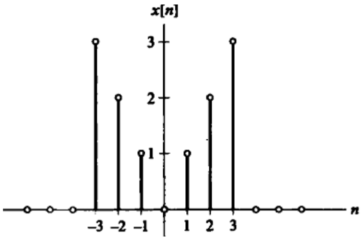
USN | | | | | | | | | | |

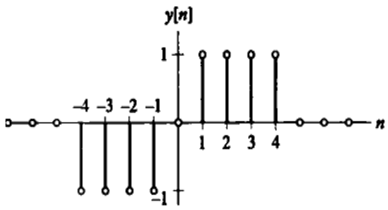
Internal Assessment Test 1 – Sept. 2017

Sub:	Signals & Systems				Sub Code:	15EE54	Branch:	EEE
Date:	20/9/17	Duration:	90 min’s	Max Marks: 50	Sem / Sec:	5 A & B		OBE

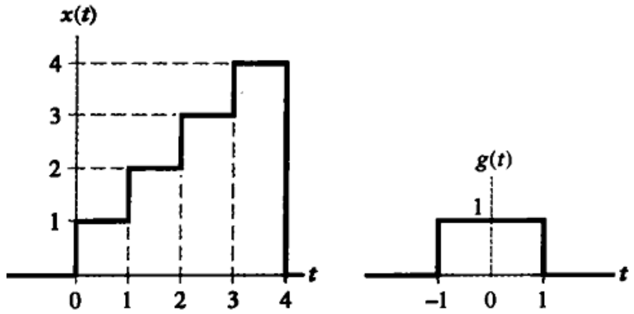
Answer any FIVE FULL Questions

		MARKS	CO	RBT
1 (a)	The sinusoidal $x(t) = 3\cos(200t + \frac{\pi}{6})$ is passed through a square law device defined by the input output relation $y(t) = x^2(t)$. Specify the dc component and fundamental frequency of the sinusoidal component of the output $y(t)$.	[05]	CO2	L3
(b)	Determine whether the following signals are periodic. If they are periodic, find the fundamental period. $i)\ x[n] = (-1)^{n^2}$ $\quad ii)\ x(t) = e^{(-1+j)t}$	[05]	CO2	L2
2 (a)	Determine the total energy of the raised-cosine pulse $x(t)$ is defined as $x(t) = \begin{cases} \frac{1}{2}[\cos(\omega t) + 1, & -\pi/\omega \le t \le \pi/\omega \\ 0, & otherwise \end{cases}$.	[05]	CO2	L2
(b)	Let $x[n]$ and $y[n]$ be given in figures. Sketch carefully the following signals $i)\ x[3-n]y[n]$ $\quad ii) x[-n]y[-n]$	[05]	CO1	L2





		MARKS	CO	RBT
3 (a)	The staircase signal $x(t)$ is viewed as superposition of four rectangular pulses. Starting with the compressed version of rectangular pulse $g(t)$ construct the waveform of fig (a) and express $x(t)$ in terms of $g(t)$.	[06]	CO2	L3



(a) (b)

		MARKS	CO	RBT
(b)	Consider the signal $x(t) = \begin{cases} \sin\left(\frac{\pi t}{T}\right), & -T \le t \le T \\ 0, & otherwise \end{cases}$. Is the signal $x(t)$ an even or odd function of time t?	[04]	CO2	L2
4 (a)	Categorize each of the following signals as energy or power signal, find the energy or time averaged power of the following signals. $i)\ x[n] = \begin{cases} n, & 0 \le n < 5 \\ 10 - n, & 5 \le n \le 10 \\ 0, & otherwise \end{cases}$ $\quad ii)\ x(t) = 5\cos(\pi t) + \sin(5\pi t), -\infty < t < \infty$	[06]	CO2	L2
(b)	Let $x[n] = \begin{cases} n & for\ n\ odd \\ 0 & otherwise \end{cases}$. Determine $y[n] = x[2n]$.	[04]	CO3	L3

5	Determine whether the following systems are linear, causal, memory less, time-invariant, and stable. $(i)\ y[n] = \log_{10}(\lvert x[n]\rvert)$ $(ii) y(t) = x(2-t)$ $(iii)\ y(t) = \cos(x(t))$ $(iv)\ y[n] = 2x[n]u[n]$	[10]	CO3	L3
6	Evaluate the discrete convolution sum $y[n] = \alpha^n\{u[n-2] - u[n-13]\} * 2\{u[n+2] - u[n-12]\}$.	[10]	CO4	L5
7	The input $x(t)$ and impulse response $h(t)$ of LTI system are respectively given by $x(t) = (t-1)[u(t-1) - u(t-3)]$ and $h(t) = u(t+1) - 2u(t-2)$. Find the output of the system.	[10]	CO4	L5
8	Evaluate the total response of the LTI system described by the differential equation $$\frac{dy(t)}{dt} + y(t) = 2\frac{dx(t)}{dt} + x(t)$$ where $x(t) = 4e^{-2t}u(t)$ and $y(0) = 2$.	[10]	CO4	L5

SIGNALS AND SYSTEMS

NAME :- BINDU. M
CLASS :- V SEM A
USN :- 1CR19EE021
ASSIGNMENT #01

1 (a) The sinusoidal $x(t) = 3\cos(200t + \pi/6)$ is passed through a square law device defined by the input output relation $y(t) = x^2(t)$ specify the dc-component & fundamental frequency of the sinusoidal component of the output $y(t)$.

Sol^n

$$y(t) = \left(3\cos(200t + \pi/6)\right)^2$$

$$= 9\cos^2(200t + \pi/6)$$

$$= \frac{9}{2}\left[\cos(400t + \pi/3)\right]$$

(a) dc component $= 9/2$

(b) Sinusoidal component $= \frac{9}{2}\cos\left(400t + \pi/3\right)$

$$\omega = 200$$

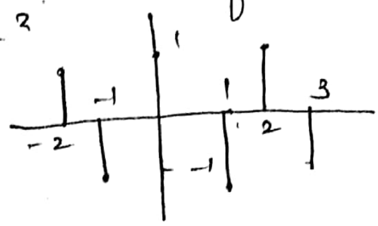
$$T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{400} = \frac{\pi}{200}$$

$\therefore$ Fundamental frequency $= \frac{200}{\pi}$ Hz

(b) Determine whether the following signals are periodic. If they are periodic, find the fundamental period

(i) $x(n) = (-1)^{n}$



$N = 2$ $x(n)$ is periodic
Fundamental period = 2 samples.

(ii) $x(t) = e^{-(1+j)t}$

$= e^{-t} \cdot e^{jt}$

$x(t)$ is non-periodic

2 (a) Determine the total energy of the raised-cosine pulse $x(t)$ is defined as

$$x(t) = \begin{cases} \frac{1}{2}[\cos(\omega t)+1], & -\frac{\pi}{\omega} \le t \le \pi/\omega \\ 0 & \text{otherwise} \end{cases}$$

soln

$$E = \int_{-\pi/\omega}^{\pi/\omega} \frac{1}{4}[\cos\omega t + 1]^2 \, dt$$

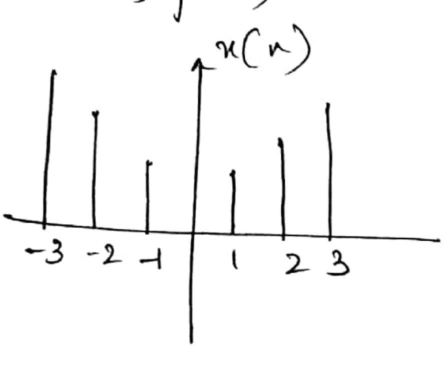
$$= \frac{1}{2} \int_{0}^{\pi/\omega} (\cos^2(\omega t) + 2\cos(\omega t) + 1] \, dt$$

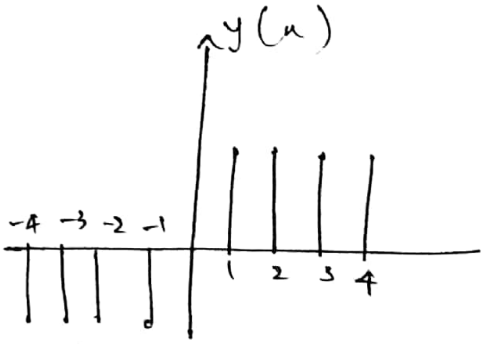
$$= \frac{1}{2} \int_{0}^{\pi/\omega} (\frac{1}{2}\cos(2\omega t) + \frac{1}{2} + 2\cos(\omega t) + 1] \, dt$$

$$= \frac{1}{2}\left(\frac{3}{2}\right)\frac{\pi}{\omega} = \frac{3\pi}{4\omega}$$

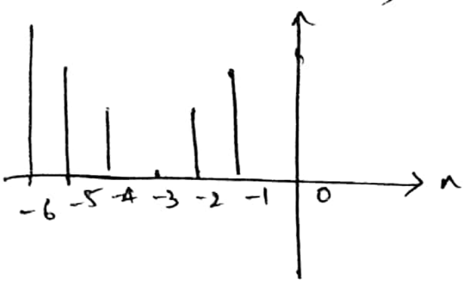
(b) Let $x(n)$ & $y(n)$ is given in figure. Sketch carefully the following signals.

(i) $x(3-n)\,y(n)$

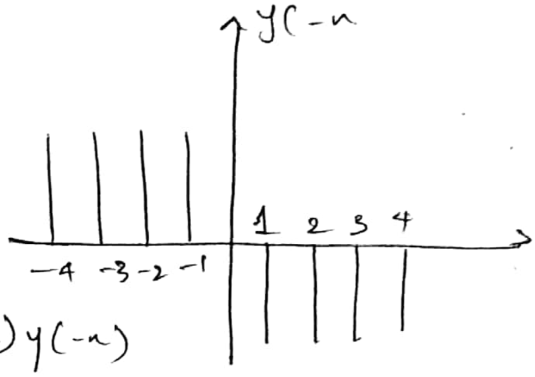




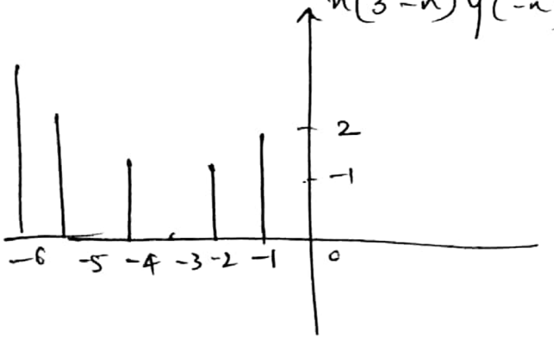
$x(n-3)$



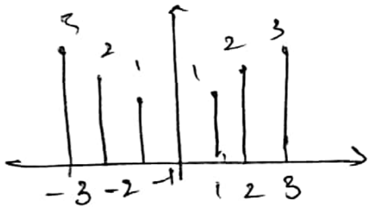
$y(-n)$



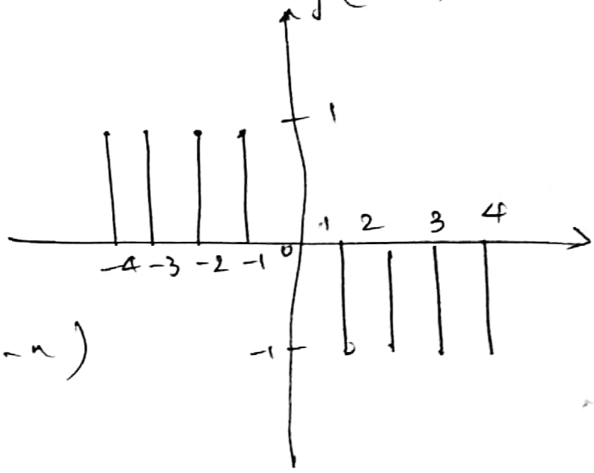
$x(3-n)\,y(-n)$



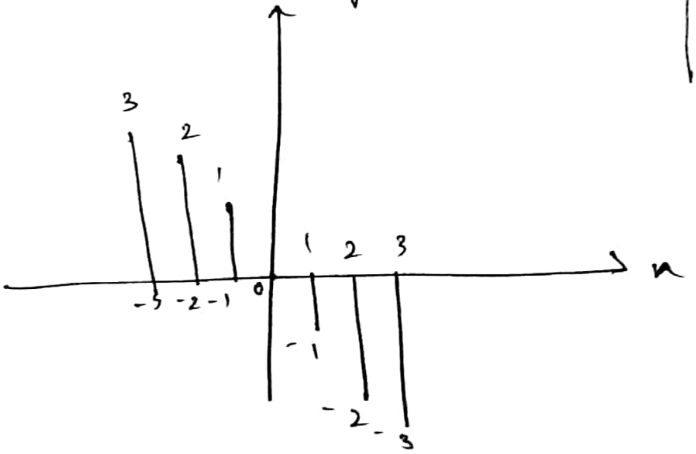
(ii) $x(-n)\,y(-n)$



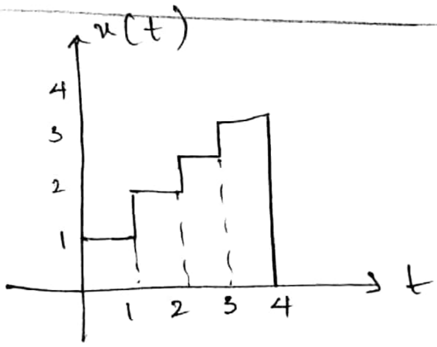
$y(-n)$

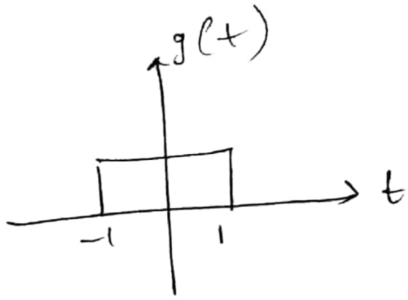


$x(-n)\,y(-n)$

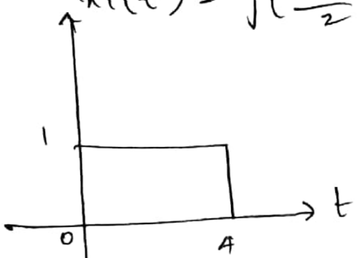


3
a. The straight line $x(t)$ is viewed as superposition of four rectangular pulses. starting with the compressed version of Rectangular $g(t)$ construct the waveform of fig (a) & express $x(t)$ in terms of $g(t)$



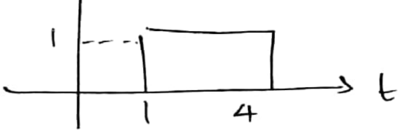


$$x_1(t) = g\left(\frac{t-2}{2}\right)$$



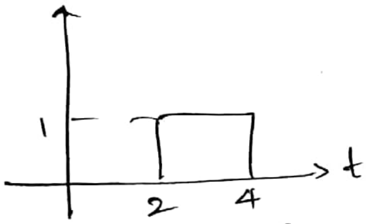
$$a = \frac{2}{4} = \frac{1}{2}$$

$$x_2(t) = g\left[\frac{2(t-2.5)}{3}\right]$$



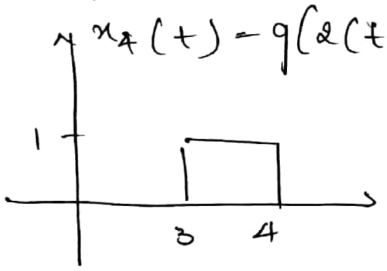
$$a = 2/3$$

$$x_3(t) = g(t-2)$$



$$a = 1$$

$$x_4(t) = g(2(t-3.5))$$



$$a = 2$$

$$\therefore x(t) = x_1(t) + x_2(t) + x_3(t) + x_4(t)$$

$$x(t) = g\left(\frac{t}{2} - 1\right) + g\left(\frac{2}{3}t - \frac{5}{3}\right) + g(t-3) + g(t_2)$$

Consider the signal $x(t) = \{\sin\left(\frac{\pi t}{T}\right) \Rightarrow -T \le t \le T$ is the signal $x(t)$ on even or odd function of time t?

$$x(t) = \sin\left(\frac{\pi t}{T}\right)$$

$$x(-t) = \sin\left(\frac{-\pi t}{T}\right) = -\sin\left(\frac{\pi t}{T}\right)$$

$\therefore x_o(0) = -x_o(0)$ Hence is odd signal

Categorise each of the following signals as energy or power signal. Find the energy or time averaged power of the following signal.

(i) $x(n) = \begin{cases} n & 0 \le n \le 5 \\ 10-n & 5 \le n \le 10 \\ 0 & \text{otherwise.} \end{cases}$

$$E = \sum_{n=\infty}^{\infty} |x(n)|^2$$

$$= \sum_{n=0}^{5} n^2 + \sum_{n=5}^{10} 10-n$$

$$= (0+1+4+9+16+25) + [5+4+3+2+1+0]$$

$\underline{E = 70 J}$ $\therefore$ Since energy is finite this is energy signal

(ii) $x(t) = 5\cos(\pi t) + \sin(5\pi t)$ $\quad -\infty < t < \infty$

It is power signal since $E = \infty$

$$P_{avg} = \lim_{t \to \infty} \frac{1}{T} \int_0^T (x(t))^2 \, dt$$

$\omega_0 = \pi$ $\quad$ $\omega_0 = 5\pi$

$T_1 = 2$ $\quad$ $T_2 = 2/5$

$\frac{T_2}{T_1} = \frac{2}{10}$

$T_2 = 5T_2 = T_1$

$T = 2$

$$P_{avg} = \frac{1}{2}\int_0^{\pi}\left[5\cos(\pi t) + \sin(5\pi t)\right]^2 dt$$

$$= \frac{5^2+1^2}{2} = \underline{\underline{13\,W}}$$

5. Determine whether the following systems are linear, causal, memory less, time invariant & stable

(i) $y(n) = \log_{10}(x(n))$

Sol

$$y_1(n) = \log(x_1(n)) \qquad y_2(n) = \log(x_2(n))$$

$$y_3(n) = \log(a_1 x_1(n) + a_2 x_2(n))$$

$$y_3'(n) = a_1 \log x_1(n) + a_2 \log(x_2(n))$$

$$y_3(n) \neq y_3'(n)$$

Hence non-linear.

$$y_2(n) = y_1(n-N_2) = \log(x_1(n-N_2))$$

$$x_1(n-N_2) = x_2(n) = \log(x_1(n-N_2)) = y_2(n)$$

Hence system is time invariant

The system is unstable since $y(n) = \infty$ when $x(n) = 2$

The system is causal & memoryless.

(ii) $y(t) = x(2-t)$

$$y_1(t) = x_1(2-t) \qquad y_2(t) = x_2(2-t)$$

$$y_3'(t) = a_1 x_1(2-t) + a_2 x_2(2-t)$$

$$y_3(t) = a_1 x_1(2-t) + a_2 x_2(2-t)$$

$$y_3(t) = y_3'(t)$$

Hence system is linear

$y(t, t_1) = x(2 - t - t_1)$

Replace t by $t - t_1$

$y(t - t_1) = x(2 - t + t_1)$

since $y(t, t_1) \neq y(t - t_1)$

system is time varient.

The system is stable as long as $x(t)$ is bounded, $y(t)$ is also bounded.

The system is anti-causal & memory

(iii) $y(t) = \cos(x(t))$

$y_1(t) = \cos(x_1(t)) \quad y_2(t) = \cos(x_2(t))$

$y_3'(t) = \cos[a_1 x_1(t) + a_2 x_2(t)]$

$y_3'(t) = a_1\cos(x_1(t)) + a_2\cos(x_2(t))$

Since $y_3(t) \neq y_3'(t)$

system is non-linear.

$y(t, t) = \cos(x(t - t))$

$y(t - k) = \cos(x(t - k))$

since $y(t, k) = y(t - k)$ system is time invariant

The system is stable. The value of $x(t)$ the cosine for thas bounded value.

The system is memoryless & causal.

(iv) $y(n) = 2x(n)u(n)$

$y_1(n) = 2x_1(n)u(n) \quad y_2(n) = 2x_2(n)u(n)$

$y_3(n) = 2a_1x_1(n)u(n) + 2a_2x_2(n)u(n)$

$= 2a_1y_1(n) + \ldots$

since $y_2(n) = y_2'(n)$ system is linear.

$y(n-k) = 2x(n-k)u(n)$

$y(n-k) = 2x(n-k)u(n-k)$

since $y(n-k) \neq y_2(n-k)$

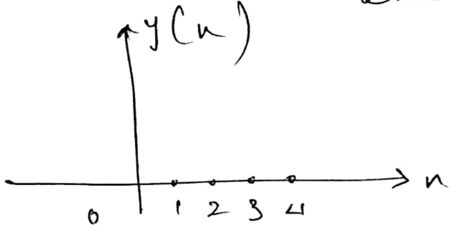
system is time variant.

WKT $u(n)=1$ for $n \geq 0$ $\therefore$ $u(n)$ is a bounded sequence as long as $x(n)$ is bounded $y(n)$ is also bounded.

Hence system is stable.

The system is causal & memoryless.

4 b. let $x(n) = \begin{cases} n & n \text{ odd} \\ 0 & \text{otherwise} \end{cases}$ Determine $y(n) = x(2n)$

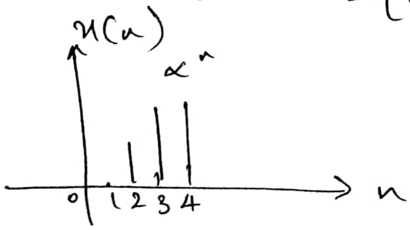


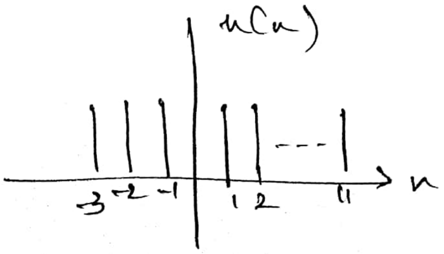
6. Evaluate the discrete convolution sum

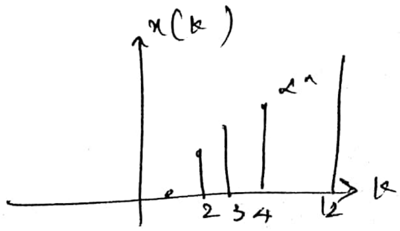
$x(n) = 2^n[u(n-2) - u(n-4)]$

$y(n) = 2[u(n+2) - u(n-12)]$

$y(n) = x(n) * h(n)$







Scanned by CamScanner

8. Evaluate $\dfrac{dy(t)}{dt} + y(t) = 2\dfrac{dx(t)}{dt} + x(t)$

$x(t) = 4e^{-2t}\,u(t) \qquad y(0) = 2$

Sol^n

$s + 1 = 0$

$s = -1$

$y_N(t) = k_1 e^{-t}$

at $y(0) = 2$

$\cancel{y_N(t) =}$ at $y(0) = 2 \rightarrow 2 = k_1$

$\therefore \boxed{y_N(t) = 2e^{-t}}$

$y_F(t) = y_N(t) + y_p(t)$

$x(t) = 4e^{-2t}$

$y_p(t) = Ce^{-2t}$

$y_p'(t) + y_p(t) = 2x'(t) + x(t)$

$-2Ce^{-2t} + Ce^{-2t} = -16e^{-2t} + 4e^{-2t}$

$\boxed{C = 12}$ $\quad y_p(t) = 12e^{-2t}$

$y_F(t) = k_1 e^{-t} + 12e^{-2t}$

$y(0) = 0$

$0 = k_1 + 12 \Rightarrow k_1 = -12$

$\boxed{y_F(t) = -12e^{-t} + 12e^{-2t}}$

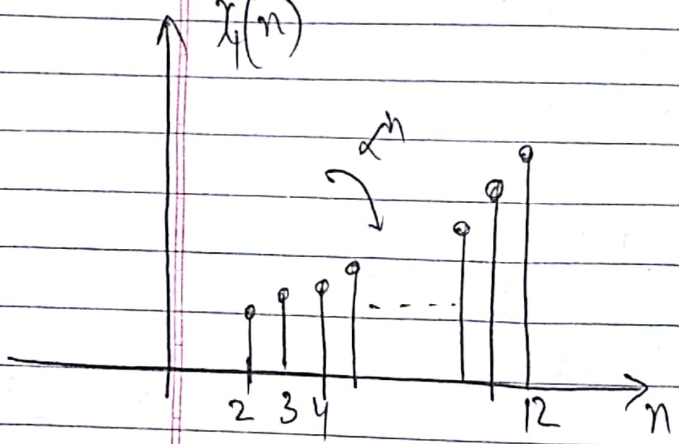
$\therefore$ Total response

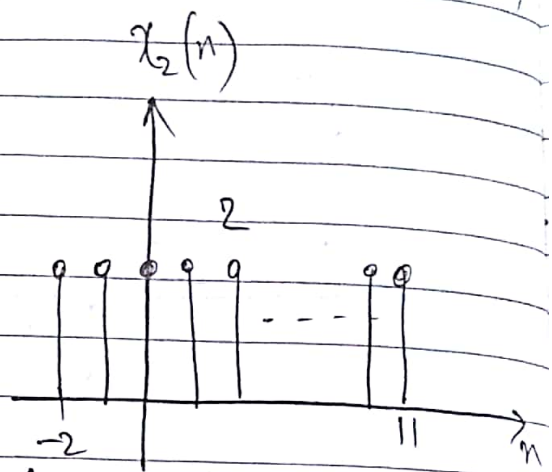
$y(t) = y_N(t) + y_F(t)$

$y(t) = 2e^{-t} - 12e^{-t} + 12e^{-2t}$

6) $y(n) = \alpha^n \{u(n-2) - u(n-13)\} * 2\{u(n+2) - u(n-12)\}$

Sol

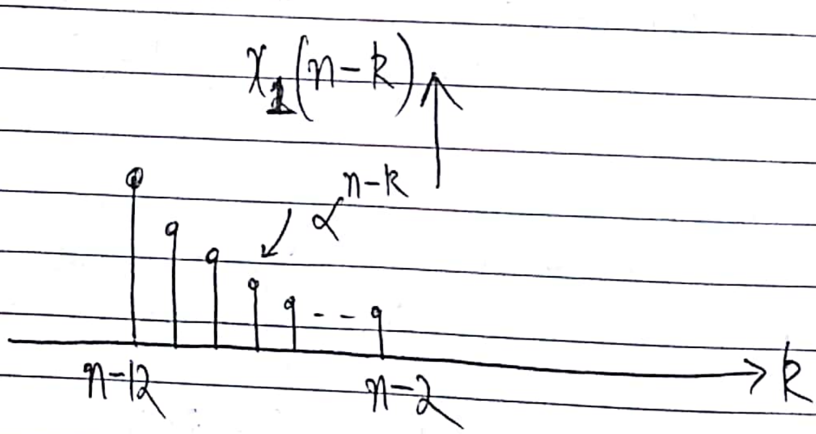




$$y(n) = x_1(n) * x_2(n) = x_2(n) * x_1(n)$$

$$y(n) = \sum_{k=-\infty}^{\infty} x_2(k)\, x_1(n-k) \longrightarrow (1)$$

-12 -2



(i) $n<0$: $y(n) = 0$ [since no overlap]

(ii) $0 \le n < 10$

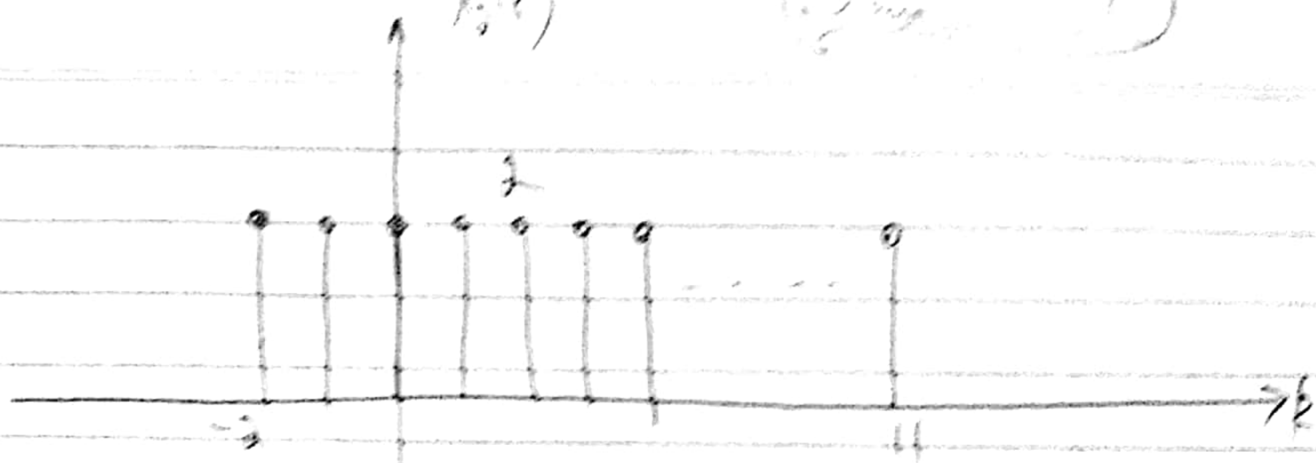
$$y(n) = \sum_{k=-2}^{n-2} 2(\alpha)^{n-k} = 2\alpha^n \sum_{k=-2}^{n-2} (\alpha^{-1})$$

Put $m = k+2$

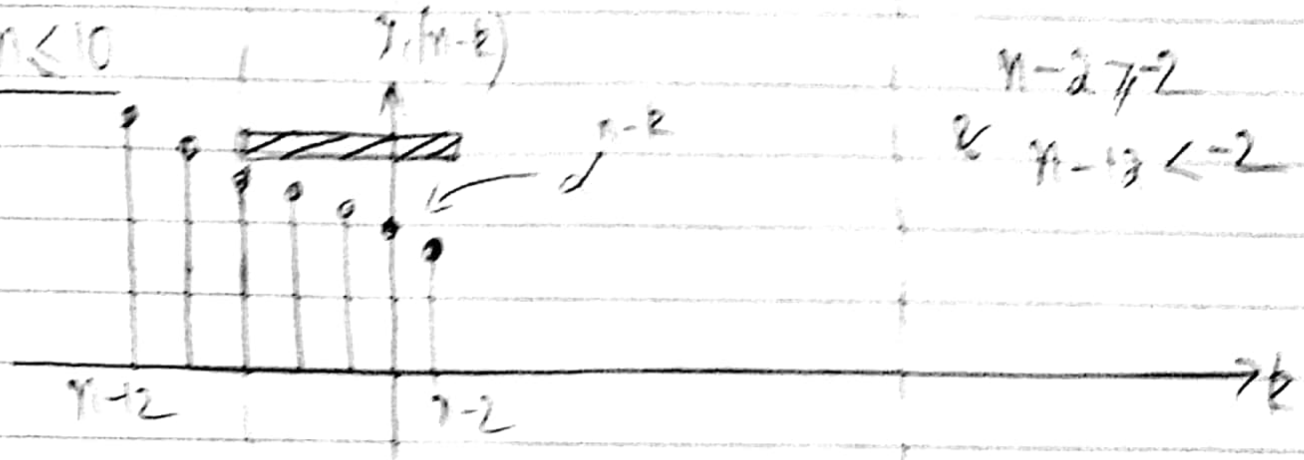
$k=-2$, $m=0$

$k=n-2$, $m = n-2+2 \Rightarrow m=n$

Scanned by CamScanner



ii) $0 \le n < 10$

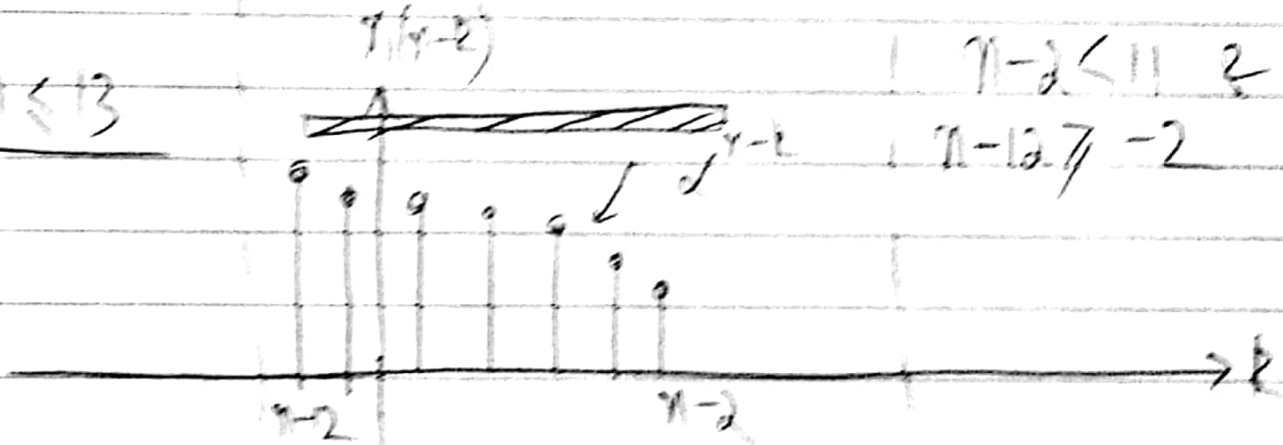


$n-2 \geq -2$

&

$n-13 < -2$

$10 \le n < 13$



$n-2 < 11$ &

$n-12 \geq -2$

$x_1(n-k)$

Date ____ Page ____

$$y(n) = 2\alpha^n \sum_{m=0}^{n} (\alpha^{-1})^{m-2} = 2\alpha^n \alpha^2 \sum_{m=0}^{n} (\alpha^{-1})^m$$

$$y(n) = 2\alpha^{n+2}\left[\frac{1-(\alpha^{-1})^{n+1}}{1-\alpha^{-1}}\right]$$

(iii) $\underline{10 \le n \le 13}$

$$y(n) = \sum_{k=n-12}^{n-2} 2\alpha^{n-k}$$

$$y(n) = 2\alpha^n \sum_{k=n-12}^{n-2} (\alpha^{-1})^k \Rightarrow y(n) = 2\alpha^n \left[\frac{(\alpha^{-1})^{n-12} - (\alpha^{-1})^{n-1}}{1-\alpha^{-1}}\right]$$

$$y(n) = 2\cancel{\alpha^n}\,\cancel{\alpha^{-n}}\,\alpha^{12}\left[\frac{1-\alpha^{-11}}{1-\alpha^{-1}}\right]$$

$m = k+12$

(iv) $\underline{13 < n \le 23}$

$$y(n) = 2\alpha^n \sum_{k=n-12}^{11} (\alpha^{-1})^k = 2\alpha^n \sum_{m=0}^{23} (\alpha^{-1})^k$$

$$y(n) = 2\alpha^n \left[\frac{1-(\alpha^{-1})^{24}}{1-\alpha^{-1}}\right]$$

$$\therefore\quad y(n) = \begin{cases} 0, & n < 0 \\ 2\alpha^{n+2}\left[\dfrac{1-\alpha^{-n-1}}{1-\alpha^{-1}}\right], & 0 \le n \le 10 \\ 2\alpha^{12}\left(\dfrac{1-\alpha^{-11}}{1-\alpha^{-1}}\right), & 10 \le n \le 13 \\ 2\alpha^{n}\left(\dfrac{1-\alpha^{-24}}{1-\alpha^{-1}}\right), & 13 \le n \le 23 \\ 0, & n > 23 \end{cases}$$

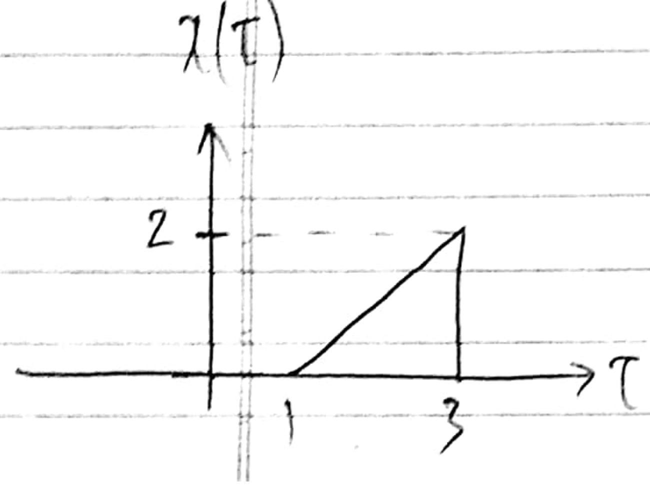
7) $x(t) = (t-1)\left[u(t-1) - u(t-3)\right]$ &

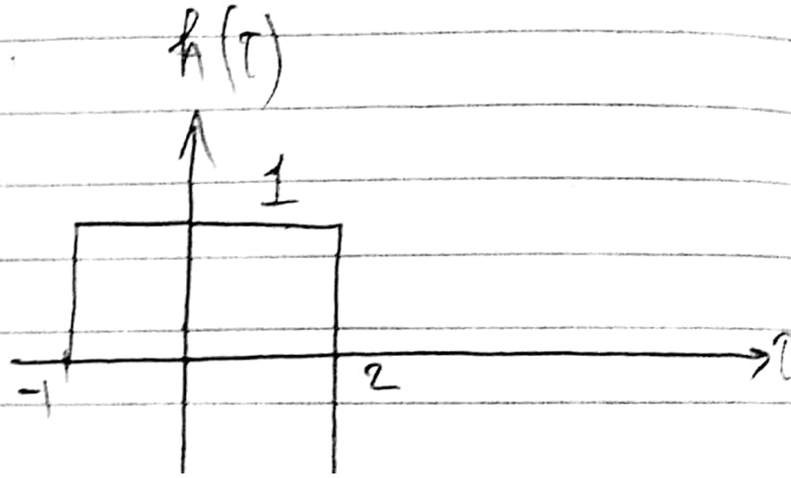
$h(t) = \left[u(t+1) - 2u(t-2)\right]$

Sol

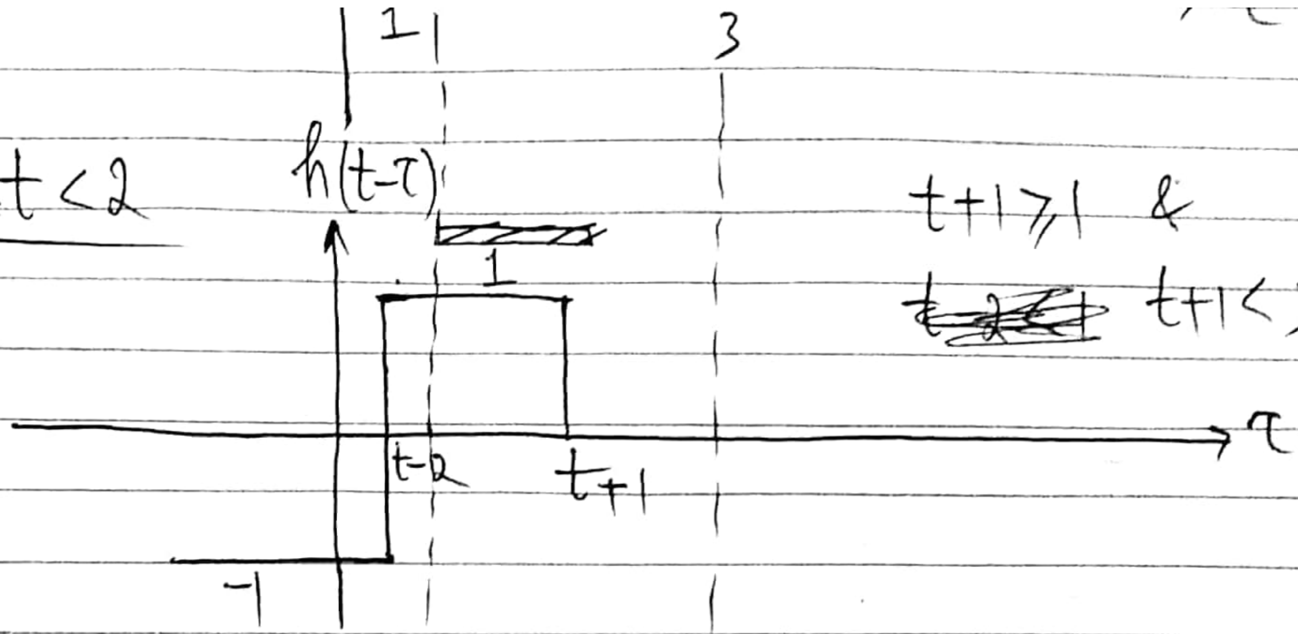
$y(t) = x(t) * h(t)$

$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \longrightarrow (1)$$





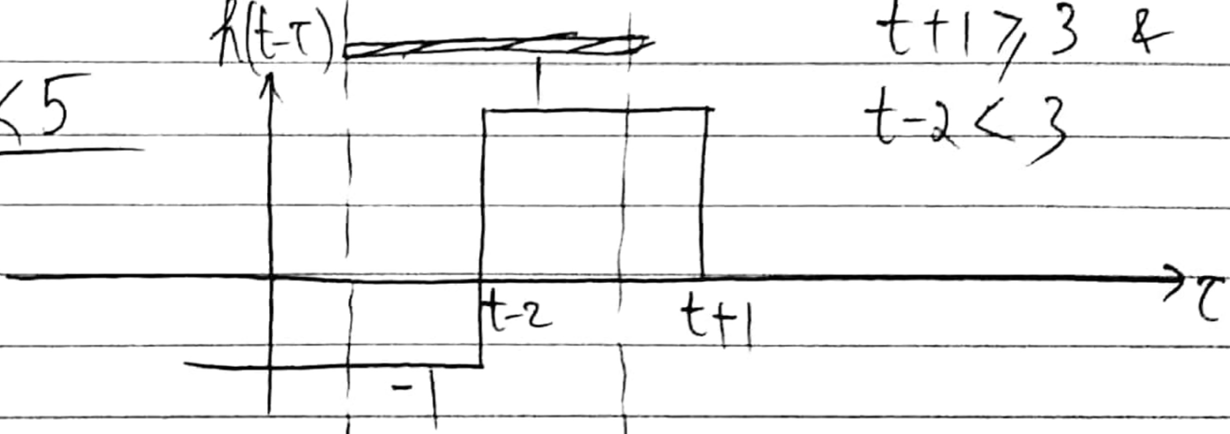
(ii) $\underline{0 \leq t < 2}$



$t+1 \geq 1$ &

$t+1 < 3$

(iii) $\underline{2 \leq t < 5}$



$t+1 \geq 3$ &

$t-2 < 3$

(iv) $\underline{t \geq 5}$

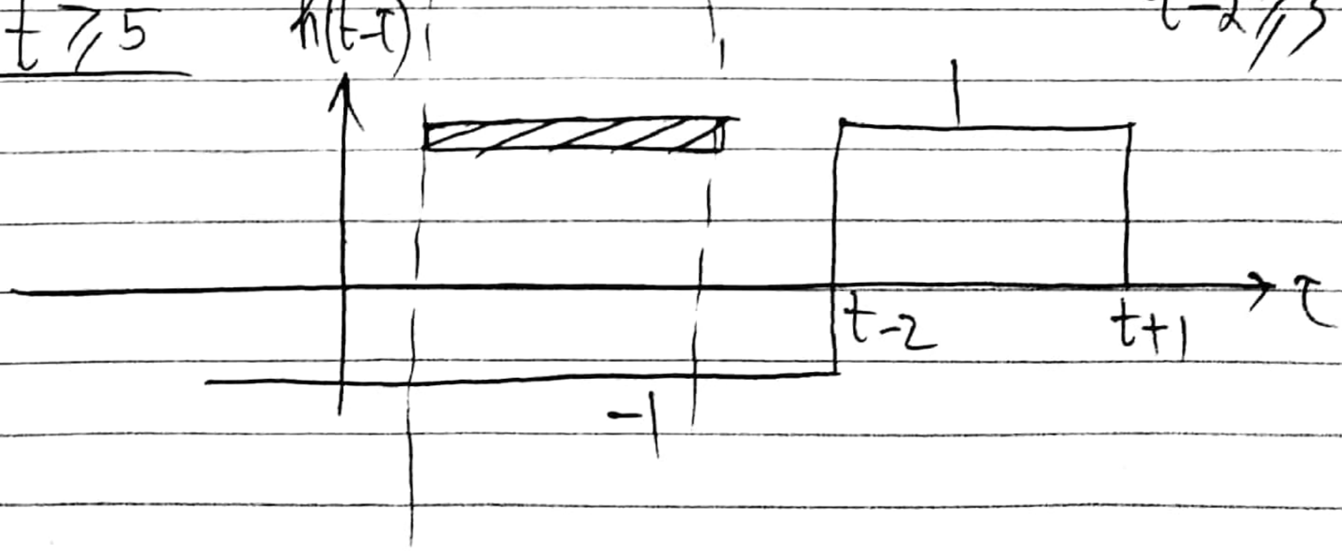
$t-2 \geq 3$

-9

$+3$

-4

1



$$y(t) = \left[\frac{(t+1)^2}{2} - (t+1)\right] - \left[\frac{1}{2} - 1\right]$$

$$y(t) = \frac{t^2+1+2t}{2} - \frac{2t+2}{2} + \frac{1}{2}$$

$$\boxed{y(t) = \frac{t^2}{2}, \quad 0 < t < 2}$$

(ii) $2 < t < 5$

$$y(t) = \int_{t-1}^{2} x(\tau)\,h(t-\tau)\,d\tau = \int_{1}^{t-2} (2-\tau)\,d\tau + \int_{t-2}^{2} (\tau-2)\,d\tau$$

$$y(t) = \left[\cancel{\tau - \frac{\tau^2}{2}}\right]_{1}^{t-2} + \left[\frac{\tau^2}{2} - \tau\right]_{t-2}^{2}$$

$$y(t) = \left[(t-2) - \frac{(t-2)^2}{2}\right] - \left[1 - \frac{1}{2}\right] + \left[\left(\frac{4}{2} - 2\right) - \left(\frac{(t-2)^2}{2} - (t-2)\right)\right]$$

$$= (2t-4) - \frac{t^2-4+4t}{2} - \frac{1}{2} + \left[\frac{2}{2} - \frac{(t^2+4-4t)}{2} + \frac{2t-4}{2}\right]$$

$$y(t) = \frac{-t^2+6t-6}{2}$$

$$y(t) = \left[\frac{-t^2+6t-8-1}{2} + \frac{2-t^2-4+4t-2t+4}{2}\right]$$

$$y(t) = -2t^2-12t$$

$$y(t) = -\left[\frac{\tau^2}{2} - \tau\right]_1^{t-2} + \left[\frac{\tau^2}{2} - \tau\right]_{t-2}^{3}$$

$$y(t) = \left[\tau - \frac{\tau^2}{2}\right]_1^{t-2} + \left[\frac{\tau^2}{2} - \tau\right]_{t-2}^{3}$$

$$y(t) = \left((t-2) - \frac{(t-2)^2}{2}\right) - \left(1 - \frac{1}{2}\right) + \left\{\left[\frac{9}{2} - 3\right] - \left[\frac{(t-2)^2}{2} - (t-2)\right]\right\}$$

$$y(t) = (t-2) - \frac{(t-2)^2}{\cancel{2}} - \frac{1}{2} + \frac{3}{2} - \frac{(t-2)^2}{\cancel{2}} + (t-2)$$

$$y(t) = 2(t-2) - \frac{\cancel{2}(t-2)^2}{\cancel{2}} + 1$$

$$y(t) = 2t - 4 - t^2 - 4 + 4t + 1$$

$$\boxed{y(t) = -t^2 + 6t - 7} \quad 3 \le t \le 5$$

(iv) $t \geq 5$ $\quad y(t) = \int_1^3 (\tau - 1)(-1)\, d\tau = \left[\tau - \frac{\tau^2}{2}\right]_1^3$

$$y(t) = \left(3 - \frac{9}{2}\right) - \left(1 - \frac{1}{2}\right) = \frac{-3}{2} - \frac{1}{2}$$

$$\boxed{y(t) = -2}$$

$$\therefore\ y(t) = \begin{cases} 0 , & t < 0 \\ \dfrac{t^2}{2} , & 0 \le t < 2 \\ -t^2 + 6t - 7 , & 3 \le t < 5 \\ -2 , & t \geq 5 \end{cases}$$