USN

CMRIT

Internal Assessment Test 2 – Nov. 2017

Sub:	FLUID MECHANICS			Sub Code:	15CV33	Branch:	CIVIL
Date:	08/11/2017	Duration: 90 min's	Max Marks: 50	Sem/Sec:	III - A & B		

Answer ALL Questions. Assume necessary data

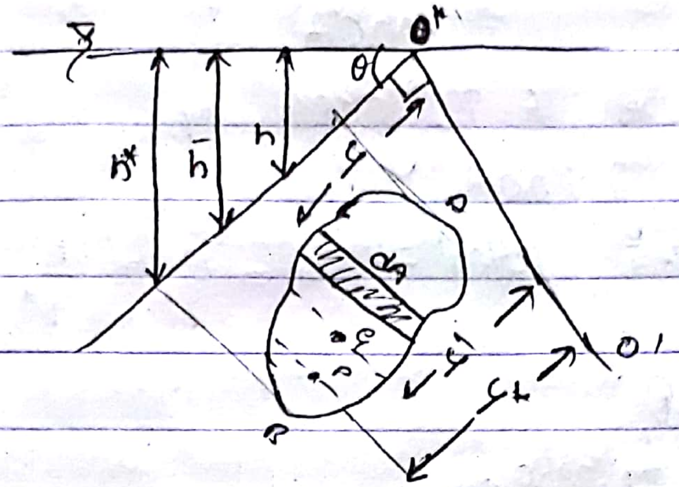
		MARKS	CO	RBT
1 (a)	Derive an expression for the total pressure and depth of centre of pressure from the free surface of liquid of an inclined plane surface submerged in the liquid.	[07]	CO2	L2
(b)	A cylindrical gate of 2.5m diameter retains 2 liquids on either side of it. On one side, there is liquid of specific gravity 0.9 upto a height of 2.5m and on the other side, there is liquid of specific gravity 0.8 upto a height of 1.25m. Estimate the resultant force due to pressure acting on unit length of the gate.	[08]	CO2	L3
2 (a)	Derive the three-dimensional continuity equation in Cartesian coordinates.	[07]	CO3	L2
(b)	If for a 2-D potential flow, the velocity potential is given by $\varphi = x(2y-1)$, determine the velocity at the point P(4,5). Determine the value of stream function ψ at the point P.	[08]	CO3	L3
3 (a)	Derive the Darcy's equation for head loss due to friction for through a pipe.	[06]	CO5	L2
(b)	Water is flowing in a pipe of 150mm diameter with a velocity of 2.5 m/s. When it is suddenly brought to rest by closing the valve, find the pressure rise assuming the pipe is elastic E = 206 GN/m^2, Poisson's ratio – 0.25 and K for water = 2.06 GN/m^2. Calculate the circumferential and longitudinal stress developed in the pipe wall.	[04]	CO5	L1
4 (a)	A 250mm diameter, 3km long straight pipe runs between 2 reservoirs of surface elevations 135 and 60m. A 1.5km long, 300mm diameter pipe is laid parallel to the 250mm diameter pipe from its mid-point to the lower reservoir. Neglecting all minor losses and assuming a friction factor of 0.02 for both pipes, find the increase in discharge caused by the addition of 300mm diameter pipe.	[10]	CO5	L

Preeti

CCI

HOD

Hydrostatic Pressure on Inclined Surface



Consider an inclined plane submerged in a liquid with an inclination of θ from the free surface. The values of h are measured from the free surface & the value of y-axis is measured from OO' axis.

let 'A' be the total area of a inclined surface, $\bar{h}$ depth of centre of gravity of inclined area from free surface, h^* Distance of centre of pressure from free surface,

let ρ be the density of liquid. let OO' be the axis $\perp^{lar}$ to the plane of the surface.

let $\bar{y}$ be the distance of centre of gravity of inclined surface from OO' axis. y^* distance of centre of pressure from OO' axis

Total pressure (F):-

Conside a small strip of Area, & a depth h from the free surface & a distance y from OO' axis as shown in fig

The pressure intensity of the strip is given by $P = \rho g h$

The force due to pressure on the small strip is given by $dF = p\,dA$

$= \rho g h\, dA$

The total pressure F is given by

$\int dF = \rho g \int h\, dA$

From the fig wkt $\sin\theta = \frac{h}{y} = \frac{\bar{h}}{\bar{y}} = \frac{h^*}{y^*}$

$h = y \sin\theta$

$F = \rho g \int y \sin\theta\, dA$

$F = \rho g \sin\theta\, A\bar{y}$ $\qquad \bar{h} = \bar{y} \sin\theta$

$F = \rho g A \bar{h}$ ✓

[$\int \rho A h \neq A\bar{h}$ because they are not 1^{los}]

Centre of Pressure (h^*):-

It is calculated by using the principle of moment which states the moment of resultant force = sum of moment about same axis.

(a) The moments are calculate w.r.t OO' axis.

let M_1 be the moment of the resultant force

$M_1 = F \times y^*$ — (1)

(b) let M_2 be the sum of all the moment w.r.t axis $00'$

$M_2 =$

$$p = \rho g h = \rho g y \sin\theta$$

$$dF = p\,dA = \rho g y \sin\theta\, dA$$

$$dM = dF \times y = \rho g y \sin\theta\, dA\, y$$

$$M_2 = \int dM = \int dF\, xy = \rho g \sin\theta \int y^2 dA$$

Pressure of the elementary strip $= \rho g h = \rho g y \sin\theta$

Force of the elementary strip $dF = p\,dA = \rho g y \sin\theta\, dA$

Moment for the elementary strip $dM = dF\, xy = \rho g y^2 \sin\theta\, dA$

The sum of the moment is obtained by integrating it

$$M_2 = \int dM = \int dF\, xy = \rho g \sin\theta \int y^2 dA.$$

$\int y^2 A = I_{00'} =$ moment of inertia of a inclined surface w.r.t $00'$ axis.

$$\rho g \sin\theta\, I_{00'} \quad - (2)$$

Equating (1) & (2)

$$M_1 = M_2$$

$$F \times y^* = \rho g \sin\theta\, I_{00'}$$

Sub (1) $\quad y^* = \dfrac{h^*}{\sin\theta} \quad - (4) \qquad (2)\ I_{00'} = I_G + A\bar{y}^2$

$$I_{00'} = I_G + \frac{A\bar{h}^2}{\sin^2\theta} \quad - (5)$$

$$y^* = \frac{h^*}{\sin\theta} \quad - (6)$$

Sub (4) (5) (6) in (3)

$$\rho g A\bar{h} \times \frac{h^*}{\sin\theta} = \rho g \sin\theta \left[I_G + \frac{A\bar{h}^2}{\sin^2\theta} \right]$$

$$h^* = \frac{\sin^2\theta}{A\bar{h}} \left[I_G + \frac{A\bar{h}^2}{\sin^2\theta} \right] = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$$

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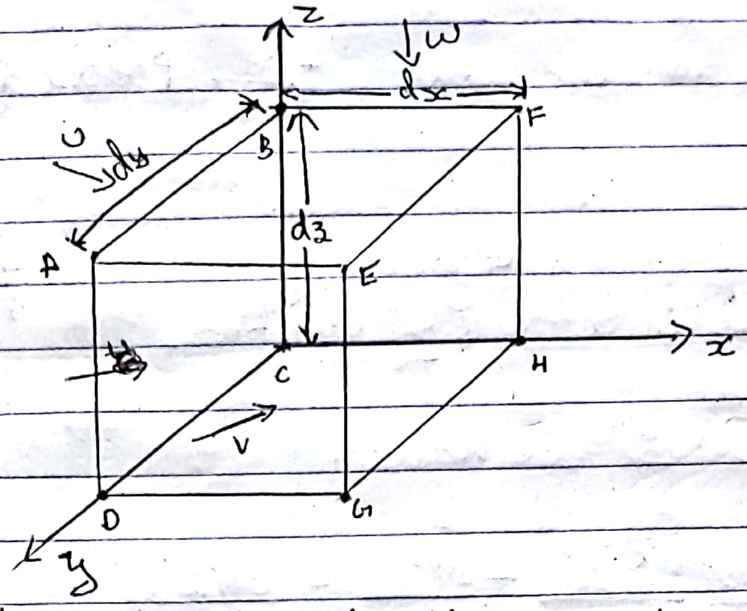
Continuity Equation for Kinematic of fluid flow (in 3D co-ordinate



Consider a fluid Element of length dx, dy, dz in the x, y & z axis respectively. Let u, v & w be the inlet velocity components in x, y & z direction respectively.

The mass of fluid Entering ABCD is given by $m = \rho \times \text{volume}$

The mass of fluid Entering ABCD/sec is given by $\frac{\rho \times \text{volume}}{\text{time (sec)}} = \rho \times \text{Area} \times \frac{\delta}{s}$

$= \rho \times dy\, dz \times u \quad \text{--- (1)} \rightarrow M_1 + \partial M_1$

The mass of fluid leaving (EFGH) the control volume/sec $= M + \frac{\partial M_1}{\partial x} dx$

$\rho \times dy \times dz \times u + \frac{\partial}{\partial x}(\rho u\, dy\, dz)\, dx$ — (2)

The gain of mass in the x direction is given by

$M/_{ABCD} - M/_{sec\ EFGH}$

$M_1 - M_1 - \partial M_1$

$= \frac{-\partial(\rho u\, dy\, dz)}{\partial x} \times dx$ — (3)

This is along x direction

III^{ly} in Y direction we have $-\frac{\partial}{\partial y}(\rho v\, dx\, dz)\, dy$ —④

III^{ly} in Z direction we have $-\frac{\partial}{\partial z}(\rho w\, dx\, dy)\, dz$ —⑤

The net gain of mass ~~is~~ given by the sum of the gain of mass in x, y & z direction.

Adding ③, ④ & ⑤ we have.

$$-dx\,dy\,dz\left[\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}\right] \quad —⑥$$

Since mass can neither be created nor destroyed the net increase of mass per unit time in the fluid Element must be Equal to the rate of increase of mass of fluid in the Element.

The net increase of mass per unit time is given by

Mass of the Element $= \rho \times dx \times dy \times dz$

increase of mass per unit time $= \frac{\partial(\rho\, dx\, dy\, dz)}{\partial t}$ —⑦

Equating ⑥ & ⑦ we have $\frac{\partial(\rho)}{\partial t}\cancel{dx\,dy\,dz} = \left[\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}\right]\cancel{dx\,dy\,dz}$

$$\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}+\frac{\partial(\rho w)}{\partial z}+\frac{\partial(\rho)}{\partial t}=0$$

This is the continuity Equation in its most general form it is

Valid for steady and unsteady flow, Compressable and incompressible flow.

If the flow is steady the Equation flow

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

for incompressible flow the Equation changes because there is no change in density.

$$\rho \left[\frac{\partial(u)}{\partial x} + \frac{\partial(v)}{\partial y} + \frac{\partial(w)}{\partial z} \right] = 0$$

$$\therefore \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

This is the Equation study in compressable flow in 3D Cartesian Co-ordinates

for 2D it reduced to $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

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If for the 2D flow the velocity potential is given by $\phi = x(2y-1)$ determine the velocity at the point $P(4,5)$. also determine the stream function ψ at the point P.

$\Rightarrow$ $\phi = x(2y-1)$

$$\frac{\partial \phi}{\partial x} = 2y-1 = -u \qquad \frac{\partial \phi}{\partial y} = 2x = -v$$

$$u = 1-2y \qquad v = -2x$$

At $P(4,5)$ $u = 1-2(5) = 9$, $v = -2(4) = -8$

$$u = 1-2y \qquad v = -2x \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\frac{\partial u}{\partial x} = 0 \qquad \frac{\partial v}{\partial y} = 0 \qquad 0 + 0 = 0$$ Therefore it is continuous.

$$\frac{\partial \psi}{\partial y} = -u = 2y-1 \qquad \frac{\partial \psi}{\partial x} = +2x$$

Integration it w.r.t x

$$\int d\psi = \int 2y-1 \cdot dx$$

$$\psi = 2yx - 1x + c \quad \text{--- (1)}$$

$$\psi = 2yx - 1x + c \quad \text{--- (2)}$$

given $\frac{d\psi}{dx} = 2x$

Differ Eq (1) w.r.t x we have $\frac{d\psi}{dx} = 2y - 1 + \frac{dc}{dx}$ --- (3)

Equating (3) & (2) we have $\frac{dc}{dx} = 2x - 2y + 1$

$c = 2$

$$\psi = 2yx - x + 2$$

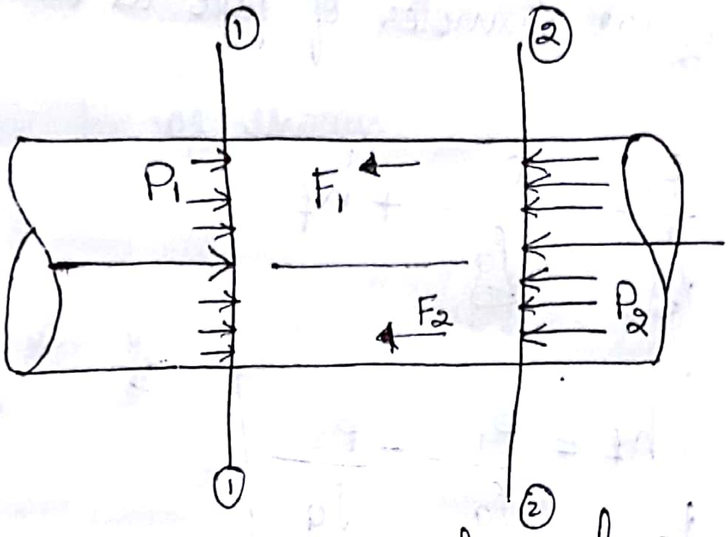
$P(4,5)$ $= 2(4)(5) - 4 + 2$

$= 38$

3) dependent on the ...

Expression for loss of head due to friction in pipe

Consider a uniform pipe, having steady flow as shown in figure.



let 1-1 and 2-2 are 2 section of pipe

let P_1 -> pressure intensity at section 1-1

V_1 -> velocity of flow at section 1-1

L -> length of the pipe b/w sec 1-1 and 2-2

d -> diameter of pipe

f' -> frictional resistance / unit ~~net~~ wetted area area / unit velocity

h_f -> loss of head due to friction

and P_2, V_2 are values of pressure intensity and velocity at section 2-2.

Applying Bernaullis Equation b/w sec 1-1 and 2-2

Total head at 1-1 = Total head at 2-2 + loss of head due to friction b/w 1-1 & 2-2

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_f$$

$Z_1 = Z_2$ as pipe is horizontal
$V_1 = V_2$ as diameter of pipe is same throughout.

$$\frac{p_1}{\rho g} = \frac{p_2}{\rho g} + h_f$$

$$\boxed{h_f = \frac{p_1}{\rho g} - \frac{p_2}{\rho g}} \quad (1)$$

But h_f is the head loss due to the friction and ~~to~~ hence intensity of pressure will be reduce in the direction of flow of frictional resistance.

Frictional resistance = frictional resistance / unit wetted area / unit velocity
wetted area $\times$ (velocity)^2

$F_1 = f' \times \pi d L \times V^2$ [wetted area $= \pi d l$.
velocity $= V = V_1 = V_2$]

Pressure

$F_1 = f' \times \vec{P} \times L \times V^2$

The force acting on the fluid between section 1-1 and 2-2 are

1) pressure force at section 1-1 $= P_1 \times A$
2) pressure force at section 2-2 $= P_2 \times A$
3) frictional force F_1 as shown

Resolving all forces in the horizontal pipe we have

$$P_1 A - P_2 A - F_1 = 0$$

$$(P_1 - P_2) A = F_1 = f' \times P \times L \times V^2$$

$$P_1 - P_2 = \frac{f' P L V^2}{A}$$

But from Equation ① we have $P_1 - P_2 = \rho g h_f$

$\therefore$ We get $\rho g h_f = \dfrac{f' P L V^2}{A}$

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L V^2 \quad \text{②}$$

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In $\frac{P}{A} = \frac{\text{Wetted Perimeter}}{\text{Area}} = \frac{\pi d}{\frac{\pi}{4} d^2} = \frac{4}{d}$ — ③

hydraulic mean depth

Substituting ③ in ② we get

$$h_f = \frac{f'}{\rho g} \times \frac{4}{d} \times L V^2$$

Paulier Equation $\Rightarrow \frac{f'}{\rho} = \frac{f}{2}$ where f is know as co-efficient of friction

$$h_f = \frac{4f}{2g} \times \frac{LV^2}{d}$$

$$h_f = \frac{4fLV^2}{2gd}$$

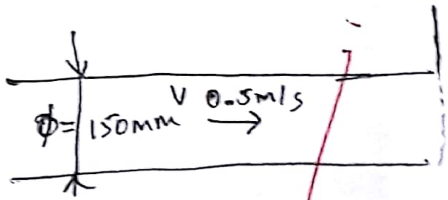
This Equation is know as Darcy-Weisbach Equation.

~~Commonly~~ Commonly used for finding loss of head due to friction in pipe.

Sometimes Equation is written as

$$h_f = \frac{fLV^2}{2gd}$$ → friction factor.

6) Water is flowing in a pipe of 150mm dia with a velocity of 0.5 m/s when it is suddenly brought to rest by closing valve. Find the pressure rise assuming the pipe is elastic $E = 206\ GN/m^2$. Poisson ratio = 0.25 ~~K for~~
K for water GN/m^2



→ Given $E = 206 \times 10^3\ N/m^2$
$\mu = 0.25$
$K = 2.06 \times 10^3\ N/m^2$

$D_1 = 150 \times 10^{-3}$

$$P = \frac{V\sqrt{\rho}}{\sqrt{\frac{1}{K} + \frac{P}{ET}}} = \frac{2.5\sqrt{1000}}{\sqrt{\frac{1}{2.06 \times 10^9} + \frac{150 \times 10^{-3}}{206 \times 10^9}}}$$

$$\boxed{P = 35.85 \times 10^3\ N/m^2}$$

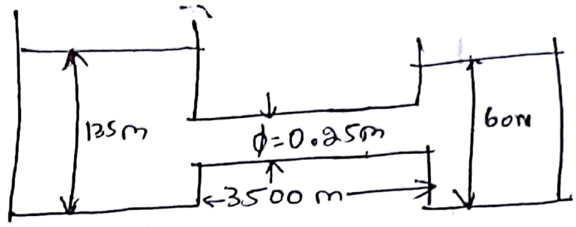
... 1000m of 0.4 and 600

At sec 1 $\Rightarrow \frac{P_1}{\rho g} + z_1 = 30.67 + 0 = 30.67\,m$

At sec 2 $\Rightarrow \frac{P_2}{\rho g} + z_2 = 0 + 15 = 15\,m$

q) A 250mm diameter, 3km long straight pipe runs b/w two reservoir of surface Elevation 135m & 60m. A 1.5 Km long 300mm diameter is laid parallel to 250mm diameter pipe from its mid point to lower reservoir. Find the increase in discharge caused by addition of 300mm diameter pipe.

Sol)



$H = 135 - 60 = 75m$

$Q_B = AV = \frac{\pi}{4} 0.25^2$
$= 0.49 m^2$

$h_f = \frac{f L V^2}{2gD}$

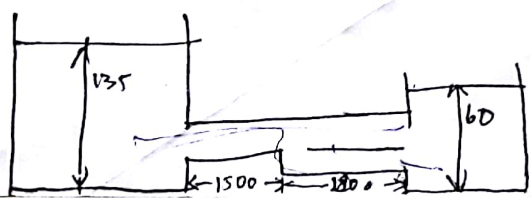
$h_f = H = 135 - 60$
$= 75m$

$$75 = \frac{0.02 \times 3000 \times V^2}{2 \times 9.81 \times 0.25}$$

$V_B = 2.47\,m/s$

$Q_B = A_1 V = 0.049 \times 2.47 = 0.121\,m^3/s$

$Q_A = Q_{A1} + Q_{A2}$



$$\frac{4fL_{A1}V_{A1}^2}{2gD_{A1}} = \frac{V_{A2}^2\, 4fL_{A1}}{2g\,D_{A2}}$$

$$\frac{V_A^2}{0.25} = \frac{V_{A2}^2}{0.3}$$

$$V_{A2}^2 = \sqrt{\frac{0.3}{0.25}}\,V_{A1} \quad \text{—— (1)}$$

$$Q = Q_{A1} + Q_{A2}$$

$$\frac{\pi}{4}\times 0.25^2 V_A = \frac{\pi}{4}\times 0.25^2 V_{A1} + \frac{\pi}{4}\times 0.3^2 V_{A2} \quad \text{— (2)}$$

sub (1) in (2)

we have $0.25^2 V_A = 0.25^2 V_{A1} + 0.3^2\sqrt{\frac{0.3}{0.25}}\,V_A$

$$\Rightarrow V_A = 2.57\,V_{A1}$$

$$V_{A1} = 0.389\,V_A$$

considering pipe XYZ $H = h_{fA} + h_{fA1}$

$$H = \frac{fL_A V_A^2}{2g D_A} + \frac{fL_1 V_{A1}^2}{2g D_1} \Rightarrow 75 = \frac{0.02\times 1500\, V_{A1}^2}{2\times 9.81\times 0.25} + \frac{0.02\times 1500\,(0.389)V_A^2}{2\times 9.81\times 0.25}$$

$$75 = \frac{0.02\times 1500}{2\times 9.81}\left[\frac{V_{A1}^2 + V_A^2\, 0.389}{0.25}\right]$$

$$V_{A1}^2 + 0.389\,V_{A1}^2 = \frac{75}{6.162}$$

$$V_A = \sqrt{\frac{1.2262}{1.389}} = 3.26\text{ m/s}$$

$$Q_A = \frac{\pi}{4}\times 0.25^2\times 3.26$$

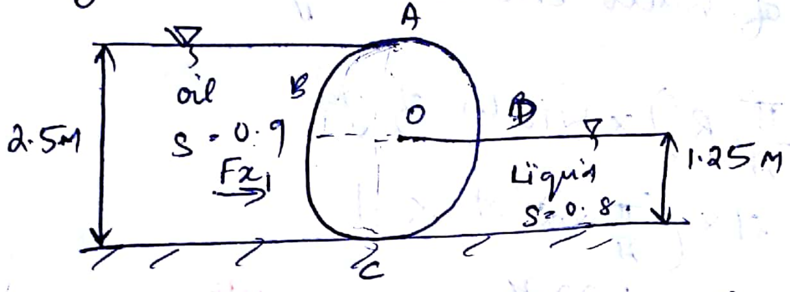
$$= 0.16\text{ m}^3/\text{s}$$

$$\%\text{ in increase} = \frac{Q_A - Q_B}{Q_B}\times 100 = \frac{0.16-0.21}{0.12}\times 100$$

$$= 32.26\,\%$$

Problems on Unit - 3.

1) A ~~cylinder~~ cylindrical gate of 2.5M dia & retains 2 liquids on either side of it as shown in fig. Estimate the resultant pressure force acting on unit length of gate. (11)



Ht:

Dia of gate = 2.5, r = 1.25M

(i) The forces acting on the left side of the cylinder are: The horiz comp, F_{x_1}

where F_{x_1} - Force on water on area projected on vert plane

= Force on area AOC

$= \rho g \bar{h} A$ $\bar{h}$ A.

$= 1000 \times 9.81 \times 1.25 \times (2.5 \times 1)$

$= \cancel{30656.25}$ N   27590.62 N

$\frac{\pi D^4}{64}$

$4 \times 4 \times 4$

F_{y_1} = Wt of water enclosed by ABCOA

$= 1000 \times 9.81 \times \left(\frac{\pi}{2} R^2\right) \times 1$ 1.25   2.455

240877.N

= ~~24087.054~~ N. ~~21669~~. 6262N

Right side of the cylinder

$F_{x_2} = \rho g A_2 \bar{h}_2$ = Force on vert area CO

$= \cancel{1000}^{800} \times 9.81 \times 1.25 \times 1 \times 0.75$

$= \cancel{9196.87\,N}$ 6131.249 N

F_{y_2} = Wt of water enclosed by DOCD

$= \rho g \left(\frac{\pi}{4} R^2\right) \times$ Width of gate

$= \cancel{1000}^{800} \times 9.81 \times \left(\frac{\pi}{4} \times 2\!\!\!\!\cancel{2}^{2}\right) \times 1$

1.2276

$= \cancel{12043.527}\,N$ $\cancel{93}$ $963\cancel{0.8}^{.944}$ N

Resultant force in the dir of x.

$F_x = F_{x_1} - F_{x_2} = 30656.25 - 9196.87$

21459.38 N

Resultant force in the dir of y

$F_y = F_{y_1} + F_{y_2}$

$= 24087.054 + 12043.527$

$= 36130.581$

Resultant force, F $= \sqrt{F_x^2 + F_y^2} = \sqrt{21459.38^2 + 36130.581^2}$

$= 37950.369$

Direction $\tan\theta = \frac{F_y}{F_x} = \frac{36130.581}{21459.38}$ 53.58°