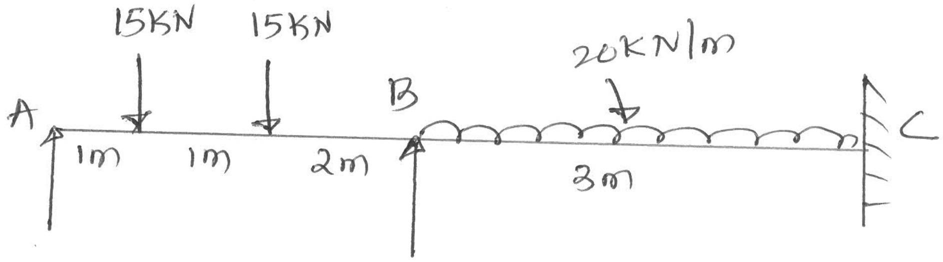
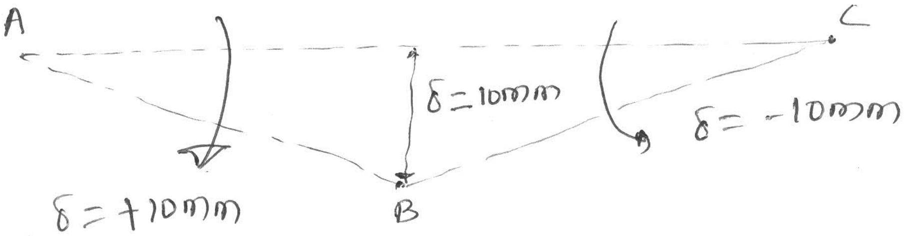
Analysis of indeterminate structures : 15CV52

Solution and scheme of valuation: IAT-I

Q 1] Analyze the continuous beam shown in fig by MD method. Draw SFD & BMD. Take EI = 4000 kN-m² support 'B' settles by 10mm.





Sol^n: I] Fixed end moments:-

$$M_{FAB} = -\left[\frac{wab^2}{l^2} + \frac{wab^2}{l^2}\right] - \frac{6EI\delta}{l^2}$$

$$= -\left[\frac{15\times1\times3^2}{4^2} + \frac{15\times2\times2^2}{4^2}\right] - \frac{6\times4000}{4^2}\left(\frac{10}{1000}\right)$$

$$= -30.9375\,kNm$$

$$M_{FBA} = +\left[\frac{wa^2b}{l^2} + \frac{wa^2b}{l^2}\right] - \frac{6EI\delta}{l^2}$$

$$= +\left[\frac{15\times1^2\times3}{4^2} + \frac{15\times2^2\times2}{4^2}\right] - \frac{6\times4000}{4^2}\left(\frac{10}{1000}\right)$$

$$= -4.6875\,kNm$$

$$M_{FBC} = -\frac{wl^2}{12} - \frac{6EI\delta}{l^2}$$

$$= -\frac{20\times3^2}{12} - \frac{6\times4000}{3^2}\left(\frac{-10}{1000}\right)$$

$$= 11.666\,kNm$$

$$M_{FCB} = +\frac{wl^2}{12} - \frac{6EI\delta}{l^2}$$

$$= \frac{20\times3^2}{12} - \frac{6\times4000}{3^2}\left(-\frac{10}{1000}\right)$$

$$= 41.666\ KNm$$

II] Distribution factors :-

Joint	member	stiffness k	ΣK	D.F
B	BA	$\frac{3}{4}\frac{I}{4} = 0.1875I$	0.521I	0.36
	BC	$I/3 = 0.333I$		0.64

III] M.D table

Joint	A	B		C
member	AB	BA	BC	CB
D.F	-	0.36	0.64	-
FEM's	-30.937	-4.687	11.666	41.666
Release A	+30.937			
C.O		15.468		
Initial val	0	10.7815	11.666	41.666
Bal		-8.0811	-14.3664	
C.O	0	-	-	-7.1832

IV] End moments

$$M_{AB} = 0$$

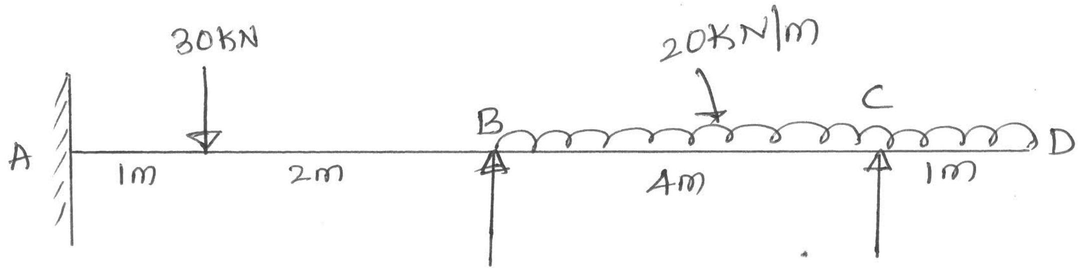
$$M_{BA} = 2.7004\ KNm$$

$$M_{BC} = -2.7004\ KNm$$

$$M_{CB} = 34.4828\ KNm.$$

2] Analyze the continuous beam shown in fig by M.D method.

Draw SFD & BMD



Soln: I] FEM's

$M_{FAB} = -\frac{wab^2}{l^2} = -\frac{30\times1\times2^2}{3^2}$ $M_{FBA} = +\frac{wa^2b}{l^2} = \frac{30\times1^2\times2}{3^2}$

$M_{FAB} = -13.333 KNm$ $M_{FBA} = 6.666 KNm.$

$M_{FBC} = -\frac{wl^2}{12}$ $M_{FCB} = +\frac{wl^2}{12}$ $M_{FCD} = 20\times1\times\frac{1}{2}$

$= -\frac{20\times4^2}{12}$ $= \frac{20\times4^2}{12}$ $= -10KNm$

$= -26.666 KNm$ $= 26.666 KNm$

II] D.F

Joint	member	stiffness K	ΣK	$D.F = \frac{K}{\Sigma K}$
B	BA	$I/3 = 0.333I$	0.5205I	0.64
	BC	$\frac{3}{4}\frac{I}{4} = 0.1875I$		0.36
C	CB	$I/4 = 0.25I$	0.25I	1
	CD	0		

III] M.D. table.

Joint	A	B		C	
member	AB	BA	BC	CB	CD
D.F	-	0.64	0.36	1	0
FEM's	-13.333	6.666	-26.666	26.666	-10
Bal		+12.8	7.2	-16.666	
C.O	6.4		-8.333		
Bal		+5.333	+3		
C.O	+2.665	-	-	+1.5	

IV] End moments

~~$M_{AB} = -9.598$ KNm~~

~~$M_{BA} = 14.133$ KNm~~

~~$M_{BC} = -14.133$ KNm~~

~~$M_{CB} =$~~

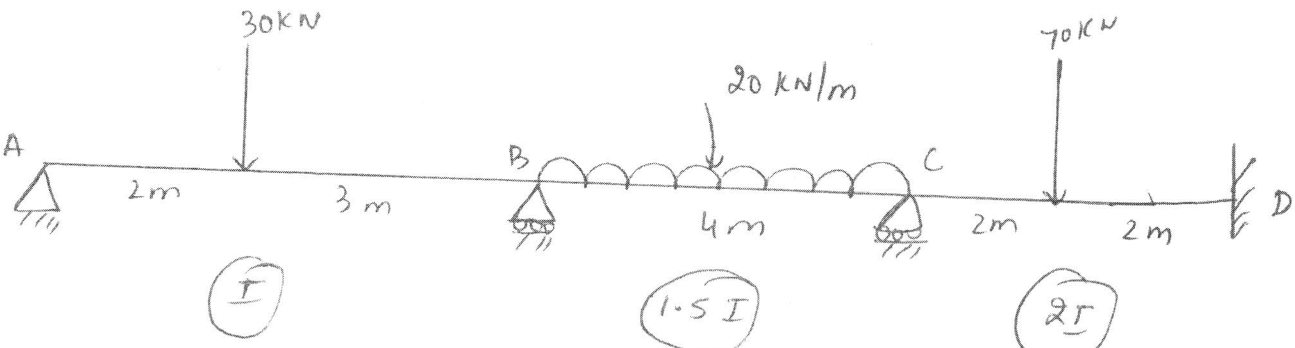
$M_{AB} = -4.268$ KNm

$M_{BA} = 24.799$ KNm

$M_{BC} = -24.799$ KNm

$M_{CB} = 10$ KNm

$M_{CD} = -10$ KNm



(i) Fixed end moment

$$MF_{AB} = -\frac{wab^2}{l^2} = -\frac{30 \times 2 \times 3^2}{5^2} = -21.6 \text{ KNm}$$

$$MF_{BA} = \frac{wa^2b}{l^2} = \frac{30 \times 2^2 \times 3}{5^2} = 14.4 \text{ KNm}$$

$$MF_{BC} = -\frac{wl^2}{12} = -\frac{20 \times 4^2}{12} = -26.66 \text{ KNm}$$

$$MF_{CB} = \frac{wl^2}{12} = \frac{20 \times 4^2}{12} = 26.66 \text{ KNm}$$

$$MF_{CD} = -\frac{wl}{8} = -\frac{70 \times 4}{8} = -35 \text{ KNm}$$

$$MF_{DC} = \frac{wl}{8} = \frac{70 \times 4}{8} = 35 \text{ KNm}$$

(ii) Slope-deflection equation

- Span AB

$$M_{AB} = MF_{AB} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\delta}{l}\right]$$

$\delta = 0, \; I = 1$

$$= -21.6 + \frac{2EI}{5}\left[2\theta_A + \theta_B\right]$$

$$= -21.6 + 0.8\,EI\theta_A + 0.4\,EI\theta_B \quad \text{——— (1)}$$

$$M_{BA} = MF_{BA} + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\delta}{l}\right]$$

$$\delta = 0, \quad I = 1$$

$$= 14.4 + \frac{2EI}{5}\left[2\theta_B + \theta_A\right]$$

$$= 14.4 + 0.8\,EI\theta_B + 0.4\,EI\theta_A \quad \text{——— (2)}$$

$$M_{BC} = MF_{BC} + \frac{2EI}{l}\left[2\theta_B + \theta_C - \frac{3\delta}{l}\right]$$

$$\delta = 0, \quad I = 1.5$$

$$= -26.66 + \frac{2 \times 1.5 \times EI}{4}\left[2\theta_B + \theta_C\right]$$

$$= -26.66 + 1.5\,EI\theta_B + 0.75\,EI\theta_C \quad \text{——— (3)}$$

$$M_{CB} = MF_{CB} + \frac{2EI}{l}\left[2\theta_C + \theta_B - \frac{3\delta}{l}\right]$$

$$\delta = 0, \quad I = 1.5$$

$$= 26.66 + \frac{2 \times 1.5 \times EI}{4}\left[2\theta_C + \theta_B\right]$$

$$= 26.66 + 1.5\,EI\theta_C + 0.75\,EI\theta_B \quad \text{——— (4)}$$

$$M_{CD} = MF_{CD} + \frac{2EI}{l}\left[2\theta_c + \theta_D - \frac{3\delta}{l}\right]$$

$\delta = 0, \; I = 2, \; \theta_D = 0$

$$= -35 + \frac{2 \times 2 \times EI}{4}\left[2\theta_c\right]$$

$$= -35 + 2EI\theta_c \quad \text{—— (5)}$$

$$M_{DC} = MF_{DC} + \frac{2EI}{l}\left[2\theta_D + \theta_c - \frac{3\delta}{l}\right]$$

$\delta = 0, \; \theta_D = 0, \; I = 2$

$$= 35 + \frac{2 \times 2 \times EI}{4}\left[\theta_c\right]$$

$$= 35 + EI\theta_c \quad \text{—— (6)}$$

(iii) equlibrium Condition

At Support 'B'

$$M_{BA} + M_{BC} = 0$$

$$14.4 + 0.8EI\theta_B + 0.4EI\theta_A + 1.5EI\theta_B + 0.75EI\theta_c - 26.66 = 0$$

$$0.4EI\theta_A + 2.3EI\theta_B + 0.75EI\theta_c = 12.26 \quad \text{—— (7)}$$

At Support 'A'

$$M_{AB} = 0$$

$$-21.6 + 0.8EI\theta_A + 0.4EI\theta_B = 0 \quad \text{—— (8)}$$

At Support 'C'

$$M_{CB} + M_{CD} = 0$$

$$26.66 + 1.5EI\theta_C + 0.75EI\theta_B - 35 + 2EI\theta_C = 0$$

$$0.75EI\theta_B + 3.5EI\theta_C = 8.34 \quad \text{— (9)}$$

$$\theta_A = \frac{27.08}{EI}, \quad \theta_B = \frac{-0.168}{EI}, \quad \theta_C = \frac{2.419}{EI}$$

(iv) End moments

Sub θ_A θ_B θ_C in (1) (2) (3) (4) (5) (6)

$$M_{AB} = -21.6 + 0.8EI\theta_A + 0.4EI\theta_B = 0$$

$$M_{BA} = 14.4 + 0.8EI\theta_B + 0.4EI\theta_A = 25.09\,KNm$$

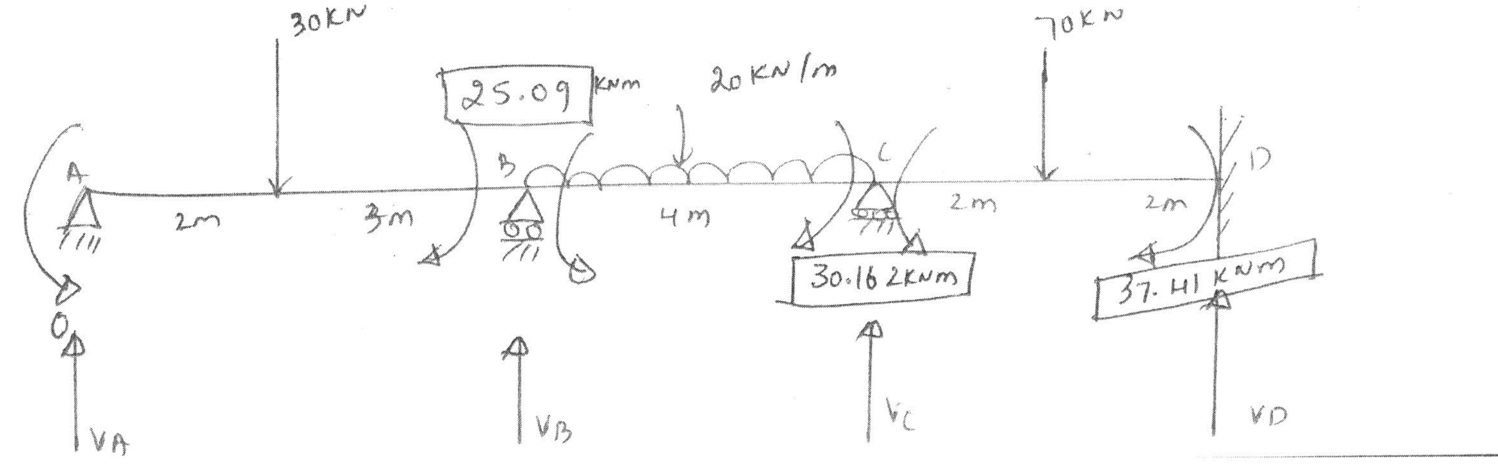
$$M_{BC} = -26.66 + 1.5EI\theta_B + 0.75EI\theta_C = -25.09\,KNm$$

$$M_{CB} = 26.66 + 1.5EI\theta_C + 0.75EI\theta_B = 30.162\,KNm$$

$$M_{CD} = -35 + 2EI\theta_C = -30.162\,KNm$$

$$M_{DC} = 35 + EI\theta_C = 37.419\,KNm$$

(v) SFD & BMD



$\Sigma M_B = 0$ [LHS]

$V_A \times 5 - 30 \times 3 + 25.09 = 0$

$V_A = 12.98\ kN$

$\Sigma M_C = 0$ (RHS)

$-V_D \times 4 + 37.41 + 70 \times 2 - 30.162 = 0$

$V_D = 36.81\ kN$

$\Sigma M_B = 0$ [RHS]

$30.162 - 30.162 + 37.41 - 25.09 + 20 \times 4 \times \frac{4}{2} - V_C \times 4 - V_D \times 8 + 70 \times 6 = 0$

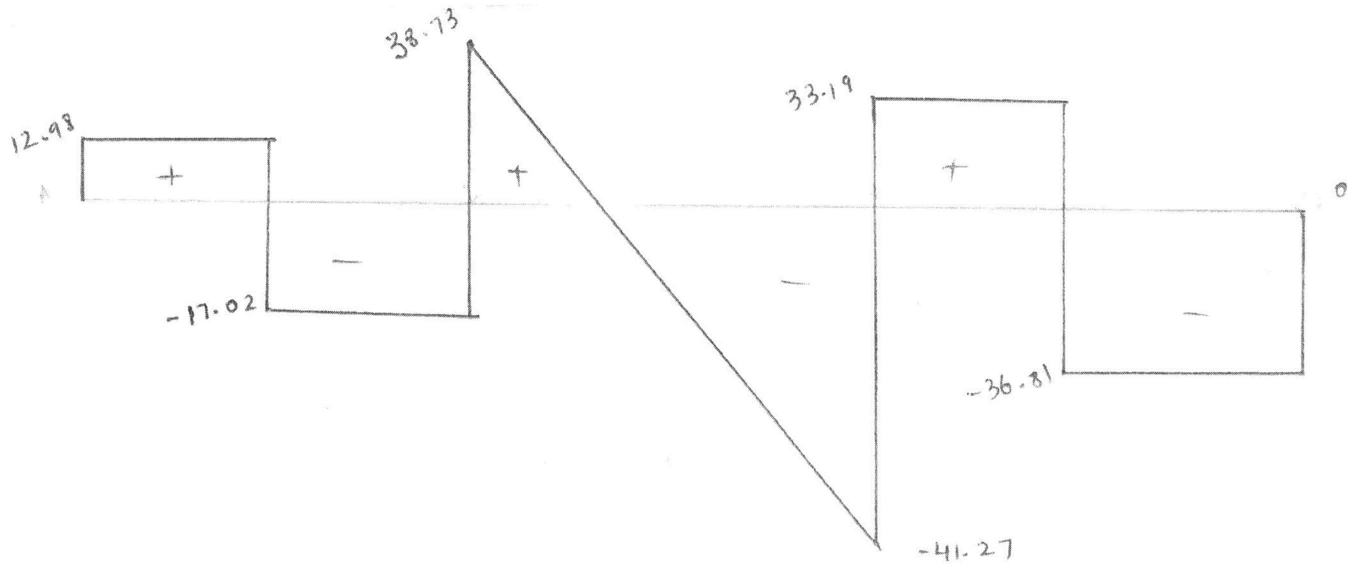
$V_C = 74.46\ kN$

$\Sigma V = 0$

$V_A + V_B + V_C + V_D = 70 + 30 + 20 \times 4$

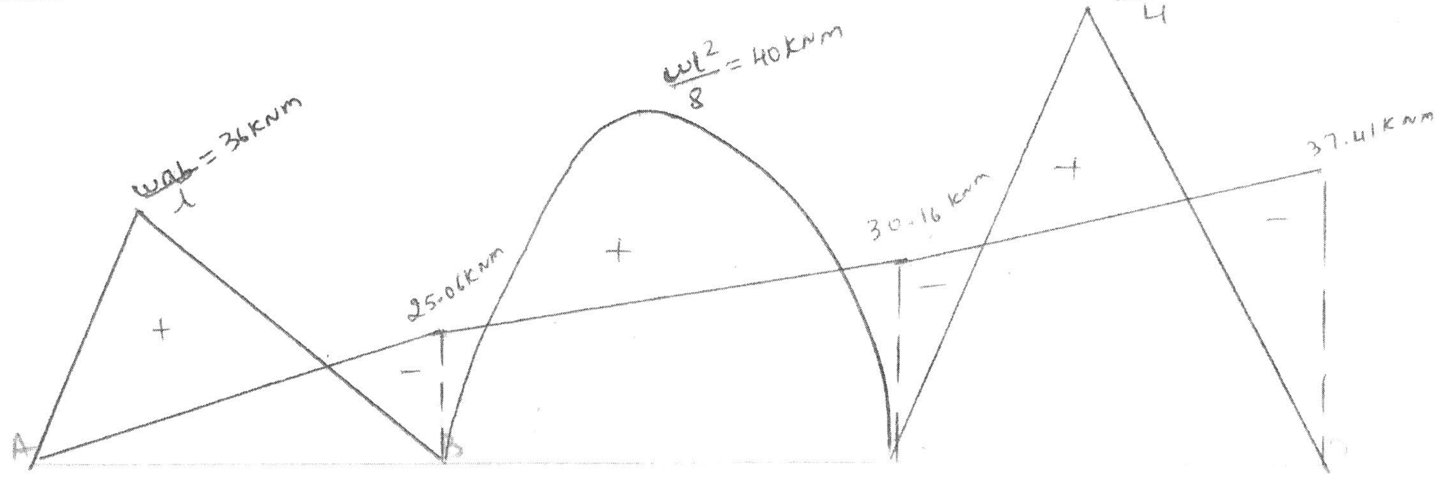
$V_B = 55.75\ kN$

SFD



12.98
A
+
-17.02
−
38.73
+
−
33.19
+
-41.27
0
−
-36.81

BMD



wab/l = 36KNm
+
A
25.06KNm
−
B
wl^2/8 = 40KNm
+
30.16 KNm
−
wl/4 = 70KNm
+
37.41KNm
−