$

$$= \int_{x=0}^{1} (x - x^2 + 2x^2 + 2x^3)\,dx$$

$$= \int_{x=0}^{1} (x + x^2 + 2x^3)\,dx$$

$$= \left(\frac{x^2}{2} + \frac{x^3}{3} + \frac{2x^4}{4}\right)_0^1$$

$$= \frac{4}{3}$$

Along AO $x = y^2$

$dx = 2y\,dy$

$y = 1$ to $y = 0$

$$\oint_C (x-y)\,dx + (x+y)\,dy$$

$$= \int_{y=1}^{0} (y^2 - y)\,2y\,dy + (y^2+y)\,dy$$

$$\int_{y=1}^{0} (2y^3 - 2y^2 + y^2 + y)\,dy$$

$$\int_{y=1}^{0} (2y^3 - y^2 + y)\,dy$$

$$= \left(\frac{2y^4}{4} - \frac{y^3}{3} + \frac{y^2}{2}\right)_1^0$$

$$= -\left(\frac{1}{2} - \frac{1}{3} + \frac{1}{2}\right) = -\frac{2}{3}$$

$$\oint_C \vec{f} \cdot d\vec{r} = \frac{4}{3} - \frac{2}{3} = \frac{2}{3}$$

$M = x - y$ $\qquad N = x + y$

$M_y = -1$ $\qquad N_x = 1$

$$RHS = \iint_R \left(N_x - M_y\right) dx\,dy$$

$$= \int_{y=0}^{1}\int_{x=y^2}^{\sqrt{y}} \left(1-(-1)\right) dx\,dy$$

$$= \int_{y=0}^{1}\int_{x=y^2}^{\sqrt{y}} 2\, dx\,dy$$

$$2\int_{y=0}^{1} \left(x\right)_{y^2}^{\sqrt{y}} dy$$

$$= 2\int_{y=0}^{1} \left(\sqrt{y} - y^2\right) dy$$

$$= 2\left\{\frac{y^{3/2}}{3/2} - y^3/3\right\}_0^1 = 2\left(\frac{2}{3} - \frac{1}{3}\right) = \frac{2}{3}$$

(5M)

Green's Theorem is verified.

6. $\vec{f} = 4xz\,\hat{i} - y^2\hat{j} + yz\,\hat{k}$

$$\int_S \vec{f}\cdot\hat{n}\,ds = \int_V \operatorname{div}\vec{f}\,dV$$

RHS $\int_{x=0}^{1}\int_{y=0}^{1}\int_{z=0}^{1} \left(4z - 2y + y\right) dz\,dy\,dx$

$$\nabla\cdot\vec{f} = \nabla\cdot\left(4xz\vec{i} - y^2\vec{j} + yz\vec{k}\right)$$

$$4z - 2y + y = 4z - y$$

$$\int_V \text{div}\,\vec{f}\,dv = \int_{x=0}^{1}\int_{y=0}^{1}\int_{z=0}^{1}(4z-y)\,dz\,dy\,dx$$

$$\int_{x=0}^{1}\int_{y=0}^{1}\left\{4\left(\frac{z^2}{2}\right)_0^1 - y(z)_0^1\right\}dy\,dx$$

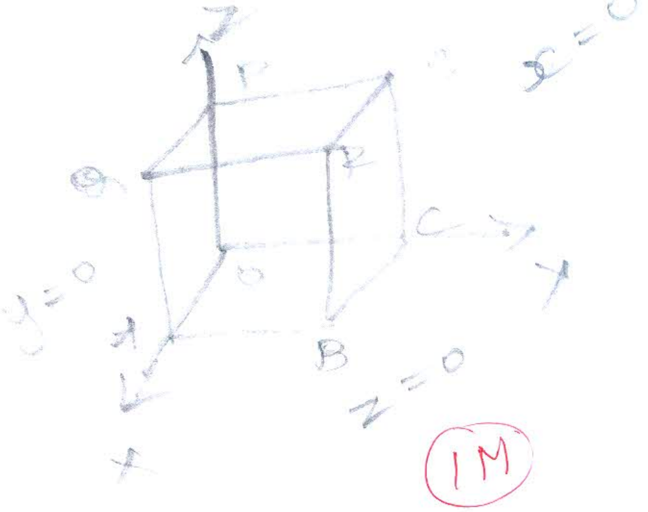
$$= \int_{x=0}^{1}\int_{y=0}^{1}(2-y)\,dy\,dx$$

$$= \int_{x=0}^{1}\left(2y - \frac{y^2}{2}\right)_0^1 dx$$

$$= \frac{3}{2}\int_{x=0}^{1}dx = \frac{3}{2} \text{ — ①}$$

3M

$$\int_S \vec{f}\cdot\hat{n}\,ds$$



1M

S_1 OABC $\hat{n} = -\hat{k}$ $z=0$

$$\int_{OABC} \vec{f}\cdot(-\hat{k})\,ds = \int_{z=0} yz\hat{k}\cdot -\hat{k}\,ds$$

$$\int_{OABC} \vec{f}\cdot\hat{n}\,ds = 0$$

S_2 OPQA $\hat{n} = -\hat{j}$

$$\vec{f}\cdot\hat{n} = \left(4xz\hat{i} - y^2\hat{j} + yz\hat{k}\right)\cdot -\hat{j} = y^2$$

$y=0$

$$\int_{OPQA} \vec{f}\cdot\hat{n}\,ds = 0$$

S_3 OCSP $\hat{n} = -\hat{i}$ $x=0$

$$\vec{f}\cdot\hat{n} = 4xz = 0$$

$$\int_{OCSP} \vec{f}\cdot\hat{n}\,ds = 0$$

S_4 PQRS $\hat{n} = \hat{k}$ $z=1$ $dz=0$

$$\vec{f}\cdot\hat{n} = yz = y$$

$$\int_{x=0}^{1}\int_{y=0}^{1} y\,dy\,dx = \left(\frac{y^2}{2}\right)_0^1 (x)_0^1 = 1/2$$

S_5 CSRB $\hat{n} = \hat{j}$ $y=1$

$$\vec{f}\cdot\hat{n} = -y^2\hat{j}\cdot\hat{j} = -y^2 = -1$$

$$\int_{z=0}^{1}\int_{x=0}^{1} -1\,dx\,dz = -1$$

S_6 ABRQ $\hat{n} = \hat{i}$ $x = 1$

$\vec{f}\cdot\hat{n} = 4xz = 4z$

$$\iint \vec{f}\cdot\hat{n}\, ds = \int_{z=0}^{1}\int_{y=0}^{1} 4z\, dy\, dz$$

$$= 4\left(\frac{z^2}{2}\right)_0^1 \left(y/2\right)_0^1 = 2$$

$$\int_S \vec{F}\cdot\hat{n}\, dS = 0+0+0+\frac{1}{2}-1+2 = 3/2$$ (3M)

Gauss Divergence theorem is verified

7. $$I(y(x)) = \int_0^1 (xy + y^2 - 2y^2y')\, dx$$

$y(0) = 1$ $y(1) = 2$

$f(x, y, y') = xy + y^2 - 2y^2y'$

Euler's eqn $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0$

$$(-4yy' + (x+2y)) - \frac{d}{dx}(-2y^2) = 0$$

~~$x + 2y +$~~

$$(x + 2y - 4yy') + 2\frac{d}{dx}(y^2) = 0$$

$$x + 2y \cancel{+} 4yy' + 2(2yy') = 0$$

$$x + 2y = 0 \Rightarrow \boxed{y = -\frac{x}{2}}$$ (5M)

does n't satisfy the end conditions (1M)

$y(0) = 1$ and $y(1) = 2$

∴ Extremum cannot be achieved on the class of continuous functions. (1M)

8. $y_{n+2} + 6y_{n+1} + 9y_n = 2^n$, $y_0 = y_1 = 0$

Taking Z transform

$$Z(y_{n+2} + 6y_{n+1} + 9y_n) = Z(2^n)$$

$$Z(y_{n+2}) + 6Z(y_{n+1}) + 9Z(y_n) = \frac{z}{z-2}$$

$$z^2\left\{\bar{y}(z) - y_0 - \frac{y_1}{z}\right\} + 6z\left\{\bar{y}(z) - y_0\right\} + 9\bar{y}(z) = \frac{z}{z-2}$$

$$(z^2 + 6z + 9)\,\bar{y}(z) = \frac{z}{z-2}$$ (3M)

$$\frac{\bar{y}(z)}{z} = \frac{1}{(z^2+6z+9)(z-2)}$$

$$= \frac{1}{(z+3)^2(z-2)}$$

$$= \frac{A}{z-2} + \frac{B}{z+3} + \frac{C}{(z+3)^2}$$

$$\frac{\bar{Y}(z)}{z} = \frac{A}{z-2} + \frac{B}{z+3} + \frac{C}{(z+3)^2}$$

$$\frac{1 \cancel{z}}{(z-2)(z+3)^2} = \frac{A}{z-2} + \frac{B}{z+3} + \frac{C}{(z+3)^2}$$

$$1 = A(z+3)^2 + B(z-2)(z+3) + C(z-2)$$

$z = -3$ $\quad 1 = -5C \Rightarrow \boxed{C = \frac{-1}{5}}$

$z = 2$ $\quad 1 = 25A \Rightarrow \boxed{A = \frac{1}{25}}$

$z = 0$ $\quad 1 = 9A - 6B - 2C$

$$1 = \frac{9}{25} - 6B - 2\left(\frac{-1}{5}\right)$$

$$1 = \left(\frac{9}{25} + \frac{2}{5}\right) - 6B$$

$$1 = \frac{19}{25} - 6B \Rightarrow 1 - \frac{19}{25} = -6B$$

$$\frac{6}{25} = -6B \Rightarrow \boxed{B = \frac{-1}{25}}$$

3M

$$\frac{\bar{Y}(z)}{z} = \frac{1}{25}\frac{1}{z-2} - \frac{1}{25(z+3)} - \frac{1}{5(z+3)^2}$$

$$\bar{y}(z) = \frac{1}{25}\left\{\frac{z}{z-2} - \frac{z}{z+3} - \frac{5z}{(z+3)^2}\right\}$$ (#9)

$$\bar{y}(z) = \frac{1}{25}\frac{z}{z-2} - \frac{1}{25}\frac{z}{z+3} - \frac{1}{25}\frac{5z}{(z+3)^2}$$

Taking the inverse Z transform

$$y_n = \frac{1}{25}\left\{2^n - (-3)^n - 5n(-3)^{n-1}\right\}$$ (1M)

This is the solution of the given equation under the given conditions.

9. $X = x - 1974$

$X = x - 1974$	y	X^2	Xy
1	8	1	8
11	10	121	110
21	12	441	252
31	10	961	310
41	16	1681	656
$\boxed{105}$	$\boxed{56}$	$\boxed{3205}$	$\boxed{1336}$

(4M)

Normal eqns $3205a + 105b = 1336$

$105a + 5b = ~~1336~~\ 56$

$a = 0.16 \quad b = 7.84$

$y = aX + b$

$= 0.16\,X +$ ~~0.784~~ 7.84

$= 0.16(x - 1974) + 7.84$ (1M)

$$\boxed{y = 0.16x - 308}$$ (1M)

For $x = 2020$

$y = 15.2$ thousand units. (1M)

Estimated production is 15.2 thousand units for the year 2020.