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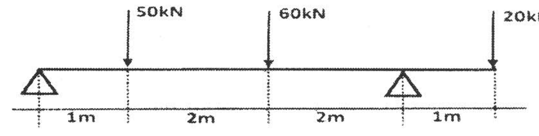
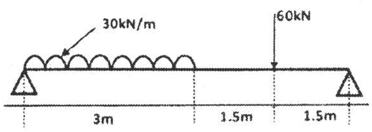
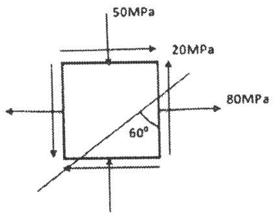
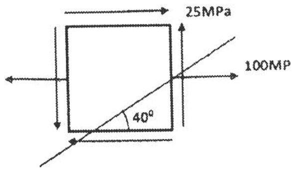
SoM

USN

Internal Assessment Test 2 – Nov. 2017		
Sub: Strength of Materials	Sub Code: 15CV32	Branch: Civil
Date: 9/11/17 · Duration: 90 min's · Max Marks: 50	Sem / Sec: 3rd sem	OBE

Answer any FIVE Full questions choosing at least ONE FULL question from each part
Assume any missing data suitably

		MARKS	CO	RBT
	PART A			
1	Define Shear Force and Bending moment. Briefly explain the sign conventions for the same with the help of neat sketches. Derive the relationship between SF, BM and load intensity.	[10]	CO2	L2
2	Draw the shear force and bending moment diagrams for the beam loaded as shown in Fig.1 below. Locate salient features in SFD and BMD if any. Indicate the positions and magnitudes of maximum positive and negative Bending moments in BMD.  Fig.1	[10]		
3	Draw the shear force and bending moment diagrams for the beam loaded as shown in fig.2 below. Locate salient features in SFD and BMD if any. Indicate the positions and magnitudes of maximum positive and negative Bending moments in BMD.  Fig.2	[10]		
	PART B			
4	What is pure bending? Derive the standard pure bending equation with the help of a neat sketch. Show that neutral axis coincides with longitudinal CG axis.	[10]		
5	A symmetric I-section is used as a simply supported beam over a span of 4m. It carries a UDL of 40kN/m over the entire span. Find the normal stresses due to bending for the maximum bending moment and plot the same. The I-section has flanges of 200mmX10mm and web of 230mmX10mm	[10]		
6	A T-section is used as a cantilever beam with its flange at the top. The beam has a span of 3m and the flange is 200X10mm & web is 240X10mm. It carries a UDL of 20kN/m over the entire span. Plot the shear stress distribution due to bending considering maximum shear force.	[10]		
	PART C			
7	Derive the expressions for the normal and tangential stresses on any given inclined plane for the case of a general two- dimensional compound stress system	[10]		

8	A 2-D compound stress system is shown in Fig.3. Find the normal, shear and the resultant stresses on the given inclined plane. Also estimate the principal stresses, their planes, maximum shear stress and its plane. Use either analytical or graphical method.  Fig.3	[10]	CO3	L3
9	A 2-D compound stress system is shown in Fig.4. Find the normal, shear and the resultant stresses on the given inclined plane. Also estimate the principal stresses, their planes, maximum shear stress and its plane. Use either analytical or graphical method.  Fig.4	[10]	CO3	L3

Internal Assessment test 2 – NOV 2017. ①

15CV32 – Strength of Materials.

Solutions to problems and scheme of evaluation.

1) SF – BM definition ——— $\boxed{2}$

Sign convention (with sketches) ——— $\boxed{3}$

Relation betn SF, BM & load intensity ——— $\boxed{5}$

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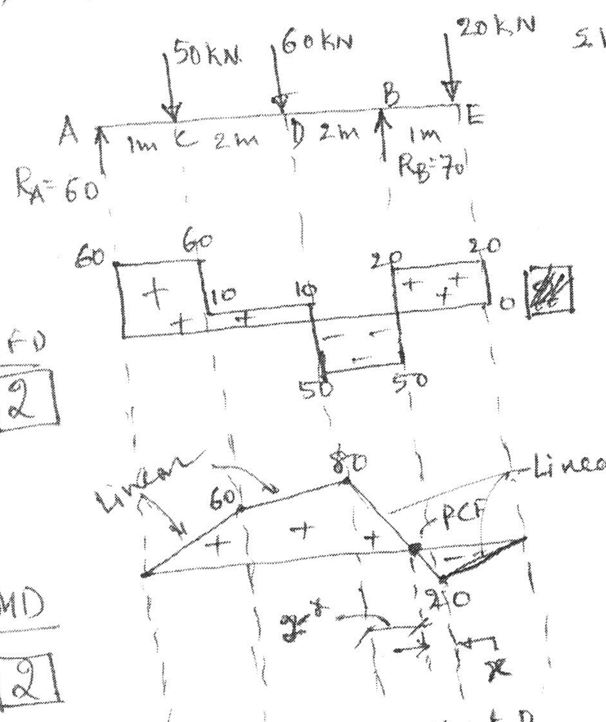
2) SFD & BMD Problem.



SFD $\boxed{2}$

BMD $\boxed{2}$

+ve M_{max} = 80 kNm at D

– ve M_{max} = 20 kNm at B

PCF at $x = 0.4$ to left of B.

i) Support Reactions

$\Sigma V = 0 \therefore R_A + R_B = 50 + 60 + 20 = 130$ —(i).

$\Sigma M_A = 0$

$R_B \times 5 = 50 \times 1 + \underset{180}{60 \times 3} + \underset{120}{20 \times 6} = 350$

$\therefore R_B = 70$ kN & $R_A = 60$ kN. —— $\boxed{2}$

ii) SF Calculations

$R_{A1} = 0$, $R_{A2} = +60^{kN}$ (ΣM_L)

$R_{C1} = +60^{kN}$ (ΣM_L without F on C)

$R_{C2} = 60 - 50$ (ΣM_L, with F on C)

$= 10$ kN

$R_{D1} = 10$ kN (ΣM_L without F on D)

$R_{D2} = 10 - 60 = -50$ kN (ΣM_L with F on D)

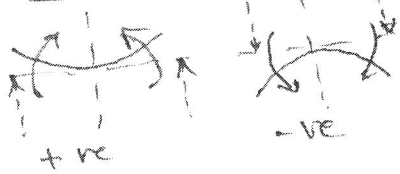
$R_{B1} = -50$ kN (ΣM_L without F on B)

$R_{B2} = -50 + 70 = +20$ (ΣM_L with F on B)

$R_{E1} = +20$ (ΣM_L without F on E) $\boxed{2}$

$R_{E2} = 0$ (ΣM_L with F on E)

iii) BM Calculations



$M_A = 0$ (ΣM_L)

$M_C = 60 \times 1 = 60$ kNm (ΣM_L)

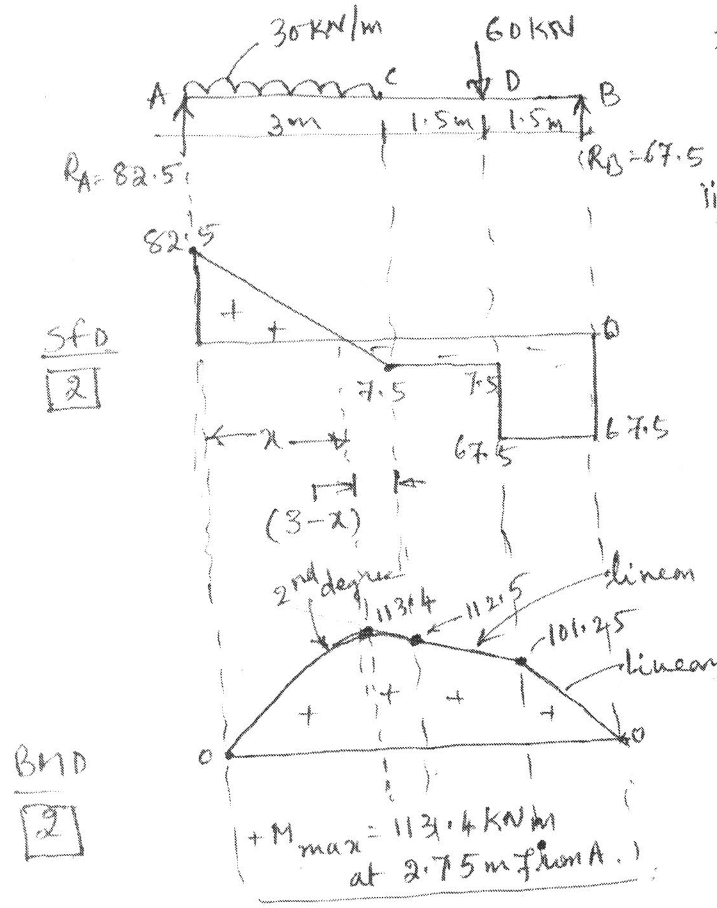
$M_D = 60 \times 3 - 50 \times 2 = 80$ kNm (ΣM_L)

$M_B = -20 \times 1 = -20$ kNm (ΣM_L $\boxed{2}$

There is a PCF at x from B. using $|||^{lar}$ $\triangle^{les}$, $\frac{80}{2-x} = \frac{20}{x}$

$\therefore x = \frac{40}{100} = \underline{0.4\,m}$.

3) SFD & BMD Problem.



i) Support Reactions. (2)

$\Sigma Y = 0 \uparrow\downarrow$, $R_A + R_B = 30\times3+60 = 150\,KN$ — ①.

$\Sigma M_A = 0$, $R_B \times 6 = 30\times3\times1.5 + 60\times4.5$

$\therefore R_B = 67.5\,KN \therefore R_A = 82.5\,KN.$ [2]

ii) SF Calculations. +ve −ve

$F_{A1} = 0$ (ΣM_L without)

$F_{A2} = +82.5\,KN$ (ΣM_L with)

$F_C = 82.5 - 30\times3 = -7.5\,KN$ (ΣM_L)

$F_{D1} = -7.5\,KN$. (ΣM_L without)

$F_{D2} = -7.5 - 60 = -67.5\,KN$ (ΣM_L with)

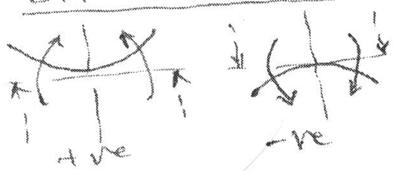
$F_{B1} = -67.5\,KN$ (ΣM_L without)

$F_{B2} = 0$ (ΣM_L with).

F = 0 ~~at x from A~~ & changes sign at x from A.

$\therefore$ using $\parallel^{lar}\ \Delta^{les}$, $\dfrac{82.5}{x} = \dfrac{7.5}{(3-x)}$ $\therefore$ $90x = 247.5$ $\therefore$ $x = 2.75\,m$. [2]

iii) BM Calculations. :



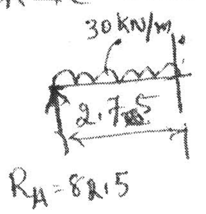
$M_A = 0$ (ΣM_L), $M_C = 67.5\times3 - 60\times1.5$ (ΣM_R)

$= 112.5\,KNm.$

$M_D = 67.5\times1.5 = 101.25\,KNm$, ($\Sigma M_R$) $M_B = 0$ (ΣM_R)

M_{max} betn A & C where F = 0 & changes sign is,

$M\Big|_{x=2.75}$



$\therefore M\Big|_{x=2.75} = 82.5\times2.75 - 30\times\dfrac{2.75^2}{2}$

$= 113.44\,KNm$ [2]

4) Pure bending – ~~the~~ Concept —— $\boxed{2}$

Derive Pure bending equation (Sketch) —— $\boxed{6}$

Show NA coincides with C.G axis —— $\boxed{2}$

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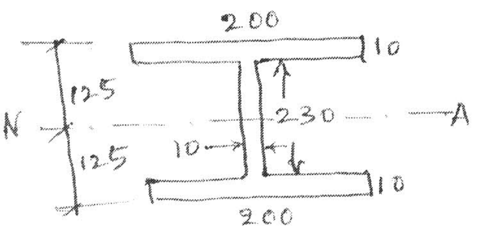
5) Bending Normal Stress Problem.

i) $M = \frac{WL^2}{8} = \frac{40 \times 4^2}{8} = 80\,kNm = 80 \times 10^6\,Nmm$ —— $\boxed{2}$

ii) To find Z $\quad$ ∵ of Symmetry about x-axis, $\bar{y} = \frac{\text{Total depth}}{2}$



$\therefore \bar{Y} = \frac{250}{2} = 125\,mm$

$\therefore I = I_{NA} = 2\left[\frac{200 \times 10^3}{12} + 200 \times 10 \times 120^2\right] + \frac{10 \times 230^3}{12}$

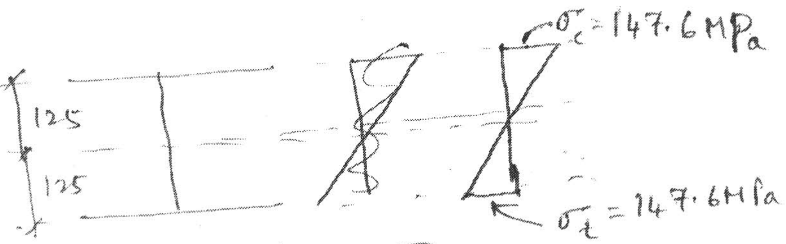
$\quad\quad \underbrace{\quad}_{33.33 \times 10^6 + 28.8 \times}\quad\quad + 10.14 \times 10^6$

$= \cancel{38.86 \times 10^6\,mm^4}\ \cancel{94.4\,mm^4}\ \boxed{67.71 \times 10^6\,mm^4}$

Since $y_c = y_t$, $Z_c = Z_t = \frac{I}{y} = \frac{\cancel{38.86} \times 10^6}{\cancel{125}} = \cancel{0.312 \times 10^6\,mm^3}$

$\therefore Z_c = Z_t = \frac{67.77 \times 10^6}{125} = 0.542 \times 10^6\,mm^3$

iii) using $M = \sigma Z$, $\sigma_c = \sigma_t = \frac{M}{Z} = \frac{80 \times 10^6}{0.542 \times 10^6} = 147.6\,N/mm^2$ (MPa).



Plot $\boxed{1}$

M	$\boxed{2}$
$\bar{Y}$	$\boxed{1}$
I	$\boxed{3}$
Z	$\boxed{1}$
σ	$\boxed{2}$
Plot	$\boxed{1}$
	10

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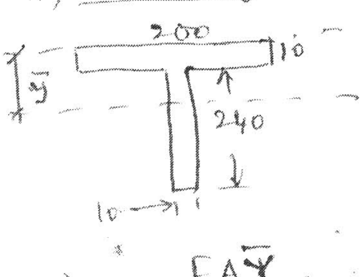
6) i) $F = WL = 20 \times 3 = 60\,kN = 60 \times 10^3\,N$.

ii) To find $\bar{y}$ & I. $\quad \bar{y} = \frac{\Sigma ay}{\Sigma a} = \frac{\overbrace{200 \times 10 \times 5}^{10000} + \overbrace{10 \times 240\,(10 + 120)}^{288000}}{\underbrace{200 \times 10 + 10 \times 240}_{4400}} = 67.73\,mm$.



$I = I_{NA} = \left.\begin{array}{l} \frac{\overset{16666}{200 \times 10^3}}{12} + 200 \times 10\,(\underbrace{67.73 - 5}_{62.73})^2 \\ + \frac{10 \times 240^3}{12} + 10 \times 240\,(\underbrace{120 - 67.73}_{52.27})^2 \end{array}\right\} = 25.97 \times 10^6\,mm^4$

$\quad 11.54 \times 10^6 \quad\quad 14.43 \times 10^6$

iii) $\tau = \frac{FA\bar{Y}}{Ib}$; $\quad$ a) $\tau_{df} = \frac{60 \times 10^3 \times 2000 \times 62.73}{25.97 \times 10^6 \times 200}$

$A = 200 \times 10 = 2000\,mm^2$
$\bar{Y} = 67.73 - 5 = 62.73$
$b_f = 200$.

$= \cancel{0.145\,N/mm^2}$.

$= 1.45\,N/mm^2$

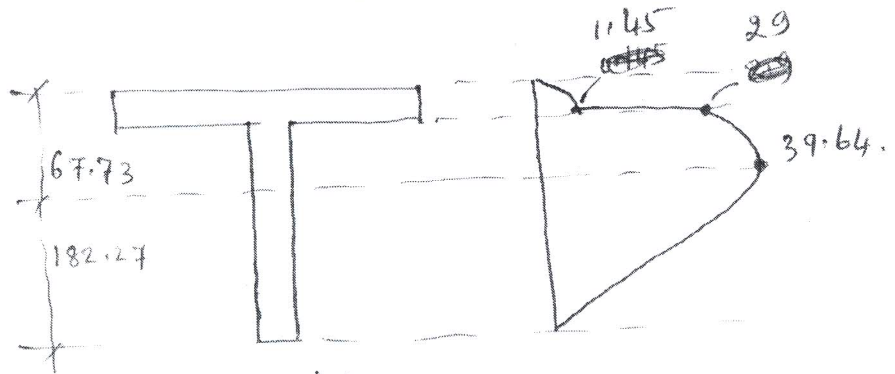
(4)

b) $\tau_{jw} = \dfrac{\checkmark \quad \checkmark \quad \checkmark}{\checkmark \times 10} = \cancel{28.9\ N/mm^2}\ 29\ N/mm^2$

c) τ_{NA} (considering portion below which is rectangle of depth $250 - 67.73 = 182.27$ mm)

$\therefore \tau_{NA} = \dfrac{60 \times 10^3 \times 1882.7 \times 91.135}{25.97 \times 10^6 \times 10}$ | $A = 10 \times 182.27 = 1882.7\ mm^2$, $\bar{Y} = \dfrac{182.27}{2} = 91.135$

$= \underline{39.64\ N/mm^2}$.



Plot

F —	1
I —	3
τ —	5
Plot —	1
	10

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7 Sketch with components — 2
Derivation for P_n — 4
— " — P_t — 4
10

— 1

8) i) Problem: $f_1 = 80$ MPa, $f_2 = -50$ MPa, $q = -20$ MPa, $\theta = +60^\circ$.

Stresses on Given Plane

$P_n = \dfrac{f_1+f_2}{2} + \dfrac{f_1-f_2}{2}\cos 2\theta + q \sin 2\theta = \dfrac{80-50}{2}^{15} + \dfrac{80-(-50)}{2}^{65} \cos 120 - 20 \times \sin 120$

$= -34.82\ N/mm^2$ (MPa) (Compressive).

$P_t = q\cos 2\theta - \dfrac{f_1 - f_2}{2}\sin 2\theta = -20 \times \cos 120 - \dfrac{80-(-50)}{2}^{65} \sin 120$

$= -46.29\ N/mm^2$

$P_r = \sqrt{P_n^2 + P_t^2} = \sqrt{(-34.82)^2 + (-46.29)^2} = \underline{57.92\ N/mm^2}$.

Principal Stresses, their planes, Max Shear Stress & its plane.

$\sigma_{1,2} = \dfrac{f_1+f_2}{2} \pm \underbrace{\sqrt{\left(\dfrac{f_1-f_2}{2}\right)^2 + q^2}}_{\tau} = \dfrac{80-50}{2} \pm \sqrt{\left(\dfrac{80-(-50)}{2}\right)^2 + (-20)^2}$

$= 15 \pm 68.007$.

σ & τ

$\therefore \sigma_1 = 15 + 68.007 = 83.007$ MPa, (tensile)

$\sigma_2 = 15 - 68.007 = -53.007$ MPa (Compressive)

$\tau = 68.007$ MPa

Recording data	1
P_n, P_t, P_r	4
σ, τ, θ	5
	10

Planes: $\theta_{P_1} = \dfrac{1}{2}\tan^{-1}\left(\dfrac{q}{\frac{f_1-f_2}{2}}\right) = \dfrac{1}{2}\tan^{-1}\left(\dfrac{-20}{65}\right)$

$= -8.55^\circ$.

$\theta_{P_2} = -8.55 + 90 = 81.45^\circ$.

$\theta_S = \theta_{P_1} + 45 = -8.55 + 45 = \underline{36.45^\circ}$

9) Problem $f_1 = 0$, $f_2 = 100$ MPa, $q = +25$ MPa, $\theta = -40°$. (5)

Stress on Given plane

$$\therefore p_n = \underbrace{\frac{0+100}{2}}_{50} + \underbrace{\frac{0-100}{2}}_{-50} \underbrace{\cos(-80)}_{.1736} + 25 \underbrace{\sin(-80)}_{-.9848} = \underline{16.7 \text{ MPa}}$$

$$p_t = 25\cos(-80) - \frac{0-100}{2}\sin(-80) = \underline{-44.9 \text{ MPa}}$$

$$\therefore p_r = \sqrt{p_n^2 + p_t^2} = \sqrt{(16.7)^2 + (-44.9)^2} = \underline{47.9 \text{ MPa}}$$

$\underline{\sigma, \tau \text{ \& } \theta_p \text{ \& } \theta_s}$.

$$\sigma_{1,2} = \underbrace{\frac{0+100}{2}}_{50} \pm \underbrace{\sqrt{\left(\frac{0-100}{2}\right)^2 + 25^2}}_{\tau}$$

$$50 \pm 55.9$$

$\therefore \sigma_1 = 50 + 55.9 = 105.9$ MPa

$\sigma_2 = 50 - 55.9 = -5.9$ MPa

$\tau = 55.9$ MPa.

$$\theta_{p_1} = \frac{1}{2}\tan^{-1}\left(\frac{25}{\frac{0-100}{2}}\right)$$

$$= -13.28°$$

$\theta_{p_2} = -13.28 + 90 = 76.72°$

$\theta_s = -13.28 + 45 = \underline{31.72°}$