USN

CMRIT

Internal Assessment Test 1 – Sept. 2017

Sub:	Fluid Mechanics					Sub Code:	15CV33	Branch:	Civil
Date:	20/09/2017	Duration:	90 min	Max Marks:	50	Sem / Sec:	III – A & B		

Answer all questions

		MARKS	OBE CO	OBE RBT
1 (a)	Define the terms – a) Surface tension b) Specific volume.	[04]	CO1	L1
(b)	A cube of 0.3m sides and weight 30 N slides down an inclined plane sloped at 30° to the horizontal. The plane is covered by an oil of viscosity 2.3×10^{-3} Pas with 0.03mm thickness. Determine the velocity with which the cube slides.	[06]	CO1	L2
2 (a)	Derive an expression for capillary rise between two parallel plates.	[04]	CO1	L2
(b)	State and prove Pascal's Law.	[06]	CO2	L2
3 (a)	A U-tube differential manometer is attached to 2 points A and B in a horizontal pipeline carrying water, 5m apart. The pressure at A is $0.07 N/mm^2$ and pressure head is 150mm of Hg. Find the Hg level difference in the manometer.	[05]	CO2	L3
(b)	The water is flowing at the rate of 60lps through a tapering pipe of length 500 mm, having diameter 400mm at the upper end and 200mm at the lower end. The pipe has a slope of 1 in 40. Find the pressure at the lower end, if the pressure at the upper end is $0.24 N/mm^2$.	[05]	CO4	L3

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4 (a)	From Euler's equation, derive the Bernoulli's equation of motion along a stream tube. List the assumptions for Bernoulli's equation	[10]	CO4	L2
5 (a)	What is a notch? Derive an expression for discharge through a V – Notch.	[06]	CO5	L2
(b)	Write short notes on a) End contractions, b) Cipolletti Notch	[04]	CO5	L1

All the Best

Sign of CI **Sign of CCI** **Sign of HOD**

Surface tension :-

It is define as tensile force acting on the surface of a liquid in contact with a gas or on the surface b/w 2 immiscible liquid such that the contact surface behave like a membrane under tension.

The magnitude of this is

$$\frac{\text{force}}{\text{unit length of the free surface}} = \frac{\text{surface Energy}}{\text{unit area}}$$

Symbol – σ

MKS – kgf/m

SI – N/m

1 N = 9.81 kgf

4) **Specific Volume :-**

It is defined as the volume of a fluid occupied by a unit mass or volume per unit mass of a fluid is called Specific volume..

It is denoted by

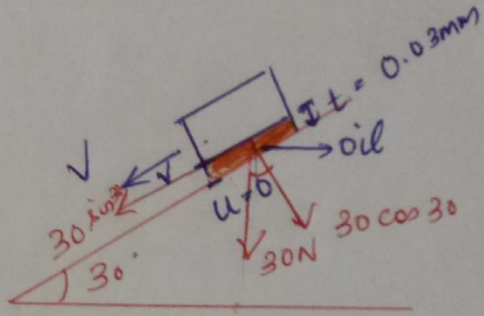
$$\text{Specific volume} = \frac{\text{Volume of the fluid}}{\text{Mass of the fluid}}$$

It's unit is m^3/kg $\left((\text{meter})^3/\text{Kilogram}\right)$

Example :- Specific Volume of water $= \frac{1}{\rho}$

$= \frac{1}{1000} = 0.001\ m^3/kg$

A cube 0.3 m sides and weight 30 N slides down an inclined plane sloped at 30° to the horizontal. The plane is covered by an oil of viscosity 2.3×10^{-3} Pa s. with 0.03 mm thickness. Determine the velocity with which the cube slides?



Given

side, $a = 0.3\,m$

Wt, $W = 30\,N$ $\qquad \mu = 2.3 \times 10^{-3}\ Ns/m^2$

$\theta = 30°$ $\qquad t = 0.03\,mm = 0.03 \times 10^{-3}\,m$.

By Newton's Law of Viscosity

$$\tau = \mu \frac{du}{dy}$$

$$\tau = \frac{F}{A} = \frac{30 \sin 30}{0.3 \times 0.3}$$ [Weight along the inclined plane] [Contact surface area.

$$= 166.67\ N/m^2$$

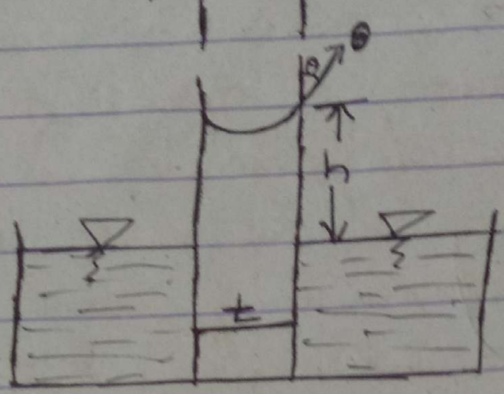
$$\mu \frac{du}{dy} = \frac{\mu(V-u)}{t} = \frac{2.3 \times 10^{-3}(V-0)}{0.03 \times 10^{-3}}$$

$u = 0$ [stationary inclined plane]

$$\therefore 166.67 = \frac{2.3 \cancel{\times 10^{-3}}\, V}{0.03 \cancel{\times 10^{-3}}}$$

$$\boxed{V = 2.17\ m/s.}$$

Capillary rise b/w two parallel vertical plates :-



let t be the distance b/w the two parallel plate, let L be the length of the plate in perpendicular direction.

F_1 = Force due to surface tension

$$F_1 = \sigma \cos\theta \times \text{circumference}$$
$$F_1 = \sigma \cos\theta \times 2l$$

F_2 = Force due to the liquid column of height h'

$$F_2 = mg$$
$$= \rho \times \text{volume} \times g$$
$$= \rho \times l t h \times g$$

$$\sigma \cos\theta \times 2\cancel{l} = \rho \cancel{l} t h g$$

$$h = \frac{2\sigma\cos\theta}{\rho g t}$$

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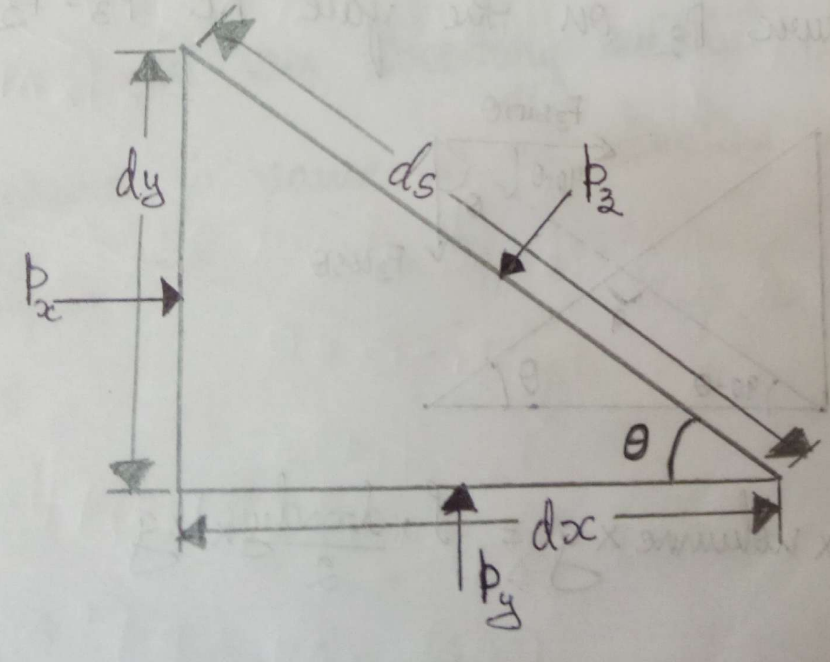
Pascal's law :- It states that the pressure or intensity of pressure in a static fluid is Equal in all direction is called Pascals law.



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Consider an arbitary fluid Element of wedge shape of fluid mass at rest. The fluid Element of very small dimension dx, dy, dz and the width of the Element $\perp^r$ to plane of the paper be unity. let P_x, P_y and P_z be the intensity of pressure acting on the faces AB, BC, CA respectively. let angle be θ at C.

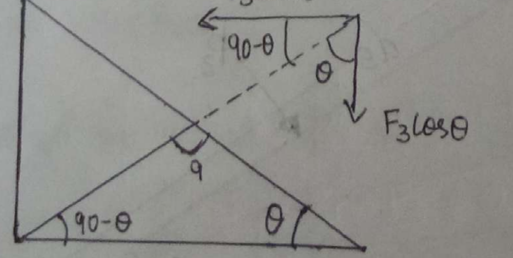
The force acting on the wedge are forces due to pressure P_x, P_y, P_y acting normal to the surface. The second force is weight of the fluid Element.

The forces are calculated below :-

force due to pressure P_x on the face AB $F_1 = P_x dy \times 1$

force due to pressure P_y on the face BC $F_2 = P_y dy \times 1$

force due to pressure P_z on the face AC $F_3 = P_z ds \times 1$



$$F_4 = mg = \rho \times \text{volume} \times g = \rho \times \frac{dx \times dy}{2} \times 1 \times g$$

for fluid at rest $\Sigma F_x = 0$

$$F_1 - F_3 \sin\theta = 0$$

$$P_x dy - P_3 ds \sin\theta = 0$$

Substitute $dy = ds \sin\theta$

$$P_x ds \sin\theta - P_3 ds \sin\theta = 0$$

$$\boxed{P_x = P_3} \quad (1)$$

|||ly resolving forces along Y $\Sigma F_y = 0$

$$F_2 - F_4 - F_3 \cos\theta = 0$$

$$P_y dy - \frac{\rho dx dy g}{2} - P_3 ds \cos\theta = 0$$

$$\boxed{P_y = P_3} \quad (2)$$

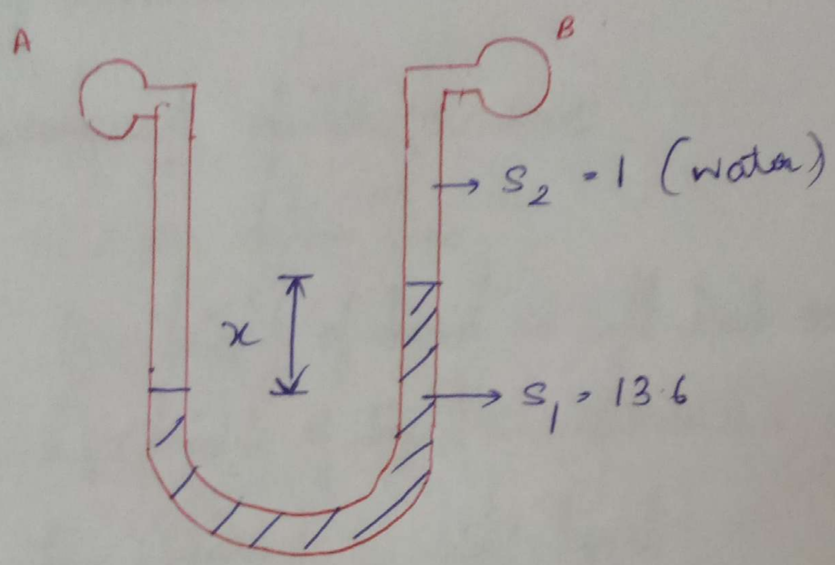
[$F_4 = 0$, since dx & dy are very small and their product can be neglected, $dy = ds \cos\theta$]

Equating (1) and (2)

$$P_3 = P_y = P_x$$

Since θ is an arbitary constant, the Equation proves that pressure is same in all direction at a point in a static liquid.

A U-tube differential manometer is attached to 2 points A and B in a horizontal pipeline carrying water, 5 m apart. The pressure at A is $0.07\,N/mm^2$ and pressure head is 150 mm of Hg. Find the Hg level difference in the manometer.



Given

$$p_A = 0.07\,N/mm^2 = 0.07 \times 10^6\,N/m^2$$

$$\frac{p_B}{\rho_{Hg}\, g} = 150\,mm = 0.15\,m$$

$$p_B = 0.15 \times 13.6 \times 1000 \times 9.81 = 20012.4\,N/m^2$$

$$= 0.02 \times 10^6\,N/m^2$$

Since $p_A > p_B$, the level of Hg connected to column A will be lower than B.

$$\frac{p_A}{\rho g} - \frac{p_B}{\rho g} = h = x\left(\frac{S_1}{S_2} - 1\right)$$

$$\frac{(0.07 - 0.02) \times 10^6}{1000 \times 9.81} = x\left(\frac{13.6}{1} - 1\right)$$

$$\boxed{x = 0.404\,m}$$

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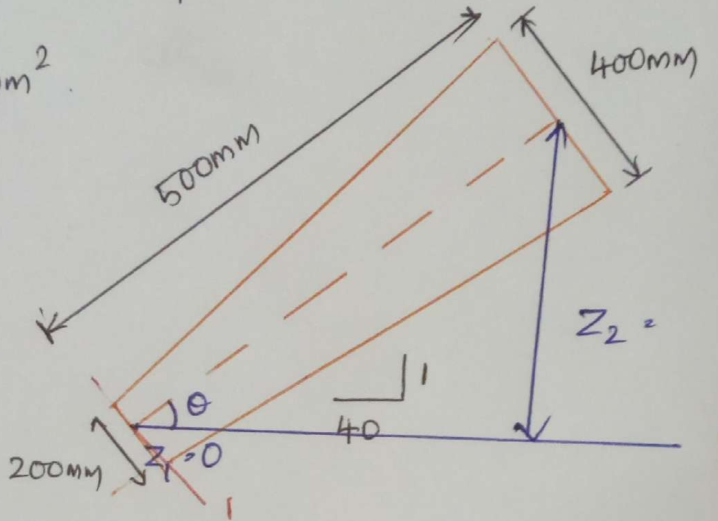
The water is flowing at the rate of 60 lps through a tapering pipe of length 500mm, having diameter 400mm at the upper end and 200mm at the lower end. The pipe has a slope of 1 in 40. Find the pressure at the lower end, if the pressure at the upper end is $0.24\ N/mm^2$



$p_2 = 0.24\ N/mm^2$

when θ is very small

$\sin\theta = \tan\theta$

$Z_2 = \frac{1}{40} \times 0.5$

Datum

$= 0.0125\ m$

Given

Discharge, $Q = 60\ l/s = 0.06\ m^3/s$

$L = 500mm = 0.5m$

$d_1 = 200mm = 0.2m$

$d_2 = 400m = 0.4m$

$A_1 = \frac{\pi}{4} \times 0.2^2 = 0.0314\ m^2$

$A_2 = \frac{\pi}{4} \times 0.4^2 = 0.1256\ m^2$

$V_1 = \frac{Q}{A_1} = 1.91\ m/s$

$V_2 = 0.477\ m/s$

$p_2 = 0.24\ N/mm^2$
$= 0.24 \times 10^6\ N/m^2$

Applying Bernoulli's Equation at (1) & (2)

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{p_1}{1000 \times 9.81} + \frac{1.91^2}{2 \times 9.81} + 0 = \frac{0.24 \times 10^6}{1000 \times 9.81} + \frac{0.477^2}{2 \times 9.81} + 0.0125$$

$p_1 = 238412.3395\ N/m^2$

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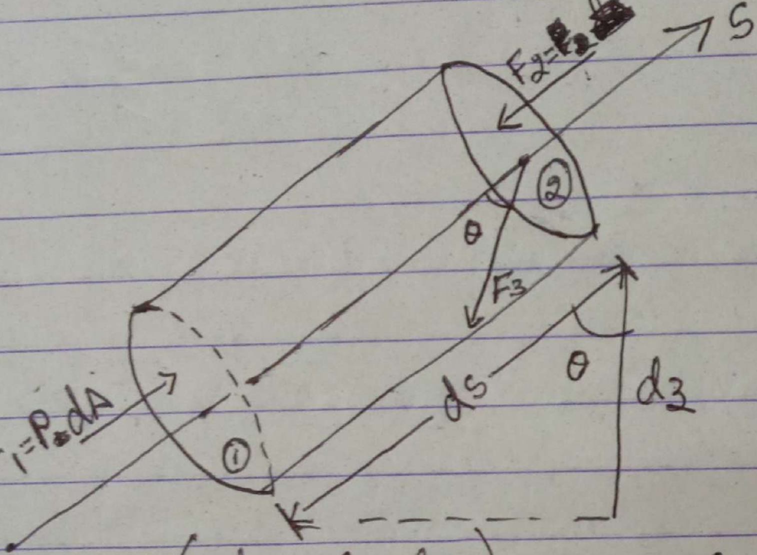
Module - 3
Fluid Dynamics

Eulers Equation along a stream line turbine



$\Rightarrow \cos\theta = \frac{dz}{ds}$ — (7)

Eulers Equation (oileri Equation) is an Equation of motion in which forces due to gravity and pressure are taken into consideration. This is derive by considering the motion of the fluid Element along a stream line.

Consider a stream line / Stream turbine in which flow is taking place in "S" direction. Consider a cylindrical Element of crossection dA and length ds. The force acting on a fluid Element are

F_1 be the force due to pressure at section ① $= P\,dA$ — ①

F_2 be the force due to pressure at section ② $= \left(P + \left(\frac{\partial P}{\partial s}\right) \times ds\right) dA$ — ②

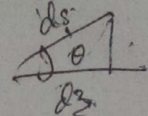
F_3 is the weight of the fluid Element $= Mg$

$= \rho \times \text{volume} \times g$

let ρ be the density of fluid

$F_3 = \rho \times (dA \times ds) \times g$ — ③

$$\frac{dy}{dx} = \frac{\partial y}{\partial z} \times \frac{\partial z}{\partial x} + \frac{\partial y}{\partial x}$$



Applying Equilibrium Equation along x-axis $\Sigma F_s = Ma_s$

$$F_1 - F_2 - F_3 \cos\theta = Ma_s \quad \text{—(4)}$$

where a_s is the acceleration along x-axis $= \frac{dv}{dt}$, where V is the function of s and t. $a_s = \frac{dv}{dt}$, V is function of s and t.

$$\frac{dv}{dt} = \frac{\partial v}{ds} \times \frac{ds}{\partial t} + \frac{\partial v}{\partial t}$$

for a steady flow, $\frac{\partial v}{\partial t} = 0$

$\frac{\partial s}{dt} = \frac{ds}{dt} = v$ ($\because$ s is only depended on t).

$$a_s = \frac{\partial v}{ds} \times v + 0 \quad \text{—(5)}$$

Substitute (1), (2), (3), (5) in (4)

$$PdA - \left(P + \frac{\partial P}{\partial s} ds\right) dA - \rho (dA \times ds) g \cos\theta = \rho (dA \times ds) \times \frac{\partial v}{\partial s} \times v$$

$$\cancel{P} - \cancel{P} - \frac{\partial P}{\partial s} ds - \rho ds g \cos\theta = \rho ds \frac{\partial v}{\partial s} v$$

Dividing the Equation by $\rho ds g$

$$-\frac{\partial P}{\partial s} \times \frac{1}{\rho g} - \cos\theta = \frac{\partial v}{\partial s} \frac{v}{g} \quad \text{—(6)}$$

replace $\frac{\partial P}{\partial s} = \frac{dP}{dP}$ $\quad \frac{\partial v}{\partial s} = \frac{dv}{ds}$ —(8)

substitute (8) & (7) in (6)

$$-\frac{dP}{\cancel{ds}} \times \frac{1}{\rho g} - \frac{dz}{\cancel{ds}} = \frac{dv}{\cancel{ds}} \times \frac{v}{g}$$

$$\boxed{0 = +\frac{dP}{\rho g} + dz + \frac{v}{g} dv}$$

→ is the Euler's Equation

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$\frac{1}{2} mv^2$

By integrating the Equation.

$$\int \frac{dP}{\rho g} + \int dz + \int \frac{v}{g} dv = 0$$

$$\frac{P}{\rho g} + z + \frac{v^2}{2g} = \text{Constant}$$

$\frac{P}{\rho g}$ is the pressure head in meters.

z is the datum or potential head.

$\frac{v^2}{2g}$ is velocity or kinetic head in meters.

Energy head $= \frac{\text{Energy}}{\text{weight}}$;

$KE = \frac{1}{2} mv^2$

$KH = \frac{\frac{1}{2} mv^2}{mg} = \frac{v}{2g}$

$Pd\ E = Mgz$

$Pd\ H = \frac{Mgz}{Mg} = z$

Bernoullis Equation :-

~~In a steady irrotational~~ incom

In an ideal incompressible fluid when a flow is steady and continues, sum of pressure head, KE H and Pd H is a constant along a stream line.

$$\frac{P}{\rho g} + z + \frac{v^2}{2g} = \text{Constant}$$

A ssumptions (for derivation of Bernoulli's eqn)

1) The liquid is ideal & incompressible
2) the flow is steady & continuous
3) The flow is along a streamline (i.e) it is one-dimensional
4. vel is uniform over a sectn & is equal to the mean vel.
5 The only forces acting on the fluid are gravity forces & pressure forces.

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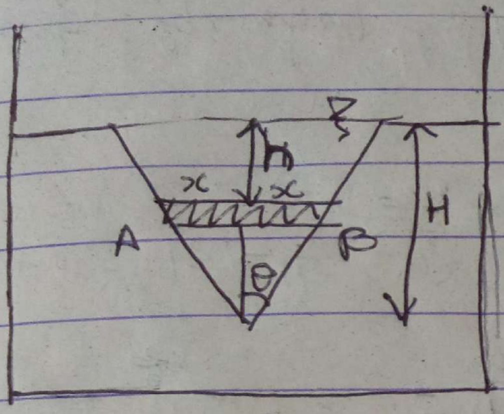
the steady flow of an incompressible



$dA = AB \times dh$

$v = \sqrt{2gh}$

$\tan\theta = \dfrac{x}{(H-h)}$

$(H-h)\tan\theta = x$

$2(H-h)\tan\theta = 2x = A$

$\therefore\ dA = 2(H-h)\tan\theta \times dh$

$dQ = dA \times v$

$dQ = 2\tan\theta\ (H-h) \times dh \times \sqrt{2gh}$

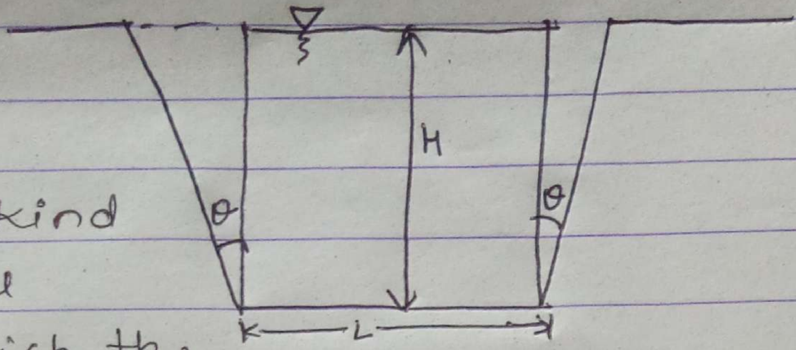
$$\int_0^Q dQ = 2\tan\theta\sqrt{2g}\int_0^H (H-h)\sqrt{h}\,.\,dh$$

$$Q = 2\tan\theta\sqrt{2g}\left(\frac{Hh^{3/2}}{3/2} - \frac{h^{5/2}}{5/2}\right)_0^H$$

$$Q = 2\tan\theta\sqrt{2g}\left(\frac{2H^{5/2}}{3} - \frac{2H^{5/2}}{5}\right)$$

$$Q = \frac{8\tan\theta\sqrt{2g}}{15}\ H^{5/2}$$

Cipolletti notch



It is a special kind of trapezoidal notch in which the sides are given as 4 vertical : 1 horizontal.

$$\Rightarrow \frac{2}{3} L\sqrt{2g}\, H^{3/2} + \frac{8}{15}\sqrt{2g}\,\tan\theta\, H^{5/2}$$

$$\tan\theta = \frac{1}{4}$$

$$= \frac{2}{3} L\sqrt{2g}\, H^{3/2} + \frac{8}{15}\times\frac{1}{4}\sqrt{2g}\, H^{5/2}$$

$$= \frac{2}{3}\sqrt{2g}\, H^{3/2}\left(L + 1/5 H\right)$$

$$= \frac{2}{3}\sqrt{2g}\, H^{3/2}\left(L + 0.2H\right)$$ [without end contraction]

With end contraction,

$$Q_{th} = \frac{2}{3}\sqrt{2g}\, H^{3/2}\left(L - 0.2H + 0.2H\right)$$

$$Q_{th} = \frac{2}{3}\sqrt{2g}\, H^{3/2} L$$

$L \rightarrow L - 0.1 \times 2H$

$L - 0.2H$