CMR INSTITUTE OF TECHNOLOGY

USN

Internal Assessment Test III

Sub: **ENGINEERING MATHEMATICS I** Code: 17MAT11

Date: 20/ 11 / 2017 Duration : 90 mins Max Marks: 50 Sem: I Sec: **B, D, F**

Q1 is compulsory. Answer any 6 questions from the rest.

	Marks	OBE CO	OBE RBT
1. Obtain the reduction formula for $\int cos^n x\, dx$ and hence evaluate $\int_0^{\pi/2} cos^n x\, dx$, where n is a positive integer.	[08]	CO7	L3
2. Find the rank of the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$.	[07]	CO7	L3
3. Solve the following system of equations by Gauss-Jordan method: $x + 2y + z = 3,\ 2x + 3y + 2z = 5,\ 3x - 5y + 5z = 2.$	[07]	CO3	L3
4. Show that the transformation $y_1 = 2x_1 + x_2 + x_3,\ y_2 = x_1 + x_2 + 2x_3,$ $y_3 = x_1 - 2x_3$ is regular and find the inverse transformation.	[07]	CO3	L3

CMR INSTITUTE OF TECHNOLOGY

USN

Internal Assessment Test III

Sub: **ENGINEERING MATHEMATICS I** Code: 17MAT11

Date: 20/ 11 / 2017 Duration : 90 mins Max Marks: 50 Sem: I Sec: **B, D, F**

Q1 is compulsory. Answer any 6 questions from the rest.

	Marks	OBE CO	OBE RBT
1. Obtain the reduction formula for $\int cos^n x\, dx$ and hence evaluate $\int_0^{\pi/2} cos^n x\, dx$, where n is a positive integer.	[08]	CO6	L3
2. Find the rank of the matrix $\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$.	[07]	CO7	L3
3. Solve the following system of equations by Gauss-Jordan method: $x + 2y + z = 3,\ 2x + 3y + 2z = 5,\ 3x - 5y + 5z = 2.$	[07]	CO7	L3
4. Show that the transformation $y_1 = 2x_1 + x_2 + x_3,\ y_2 = x_1 + x_2 + 2x_3,$ $y_3 = x_1 - 2x_3$ is regular and find the inverse transformation.	[07]	CO7	L3

5. Solve the system of equations $83x + 11y - 4z = 95$; $7x + 52y + 13z = 104$; $3x + 8y + 29z = 71$ using Gauss-Seidel method.	[07]	CO7	L3
6. Using Rayleigh's power method, find the largest eigenvalue and corresponding eigenvector of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by taking $X^{(0)} = [1,1,1]^T$ as initial eigenvector. Perform 5 iterations.	[07]	CO7	L3
7. Solve: $x\frac{dy}{dx} + y = x^3y^6$.	[07]	CO5	L3
8. Solve: $(x^2 + y^2 + x)dx + xydy = 0$.	[07]	CO5	L3
9. Find the orthogonal trajectory of the family of curves: $r^n \cos n\theta = a^n$.	[07]	CO5	L3
10. If the temperature of air is $30^o C$ and a substance cools from $100^o C$ to $70^o C$ in 15 minutes, find when the temperature reaches $40^o C$.	[07]	CO5	L3

5. Solve the system of equations $83x + 11y - 4z = 95$; $7x + 52y + 13z = 104$; $3x + 8y + 29z = 71$ using Gauss-Seidel method.	[07]	CO7	L3
6. Using Rayleigh's power method, find the largest eigenvalue and corresponding eigenvector of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ by taking $X^{(0)} = [1,1,1]^T$ as initial eigenvector. Perform 5 iterations.	[07]	CO7	L3
7. Solve: $x\frac{dy}{dx} + y = x^3y^6$.	[07]	CO5	L3
8. Solve: $(x^2 + y^2 + x)dx + xydy = 0$.	[07]	CO5	L3
9. Find the orthogonal trajectory of the family of curves: $r^n \cos n\theta = a^n$.	[07]	CO5	L3
10. If the temperature of air is $30^o C$ and a substance cools from $100^o C$ to $70^o C$ in 15 minutes, find when the temperature reaches $40^o C$.	[07]	CO5	L3

Solution

1. Let $I_n = \int \cos^n x\,dx = \int \cos^{n-1}x \cos x\,dx$

$$= \cos^{n-1}x \cdot \sin x - \int (\sin x)(n-1)\cos^{n-2}x(-\sin x)dx$$

$$= \cos^{n-1}x \sin x + (n-1)\int \cos^{n-2}x \sin^2 x\,dx$$

$$= \cos^{n-1}x \sin x + (n-1)\int \cos^{n-2}x\,(1-\cos^2 x)dx$$

$$= \cos^{n-1}x \sin x + (n-1)\int \cos^{n-2}x\,dx - (n-1)\int \cos^n x\,dx$$

$$= \cos^{n-1}x \sin x + (n-1)I_{n-2} - (n-1)I_n$$

$$(\cancel{1}+n-\cancel{1})\,I_n = \cos^{n-1}x \sin x + (n-1)I_{n-2}$$

$$I_n = \frac{\cos^{n-1}x \sin x}{n} + \frac{n-1}{n}I_{n-2} \qquad \text{— 4 mk}$$

Next, let $I_n = \int_0^{\pi/2} \cos^n x\,dx$

$$= \cos^{n-1}x \sin x\Big)_0^{\pi/2} + \frac{n-1}{n}I_{n-2}$$

$$= 0 + \frac{n-1}{n}I_{n-2} \quad \left(\because \cos\tfrac{\pi}{2} = 0 = \sin 0\right)$$

$$I_n = \frac{n-1}{n}\cdot\frac{n-3}{n-2}I_{n-4} \quad \left(\because I_{n-2} = \frac{n-3}{n-2}I_{n-4}\right)$$

$$= \frac{n-1}{n}\,\frac{n-3}{n-2}\cdot\frac{n-5}{n-4}I_{n-6}$$

$$\cdots\cdots$$

$$= \frac{n-1}{n}\,\frac{n-3}{n-2}\cdots\frac{2}{3}I_1 \text{ if } n \text{ is odd}$$

$$\text{and } = \frac{n-1}{n}\,\frac{n-3}{n-2}\cdots\frac{1}{2}I_0 \text{ if } n \text{ is even}$$

Now $I_1 = \int_0^{\pi/2} \cos x\,dx = +\sin x\Big)_0^{\pi/2} = (1-0) = 1$

and $I_0 = \int_0^{\pi/2} \cos^0 x\,dx = \int_0^{\pi/2} 1\,dx = \pi/2$

$$\therefore \int_0^{\pi/2} \cos^n x\,dx = \frac{n-1}{n}\cdot\frac{n-3}{n-2}\cdots\frac{2}{3} \text{ if } n \text{ is odd}$$

$$= \frac{n-1}{n}\,\frac{n-3}{n-2}\cdots\frac{1}{2}\,\frac{\pi}{2} \text{ if } n \text{ is even.} \qquad \text{— 4 mk}$$

2. $A = \begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ $(R_1 \leftrightarrow R_2)$

$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$ $R_2 \to R_2 + {-2}R_1$, $R_3 \to R_3 + {-3}R_1$, $R_4 \to R_4 + {-6}R_1$ —— 2 mk

$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$ $R_2 \to R_2 - R_3$

$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 66 & 44 \end{bmatrix}$ $R_3 \to R_3 + {-4}R_2$, $R_4 \to R_4 + {-9}R_2$ —— 2 mk

$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 33 & 22 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_4 \to R_4 - 2R_3$

$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & -6 & -3 \\ 0 & 0 & 1 & 2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_3 \to \dfrac{R_3}{3}$ —— 2 mk

$\rho(A) = 3$ —— 1 mk

OR $A \sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & 3/5 & 7/5 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$ $(R_2 \to R_2/5)$ —— 2 mk

$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & 3/5 & 7/5 \\ 0 & 0 & 33/5 & 22/5 \\ 0 & 0 & 33/5 & 22/5 \end{bmatrix}$ $R_3 \to R_3 + {-4}R_2$, $R_4 \to R_4 + {-9}R_2$ —— 2 mk

$\sim \begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 1 & 3/5 & 7/5 \\ 0 & 0 & 1 & 2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $R_4 \to R_4 - R_3$, $R_3 \to R_3 \times 5/33$ —— 2 mk

Rank = 3 —— 1 mk

3. $x + 2y + z = 3$

$2x + 3y + 2z = 5$

$3x - 5y + 5z = 2$

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 3 & 2 & 5 \\ 3 & -5 & 5 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 0 & -1 \\ 0 & -11 & 2 & -7 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{array}$$

— 2 mk

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & -11 & 2 & -7 \end{array}\right] \quad R_2 \to -R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 2 & 4 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 - 2R_2 \\ R_3 \to R_3 + R_2 \end{array}$$

— 2 mk

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad R_3 \to R_3/2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad R_1 \to R_1 - R_2$$

— 2 mk

$x = -1, \quad \underline{y = 1}, \quad z = 2$ — 1 mk

4. $$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & -2 \end{bmatrix}$$

$|A| = -4 - 1(-2-2) + 1(0-1) = -4 + 4 - 1 = -1 \neq 0$

$\therefore$ The transformation is regular — 2 mk

$$A^{-1} = \frac{\text{adj } A}{|A|} = -\begin{bmatrix} \begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix} & -\begin{vmatrix} 1 & 2 \\ 1 & -2 \end{vmatrix} & \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} \\ -\begin{vmatrix} 1 & 1 \\ 0 & -2 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix} \\ \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} & -\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} & \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} \end{bmatrix}^T$$

$$= -\begin{bmatrix} -2 & 4 & -1 \\ 2 & -5 & 1 \\ 1 & -3 & 1 \end{bmatrix}^T = \begin{bmatrix} 2 & -2 & -1 \\ -4 & 5 & 3 \\ 1 & -1 & -1 \end{bmatrix}$$ — 3 mk.

The inverse transformation is $X = A^{-1}Y$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -1 \\ -4 & 5 & 3 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

ie $x_1 = 2y_1 - 2y_2 - y_3$, $\underline{x_2} = -4y_1 + 5y_2 + 3y_3$, $x_3 = y_1 - y_2 - y_3$

— 2 mk.

5. $83x + 11y - 4z = 95$

$7x + 52y + 13z = 104$

$3x + 8y + 29z = 71$

$$x = \frac{1}{83}[95 - 11y + 4z]$$

$$y = \frac{1}{52}[104 - 7x - 13z]$$ — 1 mk

$$z = \frac{1}{29}[71 - 3x - 8y]$$

Let $x_0 = y_0 = z_0 = 0$

1st Iteration

$x^{(1)} = 1.1446$

$y^{(1)} = \frac{1}{52}[104 - 7 \times 1.1446 - 13 \times 0] = 1.8459$

$z^{(1)} = \frac{1}{29}[71 - 3 \times 1.1446 - 8 \times 1.8459] = 1.8207$

— 2 mk

2nd Iteration

$$x^{(2)} = \frac{1}{83}\left[95 - 11\times 1.8459 + 4\times 1.8207\right] = 0.9877$$

$$y^{(2)} = \frac{1}{52}\left[104 - 7\times 0.9877 - 13\times 1.8207\right] = 1.4119$$

$$z^{(2)} = \frac{1}{29}\left[71 - 3\times 0.9877 - 8\times 1.4119\right] = 1.9566$$

— 2mk

3rd Iteration

$$x^{(3)} = \frac{1}{83}\left[95 - 11\times 1.4119 + 4\times 1.9566\right] = 1.0518$$

$$y^{(3)} = \frac{1}{52}\left[104 - 7\times 1.0518 - 13\times 1.9566\right] = 1.3692$$

$$z^{(3)} = \frac{1}{29}\left[71 - 3\times 1.0518 - 8\times 1.3692\right] = 1.9618$$

— 2mk

$x = 1.0518$, $y = 1.3692$, $z = 1.9618$

6. $$AX^{(0)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ +2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 4 \end{bmatrix} = 6\begin{bmatrix} 1 \\ 0 \\ 0.67 \end{bmatrix} = \lambda^{(1)} X^{(1)}$$

$$AX^{(1)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0.67 \end{bmatrix} = \begin{bmatrix} 7.34 \\ -2.67 \\ 4.01 \end{bmatrix} = 7.34\begin{bmatrix} 1 \\ -0.36 \\ 0.55 \end{bmatrix} = \lambda^{(2)} X^{(2)}$$

$$AX^{(2)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.36 \\ 0.55 \end{bmatrix} = \begin{bmatrix} 7.82 \\ -3.63 \\ 4.01 \end{bmatrix} = 7.82\begin{bmatrix} 1 \\ -0.46 \\ 0.51 \end{bmatrix} = \lambda^{(3)} X^{(3)}$$

$$AX^{(3)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.46 \\ 1.51 \end{bmatrix} = \begin{bmatrix} 7.94 \\ -3.89 \\ 3.99 \end{bmatrix} = 7.94\begin{bmatrix} 1 \\ -0.49 \\ 0.5 \end{bmatrix} = \lambda^{(4)} X^{(4)}$$

$$AX^{(4)} = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -0.49 \\ 0.5 \end{bmatrix} = 7.98\begin{bmatrix} 1 \\ -0.5 \\ 0.5 \end{bmatrix}$$

— 5 mk

— 2 mk

Largest eigenvalue $= 7.98$, eigenvector $= (1, -0.5, 0.5)^T$

7. $x\frac{dy}{dx}+y=x^3y^6$ — ①

Divide by xy^6

$\frac{1}{y^6}\frac{dy}{dx}+\frac{1}{x}\cdot\frac{1}{y^5}=x^2$ — ② —— 1 mk

Put $\frac{1}{y^5}=t$ $\quad \frac{-5}{y^6}\frac{dy}{dx}=\frac{dt}{dx}$

$\frac{1}{y^6}\frac{dy}{dx}=\frac{-1}{5}\frac{dt}{dx}$ —— 2 mk

Sub in ②, $-\frac{1}{5}\frac{dt}{dx}+\frac{1}{x}t=x^2$

$\frac{dt}{dx}-\frac{5}{x}t=-5x^2$ — ③ —— 1 mk

ie, $\frac{dt}{dx}+Pt=Q$

$P=\frac{-5}{x}$, $Q=-5x^2$

IF: $e^{\int P\,dx}=e^{-5\int\frac{1}{x}dx}=e^{-5\log x}=\frac{1}{x^5}$ —— 1 mk

Soln of ③ is $\frac{t}{x^5}=\int -5x^2\cdot\frac{1}{x^5}dx+c=-5\int\frac{1}{x^3}dx+c$

$\frac{t}{x^5}=-5\frac{x^{-2}}{-2}+c$

$=\frac{5}{2x^2}+c$ —— 1 mk

Sub $t=\frac{1}{y^5}$. Soln of ① is

$\frac{1}{x^5y^5}=\frac{5}{2x^2}+c$ —— 1 mk

8. $(x^2+y^2+x)dx+xy\,dy=0$ — ①

$M=x^2+y^2+x$, $\quad N=xy$

$\frac{\partial M}{\partial y}=2y \quad \frac{\partial N}{\partial x}=y$. —— 1 mk

$\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}=y$

$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{y}{xy} = \frac{1}{x} = f(x)$ —— 2mk

IF: $e^{\int f(x)dx} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$ —— 1mk

Multiply ① throughout with x.

$x(x^2+y^2+x)\,dx + x^2y\,dy = 0$

New $M = x(x^2+y^2+x)$ $N = x^2y$.

$\frac{\partial M}{\partial y} = 2xy$ $\frac{\partial N}{\partial x} = 2xy$. $\Rightarrow$ exact

Soln is $\int M\,dx + \int N(y)\,dy = c$ —— 1mk

$\int (x^3 + xy^2 + x^2)\,dx + \int 0\,dy = c$

$\frac{x^4}{4} + \frac{x^2y^2}{2} + \frac{x^3}{3} = c$ —— 2mk

9. $r^n \cos n\theta = a^n$

Taking log, $n\log r + \log\cos n\theta = n\log a$.

Diff w.r.t θ, $\frac{n}{r}\frac{dr}{d\theta} - \frac{n\sin n\theta}{\cos n\theta} = 0$ —— 2mk

$\frac{1}{r}\frac{dr}{d\theta} = \tan n\theta$.

This is the d-e of the given fly. The d-e of the orthogonal trajectory is got by replacing $\frac{1}{r}\frac{dr}{d\theta}$ with $-r\frac{d\theta}{dr}$ —— 2mk

$-r\frac{d\theta}{dr} = \tan n\theta$

$\int \frac{d\theta}{\tan n\theta} = -\int \frac{dr}{r}$

$\int \cot n\theta\,d\theta + \int \frac{1}{r}\,dr = 0$

$\log\frac{\sin n\theta}{n} + \log r = \log c \Rightarrow \log\sin n\theta + n\log r = n\log c$

$r^n \sin n\theta = c^n$. $r^n\sin n\theta = k$. —— 3mk

or

10. $t_1 = 100°C$, $t_2 = 30°C$

$T = t_2 + (t_1 - t_2)e^{-kt}$

$= 30 + 70e^{-kt}$ —— ① . —— 2 mk

When $t = 15$, $T = 70$.

$70 = 30 + 70e^{-15k}$

$e^{-15k} = \frac{4}{7} \Rightarrow e^{15k} = \frac{7}{4}$

$k = \frac{1}{15}\ln\frac{7}{4} = 0.0373$ —— 2 mk

Sub in ①, $T = 30 + 70e^{-0.0373t}$ —— 1 mk

When $T = 40$, $40 = 30 + 70e^{-0.0373t}$

$e^{-0.0373t} = \frac{1}{7}$

$0.0373t = \ln 7$

$t = \frac{\ln 7}{0.0373} = 52.17$ min —— 2 mk

~~It will~~ the temp. reaches 40°C in 52.2 min