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Improvement Test – Nov. 2017

Sub:	Discrete Mathematical Structures			Sub Code:	15CS36	Branch:	CS and IS
Date:	20/11/2017	Duration:	90 min's	Max Marks: 50	Sem / Sec:	III CS A,B,C and III IS A, B	OBE

First question is compulsory and answer any SIX questions from question 2 to 8.

		MARKS	CO	RBT
1	Draw the Hasse diagram for the relation R defined by xRy if and only if x|y on the set of all the positive divisors of 36.	[08]	CO3	L2
2	ABC is an equilateral triangle whose sides are of length 1cm each. If we select 5 points inside the triangle, prove that at least two of these points are such that the distance between them is less than ½ cm.	[07]	CO3	L2
3	Show that if any thirty seven integers are chosen, at least two of them will have the same remainder when divided by 36.	[07]	CO3	L2
4	Let $A = \{1, 2, 3, 4, 5\}$. Define a relation R on $A \times A$ by $(x_1, y_1) R (x_2, y_2)$ iff $x_1 + y_1 = x_2 + y_2$. (i) Determine the equivalence classes $[(1,3)], [(2,4)]$ and $[(1,1)]$. (ii) Determine the partition of $A \times A$ induced by R.	[07]	CO3	L2
5	Let $f: A \to B$, $g: B \to C$ and $h: C \to D$ be three functions. Then prove that $(h \circ g) \circ f = h \circ (g \circ f)$.	[07]	CO3	

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6 Let $f : A \to B$ be defined by $f(x) = \begin{cases} 3x - 5 \; for \; x > 0 \\ -3x + 1 \; for \; x \leq 0 \end{cases}$

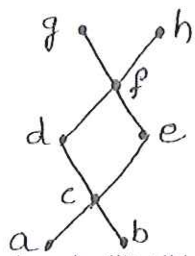
Find $f(0), f(5/3), f^{-1}(3), f^{-1}(-6), f^{-1}(0)$ and determine $f^{-1}([-5,5])$. [07] CO3 L3

OR

Let $f : A \to B$ be defined by $f(x) = \begin{cases} x + 7 \;, x \leq 0 \\ -2x + 5, \; 0 < x < 3 \\ x - 1, \; x \geq 3 \end{cases}$

Find $f(-8), f(2/3), f^{-1}(3), f^{-1}(-10), f^{-1}(0)$ and

Determine $f^{-1}([-5,-1]), f^{-1}([-2,4)), f^{-1}([11,17])$.



7 Consider the Hasse diagram of a POSET (A, R) given here: [07] CO3 L3

For B_1={c, d, e}, find (if they exist), (i) All upper bounds, (ii) All Lower bounds, (iii) All least upper bounds, (iv) All greatest lower bounds.

8 Let A = B = R, the set of all real numbers and the functions $f : A \to B$ and $g : B \to A$ be defined by $f(x) = 2x^3 - 1, \forall x \in A; g(y) = \left\{\frac{1}{2}(y+1)\right\}^{1/3}, \forall y \in B$. Show that f and g are inverses of each other. [07]

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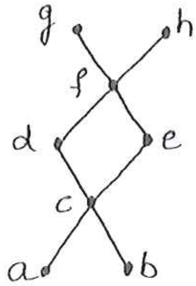

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Improvement Test - NOV 2017

Discrete Mathematical Structures - 15CS36

1. The set of all +ve divisors of 36 is

$A = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$ —— (1)

xRy if and only if x divides y.

1 is related to all elements of A

2 is related to 2, 4, 6, 12, 18, 36

3 — " — 3, 6, 9, 12, 18, 36

4 —— " —— 4, 12, 36

6 —— " —— 6, 12, 18, 36

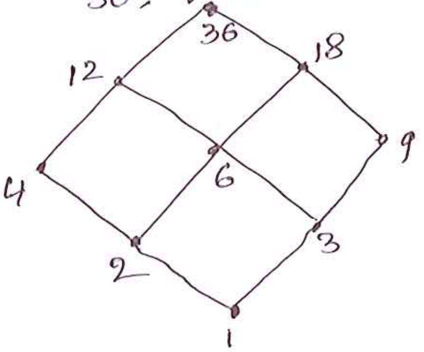
9 —— " —— 9, 18, 36

12 —— " —— 12, 36

18 —— " —— 18 & 36

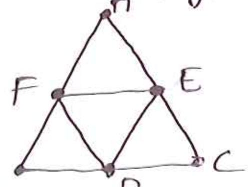
36 —— " —— 36 —— (2)

The Hasse diagram for R must exhibit all the above facts. So, the Hasse diagram is



—— (5)

2. Consider a a triangle DEF formed by the midpoints of the sides BC, CA and AB of the given triangle ABC as shown in the fig —— (1)



—— (3)

Then the triangle ABC is partitioned into four small equilateral triangles (portions), each of which has sides equal to $\frac{1}{2}$ cm. Treating each of these portions as a pigeonhole and five points chosen inside the triangle as pigeons, we find by using the pigeonhole principle that at least one portion must contain two or more points. Evidently, the distance b/w such points is less than $\frac{1}{2}$ cm. —— (4)

3. When a positive integer is divided by 36,

The possible remainders are $0, 1, 2, \ldots 35$ —— (1)

Let A_r denote the set of all positive integers that leave the remainder r when divided by 36. —— (1)

Thus every $+^{ve}$ integer belongs to one of the 36 sets — $A_0, A_1, A_2, \ldots A_{35}$. Hence, if we take any 37 positive integers, then at least two of them must ~~leave~~ belong to one of these A_r's. —— (2)

Treat A_r's as pigeonholes
& 37 as the no of pigeons. —— (2)

This proves the required result. —— (1)

4. (i) $[(1,3)] = \{(x,y) \in A\times A \mid (x,y) R (1,3)\}$

$= \{(x,y) \in A\times A \mid x+y = 1+3\}$

$= \{(1,3), (2,2), (3,1)\}$

Similarly $[(2,4)] = \{(1,5), (2,4), (3,3), (4,2), (5,1)\}$

$[(1,1)] = \{(1,1)\}$ — (3)

(ii) To determine the partition induced by R, we have to find the equivalence classes of all elements (x,y) of $A\times A$ wrt R. From what has been found above, we note that.

$[(1,1)] = \{(1,1)\} = A_1$

$[(1,2)] = \{(1,2), (2,1)\} = A_2$

$[(1,4)] = \{(1,4), (2,3), (3,2), (4,1)\} = A_3$

$[(2,5)] = \{(2,5), (3,4), (4,3), (5,2)\} = A_4$

$[(3,5)] = \{(3,5), (4,4), (5,3)\} = A_5$

$[(4,5)] = \{(4,5), (5,4)\} = A_6$

$[(5,5)] = \{(5,5)\} = A_7$

$[(1,3)] = \{(1,3), (3,1), (2,2)\} = A_8$

$[(2,4)] = \{(2,4), (4,2), (3,3), (1,5), (5,1)\} = A_9$ — (3)

Thus $A_1, A_2, A_3, \ldots A_9$ are the only distanct equivalen-ce classes of $A\times A$ wrt r. Hence the partition of $A\times A$ — (1)

induced by R is represented by

$$P = \{A_1, A_2, A_3, A_4, A_5, A_6, A_7, A_8, A_9\}.$$

5. We first note that both $(hog)of$ and $ho(gof)$ are functions from A to D.

For any $x \in A$, we have

$$[(hog)of](x) = (hog)[f(x)] = (hog)(y) \quad \text{where } y = f(x)$$

$$= h[g(y)]$$

$$= h(z) \quad \text{where } z = g(y) \quad \text{—— (1)} \qquad (3)$$

and $[ho(gof)](x) = [ho(gof)(x)]$

$$= h[(gof)(x)]$$

$$= h[g\{f(x)\}]$$

$$= h[g(y)] = h(z) \quad \text{—— (2)} \qquad (3)$$

Results (1) & (2) show that

$$[(hog)of](x) = [ho(gof)](x) \text{ for every } x \in A. \qquad (1)$$

$\therefore \ (hog)of = ho(gof)$

6. $f(0) = 1$ —— (1)

$f(5/3) = (3 \times 5/3) - 5 = 0$ —— (1)

$f^{-1}(3) = \{x \in R \mid f(x) = 3\} = \{8/3, -2/3\}$ —— (1)

$f^{-1}(-6) = \phi$ because $f(x) \neq 6$ for any $x \in R$. —— (1)

$f^{-1}\{0\} = 5/3$ ①

$f^{-1}([-5,5]) = \{x \in R \mid f(x) \in [-5,5]\}$ $\cancel{-4/3 \le x \le 2 \text{ or } 0 \le x \le 10/3}$

$= \{x \in R \mid -5 \le f \le 5\}$

When $x > 0$, we have $f(x) = 3x - 5$

$-5 \le f(x) \le 5$

$\Rightarrow -5 \le (3x-5) \le 5$

$\Rightarrow 0 \le x \le 10/3$

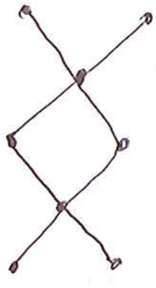
When $x \le 0$, we have $f(x) = -3x + 1$

$-5 \le f(x) \le 5$

$-5 \le (-3x+1) \le 5 \Rightarrow -4/3 \le x \le 2$ ②

Thus, $f^{-1}([-5,5]) = [-4/3, 10/3]$

7.



$B_1 = \{c, d, e\}$

(i) All of c, d, e in B_1 are related to f, g, h. $\therefore$ f, g, h are upper bounds of B_1. ②

(ii) The elements a, b and c are related to all of c, d, e which are in B_1. $\therefore$ a, b and c are lower bounds of B_1. ②

(iii) The upper bound f of B_1 is related to the other upper bounds g and h of B_1. $\therefore$ f is the LUB of B_1. (1½)

The lower bounds a and b of B are related to the lower bound c of B_1. $\therefore$ c is the GLB of B_1. —— $1\frac{1}{2}$

8. For any $x \in A$,

$$(g \circ f)(x) = g(f(x)) = g(y) = \left\{\frac{1}{2}(y+1)\right\}^{1/3} \text{ where } y = f(x).$$

$$= \left\{\frac{1}{2}(2x^3 - 1 + 1)\right\}^{1/3} \quad \because y = f(x) = 2x^3 - 1$$

$$= x$$ —— 2

Thus, $g \circ f = I_A$. —— 1

Next, for any $y \in B$,

$$(f \circ g)(y) = f(g(y)) = f\left(\left\{\frac{1}{2}(y+1)\right\}^{1/3}\right)$$

$$= 2\left[\left\{\frac{1}{2}(y+1)\right\}^{1/3}\right]^3 - 1$$

$$= 2\left[\frac{1}{2}(y+1)\right] - 1 = y$$ —— 2

—— 1

Thus, $f \circ g = I_B$.

Accordingly, each of f and g is an invertible func^n & further more each is the inverse of the other. —— 1

6 <u>2nd part</u>

$f(-8) = -8+7 = -1$

$f(2/3) = -2(2/3)+5 = -4/3 + 5 = 4/3$

Let

$f^{-1}(3) = x$

$\Rightarrow f(x) = 3$

$x+7 = 3$	$-2x+5 = 3$	$x-1 = 3$
$x = -4$	$-2x = -2$	$x = 4$
	$x = 1$	

$\therefore f^{-1}(3) = \{-4, 1, 4\}$

Let

$f^{-1}(-10) = x$

$f(x) = -10$

$x+7 = -10$	$-2x+5 = -10$	$x-1 = -10$
$x = -17$	$-2x = -15$	$x = -9 \notin [3,\infty)$
	$x = 15/2 \notin (0,3)$	

~~within~~

$\therefore f^{-1}(-10) = \{-17\}$

Let $f^{-1}(0) = \{x\}$

$\Rightarrow f(x) = 0$

$x+7 = 0$	$-2x+5 = 0$	$x-1 = 0$
$x = -7$	$-2x = -5$	$x = 1 \notin [3,\infty)$
	$x = 5/2$	

$\therefore f^{-1}(0) = \{-7, 5/2\}$

$f^{-1}([-5,-1])$

When $x \leq 0$

$-5 \leq x+7 \leq 1$

$\Rightarrow -12 \leq x \leq 0$

$\therefore x \notin [-12, 0]$

$0 < x < 3$

$-5 \leq -2x+5 \leq -1$

$-5 \leq -x \leq -3$

$3 \leq x \leq 5$

$x \notin (0,3)$

$x \geq 3$

$-5 \leq x-1 \leq -1$

$-4 \leq x \leq 0$

$x \notin [3, \infty)$

$\therefore f^{-1}([-5,-1]) = [-12, 0]$

$f^{-1}([-2,4))$

When $x \leq 0$

$-2 \leq x+7 < 4$

$-9 \leq x < -3$

$x \in [-9,-3)$

$0 < x < 3$

$-2 \leq -2x+5 < 4$

$-\frac{7}{2} \leq -x < -\frac{1}{2}$

$\frac{1}{2} < x \leq \frac{7}{2}$

$x \in (\frac{1}{2}, 3)$

$x \geq 3$

$-2 \leq x-1 < 4$

$-1 \leq x < 5$

$x \in [3,5)$

$\therefore f^{-1}([-2,4)) = [-9,-3) \cup (\frac{1}{2}, 3) \cup [3,5)$

$= [-9,-3) \cup (\frac{1}{2}, 5)$

$f^{-1}([11,17])$

When $x \leq 0$

$11 \leq x+7 \leq 17$

$4 \leq x < 10$

$x \notin (-\infty, 0]$

$0 < x < 3$

$11 \leq -2x+5 \leq 17$

$-6 \leq x \leq -3$

$x \notin (0,3)$

$x \geq 3$

$11 \leq x-1 \leq 17$

$12 \leq x \leq 18$

$x \in [12,18]$

$\therefore f^{-1}([11,17]) = [12,18]$