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# Internal Assessment Test 1 – Sept. 2017

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|---|---|---|---|---|---|
| Sub: | Discrete Mathematical Structures | | Sub Code: | 15CS36 | Branch: CS, IS |
| Date: | 21-09-2017 | Duration: 90 min's | Max Marks: 50 | Sem / Sec: | III/CSE- A,B,C & ISE-A,B |

**Question 1 is compulsory and answer any six from Q.2 to Q.9**

| | | | OBE | |
|---|---|---|---|---|
| | | MARKS | CO | RBT |
| 1 | Obtain an optimal prefix code for the message TAKE CARE. Indicate the code for the message. | [08] | CO5 | L3 |
| 2 | Check the following graphs for Isomorphism. | [07] | CO5 | L3 |

| | | | | |
|---|---|---|---|---|
| 3 | Prove that a tree with n vertices has n-1 edges. | [07] | CO5 | L3 |
| 4 | Determine the order \|V\| of the graph G = (V,E) in the following cases<br>(i) G is a cubic graph of 9 edges.<br>(ii) G is regular with 15 edges.<br>(iii) G has 10 edges with 2 vertices of degree 4 and all other vertices of degree 3. | [07] | CO5 | L3 |
| 5 | Write a short note on Konigsberg Bridge Problem. | [07] | CO5 | L2 |


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5 Write a short note on Konigsberg Bridge Problem. [07] CO5 L2

6 Define the following with an example i) regular graph ii) bipartite graph iii) spanning subgraph. [07] CO5 L1

7 A person invests Rs. 10,000 at 10.5% interest (per year) compounded monthly. Find and solve the recurrence relation for the value of the investment at the end of n months. What is the value of the investment at the end of first year? [07] CO6 L3

8 An apple, a banana, a mango and an orange are to be distributed to four boys B1, B2, B3 B4. The boys B1 and B2 do not wish to have an apple, the boy B3 does not want banana or mango and B4 refuses orange. In how many ways the distribution can be made so that no boy is displeased? [07] CO2 L3

9 There are n pairs of children's gloves in a box. Each pair is of a different colour. Suppose the right gloves are distributed at random to n children, and thereafter the left gloves are also distributed to them at random. Find the probability that [07] CO2 L3

(i) no child gets a matching pair
(ii) every child gets a matching pair
(iii) exactly one child gets a matching pair.

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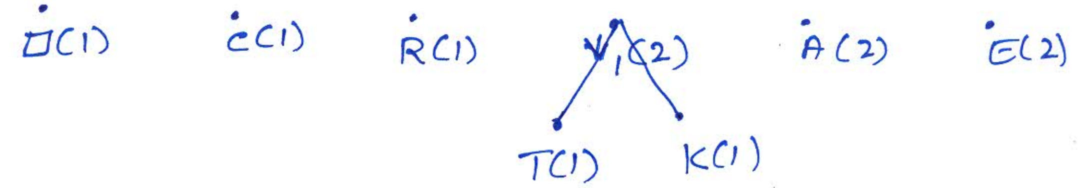
Discrete Mathematical Structures

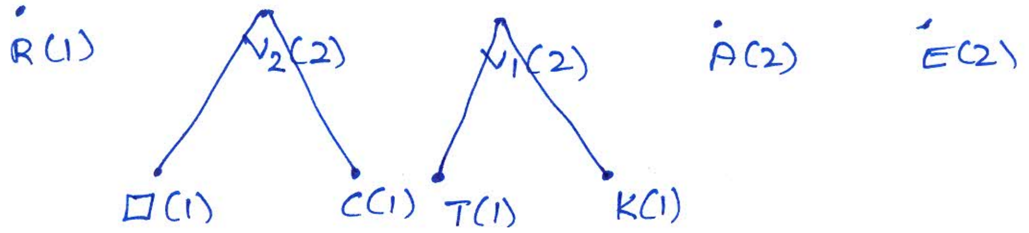
IAT - 01

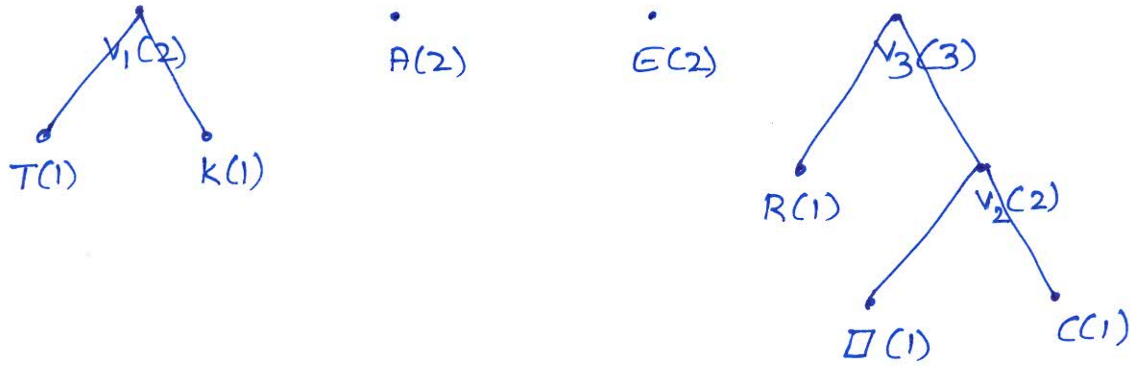
Q1 In the message TAKE CARE the letters T, A, K, E, □, C, R occur with the frequencies 1, 2, 1, 2, 1, 1, 1. where □ denote the blank space.

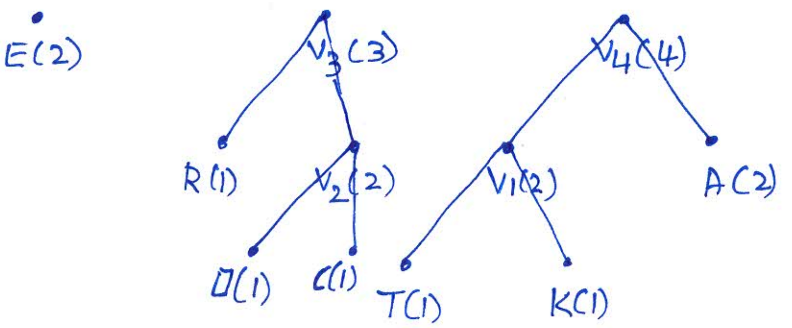
T(1) K(1) □(1) C(1) R(1) A(2) E(2)

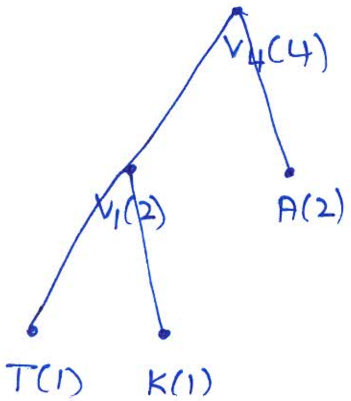
——(1)

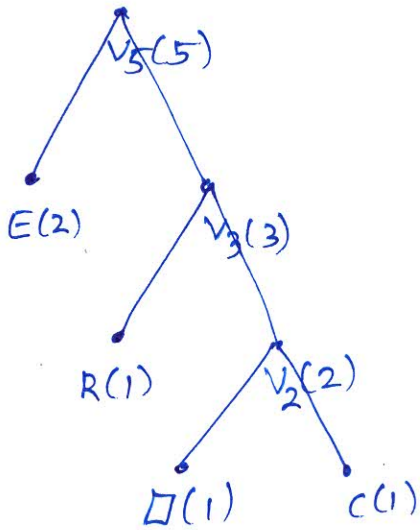


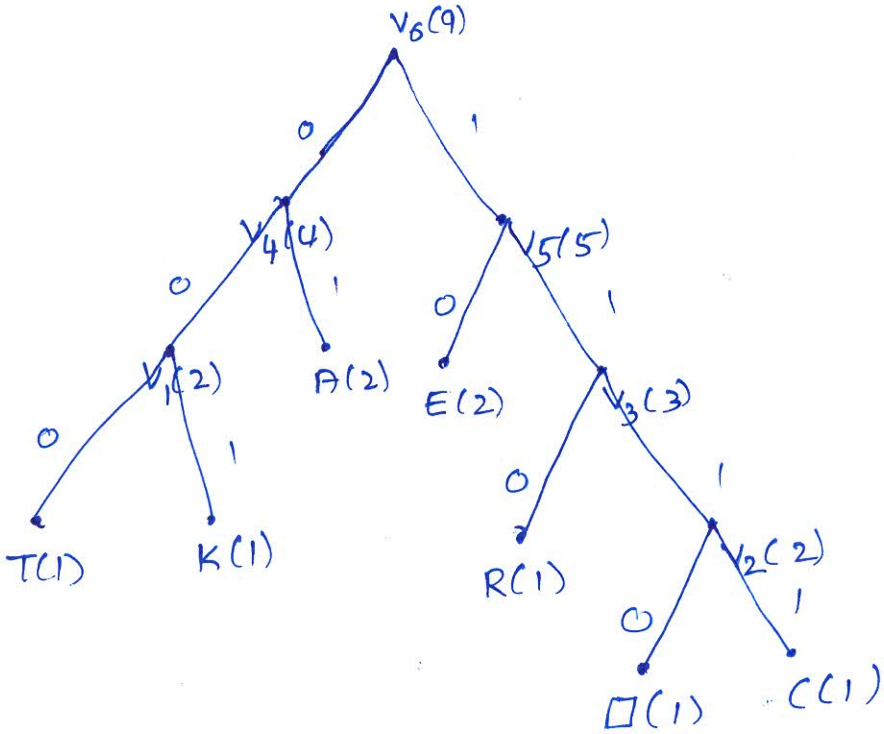






—— (4)

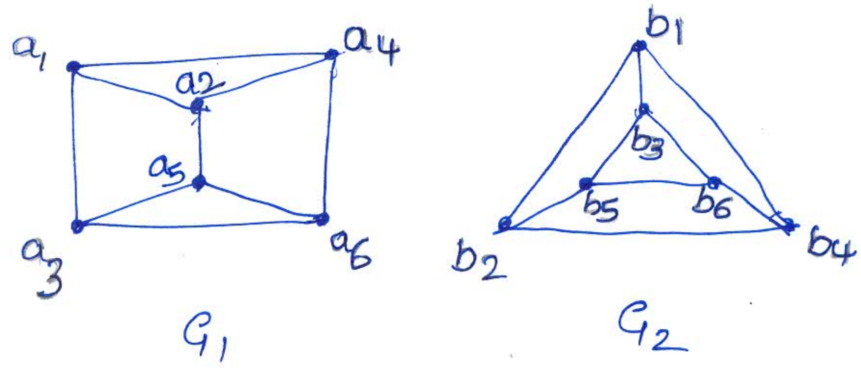
T : 000 , K : 001 , A : 01 , E : 10 , R : 110 , □ : 1110 , C : 1111

—— (1)

Code for TAKE CARE is

00001001 10 1110111101110 10 —— (1)

2)




Both the graphs $G_1$ & $G_2$ have 6 vertices & 9 edges.

Both $G_1$ & $G_2$ are 3-regular (cubic) graphs. — (1)

consider the one-to-one mappings $a_i \longleftrightarrow b_i$ between the vertices of $G_1$ & $G_2$ for $i = 1, 2, 3, 4, 5, 6$. — (1)

The edges in the two graphs correspond with each other as indicated below:

$\{a_1, a_4\} \longleftrightarrow \{b_1, b_4\}$

$\{a_1, a_2\} \longleftrightarrow \{b_1, b_2\}$

$\{a_1, a_3\} \longleftrightarrow \{b_1, b_3\}$

$\{a_3, a_6\} \longleftrightarrow \{b_3, b_6\}$

$\{a_4, a_6\} \longleftrightarrow \{b_4, b_6\}$ — (2)

$\{a_3, a_5\} \longleftrightarrow \{b_3, b_5\}$

$\{a_2, a_5\} \longleftrightarrow \{b_2, b_5\}$

$\{a_5, a_6\} \longleftrightarrow \{b_5, b_6\}$

$\{a_2, a_4\} \longleftrightarrow \{b_2, b_4\}$

— (2)

Adjacency of the vertices is preserved. — (1)

$\therefore$ The two graphs are isomorphic.

Q3. We prove the theorem by induction method.

—(1)

The theorem is obvious for $n=1$, $n=2$ & $n=3$.

Assume that the theorem is true for all trees with $n$ vertices where $n \leq k$, for a specified +ve integer $k$. —(1)

Consider a tree $T$ with $k+1$ vertices. Let $e$ be an edge with end vertices $u$ and $v$. ~~Since~~ Deletion of an edge will disconnect the graph & $T-e$ consists of exactly two components, say $T_1$ & $T_2$. Now $T_1$ & $T_2$ are trees. Both $T_1$ & $T_2$ have less than $k+1$ vertices each, and therefore, they satisfy the assumption made. Let $T_1$ & $T_2$ contain $k_1$ & $k_2$ vertices respectively. Then $T_1$ & $T_2$ have $k_1-1$ & $k_2-1$ edges resp'ly. — (3)

$\therefore$ Total no. of edges in $T_1$ & $T_2$ (taken together) =

$$k_1 - 1 + k_2 - 1$$
$$= (k_1+k_2) - 2$$
$$= (k+1) - 2$$

$\therefore$ Total no. of edges in $T-e$ $= k-1$

—(2)

$\Rightarrow$ Total no. of edges in $T = (k-1)+1 = k$.

Hence by M.I., the result is true for all +ve int $n$.

Q4. (i) Suppose the order of $G$ is $n$. Since $G$ is a cubic graph, all vertices of $G$ have degree 3, & therefore the sum of the degrees of vertices is $3n$. Since $G$ has 9 edges, we should have $3n = 2 \times 9$

$\Rightarrow n = 6$

$\therefore |V| = 6$ — (2)

(ii) Let $G$ be a $k$-regular graph.

$\therefore \sum_{v_i \in V} \deg(v_i) = kn$

$\Rightarrow 2|E| = kn$ Using Hand shaking prop

$\Rightarrow k = 30/n$ — (3)

Since $k$ is a +ve integer, $n$ must be a divisor of 30.

$\Rightarrow n = 1, 2, 3, 5, 6, 10, 15$ & $30$.

(iii) $\sum_{v_i \in V} \deg(v_i) = (2 \times 4) + (n-2) \times 3$

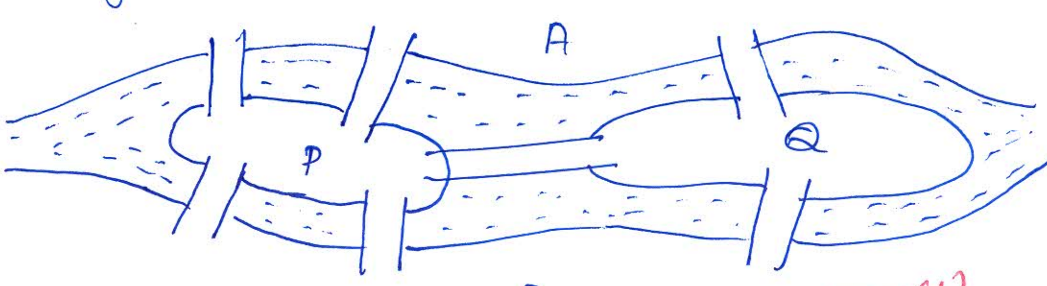
$\Rightarrow 8 + 3n - 6 = 2|E| = 2(10) = 20$

$\Rightarrow n = 6$ — (2)

Q5. In the $18^{th}$ century, city named Konigsberg in East Prussia, there flowed a river named Priegel River which divided the city into four parts. Two of these parts were the banks of the river & two were islands. These parts were connected with each other through seven bridges.

The problem was, by starting at any of the four land areas, can we return to that area after crossing each of the seven bridges exactly once?

This problem remained unsolved for several years. In the year 1736, Euler analyzed the problem with the help of a graph and gave the solution. This was the starting point for the development of graph theory.



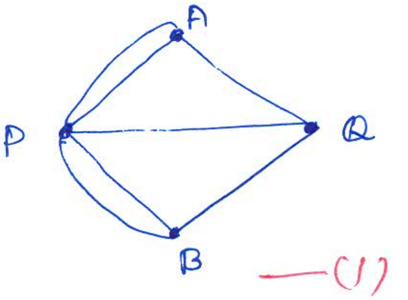

— (1)

Denote the land areas of the city by A, B, P, Q.

A, B are the banks of the river & P, Q are the islands.

Construct a graph by treating the four land areas as four vertices & the seven bridges connecting them as

~~as~~ seven edges.




— (1)

In this graph,

deg (A) = deg (B) = deg (Q) = 3,

deg (P) = 5 which are not even.

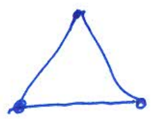
$\therefore$ the graph does not have an Euler ~~grap~~ circuit.

$\Rightarrow$ there does not exist a closed walk that contains all the edges exactly once. This amounts to saying that it is not possible to walk over each of the seven bridges exactly once & return to the starting point. — (5)

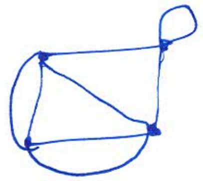
Q6

i) A graph in which all the vertices are of same degree is called a regular graph of degree k or a k-regular graph.

Eg:-



This is a 2-regular graph



This is a 4 regular graph

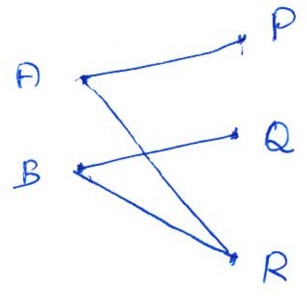
— (2)

ii) Suppose a simple graph G is such that its vertex set V is the union of two of its mutually disjoint non empty subsets $V_1$ & $V_2$ which are such that each edge in G joins a vertex in $V_1$ & a vertex in $V_2$, then G is called a bipartite graph.

Eg:- $V = \{A, B, P, Q, R\}$

$E = \cancel{\{A, P, \}}\ \{AP, AR, BQ, BR\}$

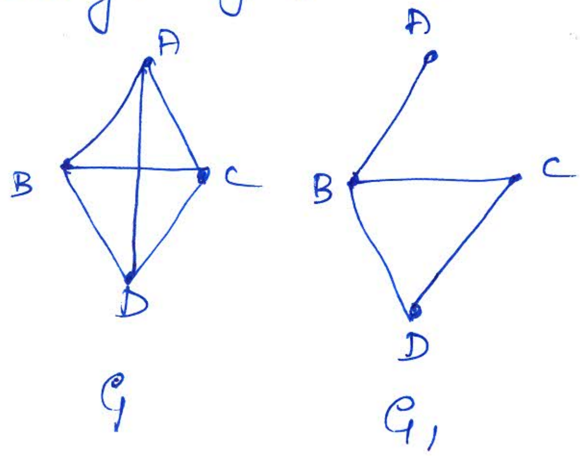
$V_1 = \{A, B\}$ , $V_2 = \{P, Q, R\}$



—(2.5)

(iii) Given a graph $G = (V, E)$, if there is a subgraph $G_1 = (V_1, E_1)$ of $G$ such that $V_1 = V$, then $G_1$ is called a spanning subgraph of $G$.

Eg:-



Here, $G_1$ is a subgraph of $G$.

—(2.5)

Q7. Since the annual interest is 10.5% , the monthly interest $= \frac{(10.5\%)}{12} = 0.875\% = 0.00875$ —(1)

Let $a_0$ be investment made. $\therefore a_0 = 10000$ —(1)

& let $a_n$ denote the value of the investment at the end of n months.

Sol, $a_1 = a_0 + (0.00875)a_0 = (1.00875)\,a_0$ — (1)

$a_2 = a_1 + (0.00875)a_1 = (1.00875)\,a_1$ — (1)

$\vdots$

$a_n = a_{n-1} + (0.00875)\,a_{n-1} = (1.00875)\,a_{n-1}$ — (1)

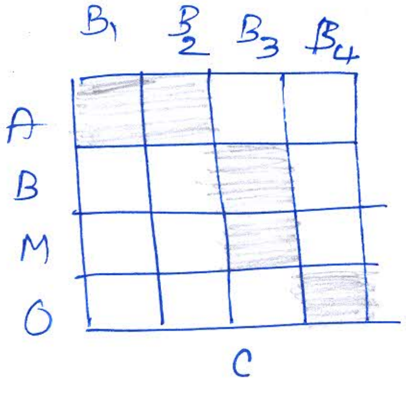
This is the required recurrence rel$^n$ for $n \geq 1$.

Sol$^n$ is : $a_n = c^n a_0 = (1.00875)^n \times 10{,}000$ — (1)

$\therefore$ At the end of first year (i.e. after 12 months),

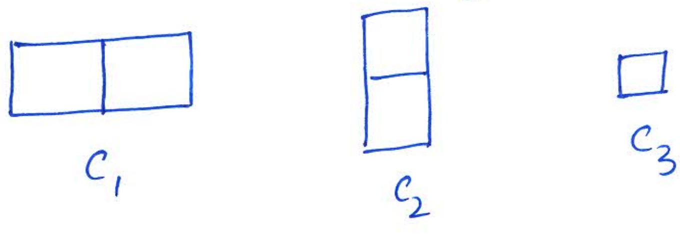
$a_{12} = (1.00875)^{12} \times 10{,}000 \simeq 11{,}102$ — (1)

Q8.




Four rows represent apple, banana, mango & orange. — (1)

Let's consider 3 mutually disjoint boards $C_1, C_2, C_3$




$r(C_1, x) = 1+2x$ , $r(C_2, x) = 1+2x$ , $r(C_3, x) = 1+x$

$\therefore$ by product formula -

$r(C, x) = r(C_1, x) \times r(C_2, x) \times r(C_3, x)$

$r(C,x) = (1+2x)^2(1+x) = 1+5x+8x^2+4x^3$ — (2)

Here $r_1 = 5,\ r_2 = 8,\ r_3 = 4$

& $S_0 = n! = 4! = 24, \quad S_k = (n-k)!\, r_k$

$S_1 = (4-1)! \times r_1 = 30, \quad S_3 = (4-3)! \times r_3 = 4$ — (3)

$S_2 = (4-2)! \times r_2 = 16,$

$\therefore$ No. of ways of distributing the fruits under the given constraints

$$\bar{N} = S_0 - S_1 + S_2 - S_3 = 24 - 30 + 16 - 4 = 6$$ — (1)

Que-9 Any one distribution of $n$ right gloves to $n$ children determines a set of $n$ places for the $n$ pairs of gloves. Let us take these as natural places for the pairs of gloves. The left gloves can be distributed to $n$ children in $n!$ ways. — (1)

(i) No child gets a matching pair is derangement of $n$ objects i.e. $d_n$

$\therefore$ the required prob. $= \frac{d_n}{n!} = \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \frac{1}{n!}\right)$

— (2)

(ii) Every child gets matching pair, it is only one possible event. $\therefore$ Prob. $= \frac{1}{n!}$ — (2)

(iii) Exactly one child gets matching pair is equivalent to saying that $(n-1)$ are in wrong places i.e. $d_{n-1}$

— (2)

$\therefore$ Prob. is $= \frac{d_{n-1}}{n!} = \frac{1}{n}\left[1 - \frac{1}{1!} + \frac{1}{2!} - \dots + (-1)^{n-1}\frac{1}{(n-1)!}\right]$