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Internal Assessment Test 2 – Nov. 2017

Sub:	Discrete Mathematical Structures			Sub Code:	15CS36	Branch:	CS and IS
Date:	08/11/2017	Duration:	90 min's	Max Marks: 50	Sem / Sec:	III CS A, B, C & IS A, B	OBE

Question 1 is compulsory and answer any SIX from questions 2 to 9

		MARKS	CO	RBT
1	Establish the validity of the argument $\forall x,[p(x)\vee q(x)]$ $\exists x,\neg p(x)$ $\forall x,[\neg q(x)\vee r(x)]$ $\forall x,[s(x)\rightarrow \neg r(x)]$ $\therefore \exists x,\neg s(x)$	[08]	CO1	L5
2	Define Tautology and Contradiction. For any propositions p, q, r, verify whether the compound proposition $[(p\wedge\neg q)\rightarrow r]\rightarrow[p\rightarrow(q\vee r)]$ is a tautology.	[07]	CO1	L3
3	Give direct proof and proof by contradiction for the statement "If n is an odd integer then n+9 is an even integer."	[07]	CO4	L3
4	Prove the following using laws of logic (i) $p\rightarrow(q\rightarrow r)\Leftrightarrow(p\wedge q)\rightarrow r$ (ii) $[\neg p\wedge(\neg q\wedge r)]\vee[(q\wedge r)\vee(p\wedge r)]\Leftrightarrow r$	[07]	CO1	L3

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5 Write the following propositions in symbolic form and find their negation: [3+4] CO1 L3

(i) For all x, if x is odd then $x^2 - 1$ is even.

(ii) All integers are rational numbers and some rational numbers are not integers.

6 Prove that $4n < (n^2 - 7)$ for all positive integers $n \geq 6$. [07] CO4 L3

7 Find an explicit definition of the sequence defined recursively by $a_1 = 7,\ a_n = 2a_{n-1} + 1$ for $n \geq 2$. [07] CO6 L3

8 [3+4] CO2 L3

(i) Find the number of non-negative integer solutions of the equation $x_1 + x_2 + x_3 + x_4 = 15$.

(ii) A woman has 11 close relatives and she wishes to invite 5 of them to dinner. (i) In how many ways can she invite them? (ii) In how many ways can she invite them if two particular persons will not attend separately?

9 Find the coefficient of (i) x^9y^3 in the expansion of $(2x - 3y)^{12}$ and (ii) $w^3x^2yz^2$ in the expansion of $(2w - x + 3y - 2z)^8$. [07] CO2 L3

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IAT-2

Discrete Mathematical Structures – 15CS36

1. $\forall x, [p(x) \vee q(x)]$

$\exists x, \neg p(x)$

$\forall x, [\neg q(x) \vee r(x)]$

$\forall x, [s(x) \rightarrow \neg r(x)]$

$\therefore \exists x, \neg s(x)$

$\Rightarrow$

$p(a) \vee q(a)$

$\neg p(a)$

$\neg q(a) \vee r(a)$ —— 1

$s(a) \rightarrow \neg r(a)$

$\therefore \exists x, \neg s(x)$

$\Rightarrow$ $q(a)$ disjunctive syllogism

$\neg q(a) \vee r(a)$

$s(a) \rightarrow \neg r(a)$ —— 2

$\therefore \exists x, \neg s(x)$

$\Rightarrow$ $r(a)$

$s(a) \rightarrow \neg r(a)$ —— 2

$\therefore \exists x, \neg s(x)$

$\Rightarrow$ $\neg s(a)$ Modus Tollens.

$\therefore \exists x, \neg s(x)$ —— 2

Above argument is valid. —— 1

2 A compound proposition which is true for all possible truth values of its components is called a tautology.

A compound proposition which is false for all possible truth values of its components is called a contradiction.

p	q	r	$\neg q$	$p \wedge \neg q$	① $(p\wedge\neg q)\to r$	$q \vee r$	② $p \to (q\vee r)$	① $\to$ ②
0	0	0	1	0	1	0	1	1
0	0	1	1	0	1	1	1	1
0	1	0	0	0	1	1	1	1
0	1	1	0	0	1	1	1	1
1	0	0	1	1	0	0	0	1
1	0	1	1	1	1	1	1	1
1	1	0	0	0	1	1	1	1
1	1	1	0	0	1	1	1	1

6

Since all the entries of the last column are zeroes, the given compound proposition $[(p\wedge\neg q)\to r] \to [p \to (q\vee r)]$ is a tautology. 1

3. Let p: n is an odd integer

q: $n+9$ is an even integer

Given $p \to q$

Direct proof: Assume p is true.

$\Rightarrow$ n is an odd integer.

$\Rightarrow$ $n = 2k+1 \quad k \in \mathbb{Z}$ 3

$\Rightarrow$ $n+9 = 2k+1+9$ $\qquad l = k+5 \in \mathbb{Z}$

$= 2k+10 = 2(k+5) = 2l \quad l \in \mathbb{Z}$ which is a multiple of 2.

$\Rightarrow$ $n+9$ is an even integer.

Method of contradiction

Assume that $p \to q$ is false ——— (1)

$\Rightarrow$ p is true and q is false.

$\Rightarrow$ n is an odd integer & $n+9$ is an odd integer.

$\Rightarrow n+9 = 2k+1 \quad k \in \mathbb{Z}$

$\Rightarrow n = 2k+1-9 = 2k-8$

$= 2l \quad l = k-4 \in \mathbb{Z}$

which is a multiple of 2.

$\Rightarrow$ n is an even integer

which is a contradiction. ——— (2)

This contradiction is because of our wrong assumption.

$\therefore p \to q$ must be true. ——— (1)

4. (i) $p \to (q \to r)$ wkt $p \to q \Leftrightarrow \neg p \vee q$

$\Leftrightarrow \neg p \vee (q \to r)$ ——— (1)

$\Leftrightarrow \neg p \vee (\neg q \vee r)$

$\Leftrightarrow (\neg p \vee \neg q) \vee r$ associative law ——— (1)

$\Leftrightarrow \neg (p \wedge q) \vee r$ De-Morgan's law

$\Leftrightarrow (p \wedge q) \to r$ using the law mentioned above.

——— (1)

4) (ii) $[\neg p \wedge (\neg q \wedge r)] \vee [(q \wedge r) \vee (p \wedge r)] \Leftrightarrow r$

LHS = $\cancel{(\neg p \wedge \neg q) \vee (\neg p \wedge r) \vee (q \wedge r) \vee (p \wedge q)}$

$\cancel{\Leftrightarrow \neg (p \vee q)}$

$\Leftrightarrow [(\neg p \wedge \neg q) \wedge r] \vee [(q \vee p) \wedge r] \rightarrow$ Associative law & Distributive law 1

$\Leftrightarrow [\neg (p \vee q) \vee (q \vee p)] \wedge r \rightarrow$ De-Morgan's Law & Distributive law 1

$\Leftrightarrow [\neg (p \vee q) \vee (p \vee q)] \wedge r \rightarrow$ Commutative law 1

$\Leftrightarrow T_0 \wedge r \Leftrightarrow r$ Inverse law 1

using Identity law

5. (i) Let $p(x)$: x is odd

$q(x)$: $x^2 - 1$ is even

Let the universe be the set of all integers.

Given statement is

$$\forall x,\ p(x) \rightarrow q(x)$$ 1

Negⁿ: $\neg \{\forall x,\ p(x) \rightarrow q(x)\}$

$\Leftrightarrow \exists x,\ \neg \{p(x) \rightarrow q(x)\}$

$\Leftrightarrow \exists x,\ p(x) \wedge \neg q(x)$ 1

There exists an integer x such that x is odd & $x^2 - 1$ is not even. 1

5 (ii) Let $p(x)$: x is a rational number

$q(x)$: x is an integer

& Z: Set of all integers

Q: Set of all rational numbers —— (1)

In symbolic form, given proposition reads

$\{\forall x \in Z, p(x)\} \wedge \{\exists x \in Q, \neg q(x)\}$ —— (1)

Negation of this is:

$\neg\{\forall x \in Z, p(x)\} \vee \neg\{\exists x \in Q, \neg q(x)\}$

$\equiv \{\exists x \in Z, \neg p(x)\} \vee \{\forall x \in Q, q(x)\}$ —— (1)

In words:

"Some integers are not rational number or every rational number is an integer." —— (1)

6. Let $S(n)$: $4n < (n^2 - 7)$

Basis step: For $n = 6$

$S(6)$: $(4 \times 6) < (6^2 - 7)$ is true.

Thus $S(n)$ is true for $n = 6$. —— (1)

Induction step: We assume that $S(n)$ is true for $n = k$ where $k \geq 6$.

ie $4k < (k^2 - 7)$ for $k \geq 6$. —— (1)

Then $4(k+1) = 4k+7$

$$< (k^2-7)+4$$

$$< (k^2-7) + (2k+1)$$ because when $k \geq 6$

$$= (k+1)^2 - 7$$ $2k+1 \geq 13 > 4$ (2)

$\therefore$ $S(k+1)$ is true. (1)

By M.I., $S(n)$ is true for all +ve int $n \geq 6$. (1)

7. Given $a_n = 2a_{n-1}+1$

By repeated use of given recursive def^n,

$$a_n = 2a_{n-1}+1 = 2\{2a_{n-2}+1\}+1$$

$$= 2[\{2(2a_{n-3}+1)+1\}]+1$$

$$= 2^3 a_{n-3} + 2^2 + 2 + 1$$ (3)

. . . . . .

. . . . . .

$$= 2^{n-1} a_{n-(n-1)} + 2^{n-2} + 2^{n-3} + \cdots + 2^2 + 2 + 1$$

$$= 2^{n-1} a_1 + (1 + 2 + 2^2 + \cdots + 2^{n-3} + 2^{n-2})$$ (2)

Using $a_1 = 7$ & the result $1 + a + a^2 + \cdots + a^{n-1} = \dfrac{a^n - 1}{a-1}$ for $a > 1$.

$$\Rightarrow a_n = 7 \times 2^{n-1} + (2^{n-1} - 1)$$

$$a_n = 8 \times 2^{n-1} - 1 = 2^{n+2} - 1$$ (2)

8 (i) $x_1+x_2+x_3+x_4=15$

Here $n=4$, $r=15$

The no of non-negative solutions of the given eq^n is

$${}^{n+r-1}C_r = {}^{4+15-1}C_{15} = {}^{18}C_{15}$$ (3)

8 (ii) (i) The number of ways of inviting 5 from 11 is given by ${}^{11}C_5$

(ii) Let them be A and B

Suppose both are attending the party, then 3 have to be selected out of 9. This can be done in ${}^{9}C_3$ ways.

Suppose both are not attending, then 5 have to be selected out of 9. This can be done in ${}^{9}C_5$ ways. (3)

So, the required answer $= {}^{9}C_3 + {}^{9}C_5$

$= 84 + 126$

$= 210$ (1)

9 (i) $$(2x-3y)^{12} = \sum_{r=0}^{12} {}^{12}C_r (2x)^r (-3y)^{12-r}$$ (1)

$$= \sum_{r=0}^{12} {}^{12}C_r \, 2^r (-3)^{12-r} x^r y^{12-r}$$

In this expansion, the coeff^t of x^9y^3 is

$$^{12}C_9\, 2^9 (-3)^3 = -(2^9 \times 3^3) \times \frac{12!}{9!\,3!}$$

$$= -2^{10} \times 3^3 \times 11 \times 10 \qquad 2$$

9(ii) The general term in the expansion of $(2w - x + 3y - 2z)^8$ is

$$\frac{8!}{n_1!\, n_2!\, n_3!\, n_4!} (2w)^{n_1} (-x)^{n_2} (3y)^{n_3} (-2z)^{n_4} \qquad 1$$

$$= \frac{8!}{n_1!\, n_2!\, n_3!\, n_4!} 2^{n_1} (-1)^{n_2} 3^{n_3} (-2)^{n_4} w^{n_1} x^{n_2} y^{n_3} z^{n_4} \qquad 1$$

For $n_1 = 3, n_2 = 2, n_3 = 1, n_4 = 2$, the coeff^t of

$w^3x^2y\,z^2$ is 1

$$\frac{8!}{3!\,2!\,1!\,2!} 2^3 (-1)^2 3^1 (-2)^2 \qquad 1$$

$$= 161280$$