

Internal Test –November 2017

Sub:	High Voltage Engineering						Code:	10EE73	
Date:	08/11/2017	Duration:	90 mins	Max Marks:	50	Sem:	VII	Branch:	EEE
Note: Answer <b>Question 1 and 2</b> . Answer any <b>TWO</b> questions from the rest.									

	Marks	OBE	
		CO	RBT
1. Explain with neat sketches Cockroft-Walton voltage multiplier circuit. Derive the expression for a) high voltage regulation, b) ripple, c) optimum no of stages when the circuit is (i) unloaded (ii) loaded.	[5+5 +5]	CO4	L2
OR			
Draw the circuit and derive the necessary formula for an impulse generator which has eight stages with each condenser rated for 0.16 $\mu$ F and 125 kV. The load capacitor available is 1000 pF. Find the series resistance and the damping resistance needed to produce 1.2/50 $\mu$ s impulse wave. What is the maximum output voltage of the generator, if the charging voltage is 120kV?	[8+7]	CO4	L3

P.T.O

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2.	Write a short notes on any <b>THREE</b> of the following: (a) Chubb-Fortesque Method (b) Trigatron Gap (c) series resistance with micro-ammeter for measuring high DC voltages, (d) Rogowsky coil.	[5+5 +5]	CO5	L1
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4.	Explain with a neat diagram sphere gap method for measuring high voltage. Explain the factors affecting the measurement	[5+5]	CO5	L3
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## Cockroft Walton Voltage Multiplier Circuit.

- ⇒ C and W suggested an improvement over the circuit developed by Greinacher for producing high D.C. voltages.
- ⇒ Fig. shows a multistage single phase cascade circuit.

### No load Operation!

- ⇒ The portion  $ABM'MA$  is exactly identical to Greinacher voltage doubled circuit and the voltage across  $C'$  becomes  $2V_{max}$  when  $M$  attains a voltage  $2V_{max}$ .
- ⇒ During next half cycle when  $B$  becomes +ve w.r.t  $A$ , potential of  $M$  falls,
- ⇒ potential of  $N$  also falls becoming less than potential at  $M'$ , hence  $C_2$  is charged through  $D_2$ .
- ⇒ Next half cycle  $A$  becomes more positive. potential of  $M$  and  $N$  rise thus charging  $C_2'$  through  $D_2$ .
- ⇒ Finally all the capacitors  $C_1', C_2', C_3', C_1, C_2$  and  $C_3$  are charged.

## Generator Loaded :-

- ⇒ When generator is loaded o/p voltage will never reach the value  $2m V_{max}$ .
- ⇒ o/p wave will consist of ripples on the voltage.
- ⇒ Thus, we have to deal with two quantities. the voltage drop  $\Delta V$  and the ripple  $\delta V$ .
- ⇒ Suppose, a charge  $q$  is transferred to the load per cycle.
- ⇒ This charge is  $q = I/f = IT$
- ⇒ The charge comes from smoothing column (series connection of  $C_1, C_2, \dots, C_n$ ).
- ⇒ If no charge were transferred during  $T$  from this stack via  $D_1, D_2, D_3, \dots$  to oscillating column.
- ⇒ then peak to peak ripple would be
- $$2\delta V = IT \sum_{n=0}^{\infty} \frac{1}{C_n T}$$
- But in practice charge is transferred.

⇒ Similarly  $C_1'$  must acquire for stability reasons a charge  $3q$  so that it can supply a charge  $q$  to the load and  $2q$  the capacitor  $C_2$  next transfer half

⇒ During the transfer cycle (fig b) the diodes  $D_1, D_2, D_3$  conduct when  $B$  is positive com.

⇒ Here  $C_2'$  transfers  $q$  charge to  $C_3$ .

$C_1'$      $n$      $2q$      $n$      $C_2$

and transformer provides a charge  $3q$ .

For  $n$ -stage circuit, the total ripple will be

$$2\delta v = \frac{I}{f} \left( \frac{1}{C_1'} + \frac{2}{C_2'} + \frac{3}{C_3'} + \dots + \frac{n}{C_n'} \right)$$

$$\delta v = \frac{I}{2f} \left( \frac{1}{C_1'} + \frac{2}{C_2'} + \frac{3}{C_3'} + \dots + \frac{n}{C_n'} \right)$$

①

⇒ The voltage across the column of capacitors consisting of  $C_1, C_2, C_3$  keeps on oscillating ⇒ Known as **oscillating column**.

⇒ Voltage across  $C_1', C_2'$  and  $C_3'$  remains constant and is known as **smoothing column**.

⇒ The voltages at  $M', N'$  and  $O'$  are  $2V_{max}, 4V_{max}$  and  $6V_{max}$ .

⇒ Therefore voltage across all the capacitors is  $2V_{max}$  except for  $C_1$  ⇒ it is  $V_{max}$  only.

⇒ The total o/p voltage is  $2mV_{max}$ , where  $m$  is the number of stages.

⇒ Use of multistages arranged in the manner enables very high voltage to be obtained.

⇒ The equal stress of the elements (both caps and diodes)

From equation 1) it is clear that in a multistage circuit the lowest caps are responsible for most ripple  $\Rightarrow$  increase cap in lower stages.

$\Rightarrow$  Disadv ! If load is large load voltage goes down smaller caps (within column) will be over stressed.

Generally  $C_n' = C_{n-1}' = \dots = C_1' = C$

$$\therefore \Delta V = \frac{I}{2fc} \times \frac{n(n+1)}{2} = \frac{In(n+1)}{4fc}$$

To evaluate voltage drop  $\Delta V$  which is the difference between theoretical no load voltage  $2nV_{max}$  and on load voltage.

$\Rightarrow$  Here  $C_1'$  is not charged upto full voltage  $2V_{max}$  but only to  $2V_{max} - 3q/c$  because of the charge given up through  $C_1'$  is only in one cycle gives a voltage drop of  $3q/c$   
 $= 3I/fc$ .

$\Rightarrow$  The voltage drop in transformer is assume to be negligible

$$\left\{ 2V_{max} - \frac{3I}{fc} \right\} - \frac{3I}{fc}$$

Since the reduction in voltage across again is  $2I/fC$ .

$\therefore C_2$  attain a voltage.

$$\Delta V_{\max} = \left( \frac{2I + 2I + 2I}{fC} \right)$$

In a three stage generator.

$$\Delta V_1 = \frac{2I}{fC}$$

$$\Delta V_2 = \left\{ 2 \times 3 + (3-1) \right\} \frac{I}{fC}$$

$$\Delta V_3 = \left\{ 2 \times 3 + 2 \times 2 + (2-1) \right\} \times \frac{I}{fC}$$

In general for  $n$ -stage generator

$$\Delta V_n = \frac{nI}{fC}$$

$$\Delta V_{n-1} = \frac{I}{fC} \left\{ 2n + (n-1) \right\}$$

$$\Delta V_{n-2} = \frac{I}{fC} \left\{ 2n + 2(n-1) + (n-2) \right\}$$

$$\Delta V_1 = \frac{I}{fC} \left[ 2n + 2(n-1) + \dots + 2 \times 3 + 2 \right]$$



After omitting I/fc, the series can be rewritten as.

$$T_n = n.$$

$$T_{n-1} = 2n + (n-1).$$

$$T_{n-2} = 2n + 2(n-1) + (n-2).$$

$$T_{n-3} = 2n + 2(n-1) + 2(n-2) + (n-3).$$

⋮

⋮

$$T_1 = 2n + 2(n-1) + 2(n-2) + \dots + 2 \times 3 + 2 \times 2 + 1.$$

$$\therefore T = T_n + T_{n-1} + \dots + T_1.$$

To sum up we add the last term of all the terms ( $T_n$  through  $T_1$ ) and again add the last term of the remaining term and so on. i.e.

$$\left[ n + (n-1) + (n-2) + \dots + 2 + 1 \right],$$

$$+ \left[ 2n + 2(n-1) + 2(n-2) + \dots + 2 \times 2 \right],$$

$$+ \left[ 2n + 2(n-1) + \dots + 2 \times 4 + 2 \times 3 \right].$$

$$+ \left[ 2n + 2(n-1) + \dots + 2 \times 4 \right].$$

$$+ \left[ 2n + 2(n-1) + 2 \times 5 \right] + \dots + \left[ 2n \right].$$

Rearranging the above terms we get

$$n + (n-1) + (n-2) + \dots + 2 + 1.$$

$$+ [2n + 2(n-1) + 2(n-2) + \dots + 2 \times 2 + 2 \times 1] - 2 \times 1.$$

$$+ [2n + 2(n-1) + 2(n-2) + \dots + 2 \times 3 + 2 \times 2 + 2 \times 1] - 2 \times 2.$$

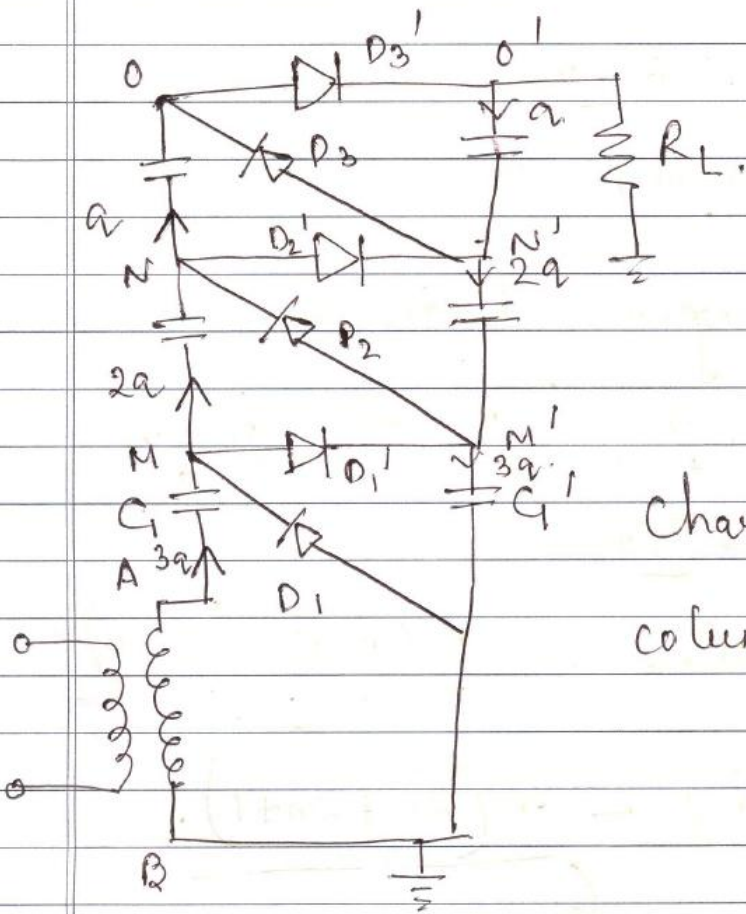
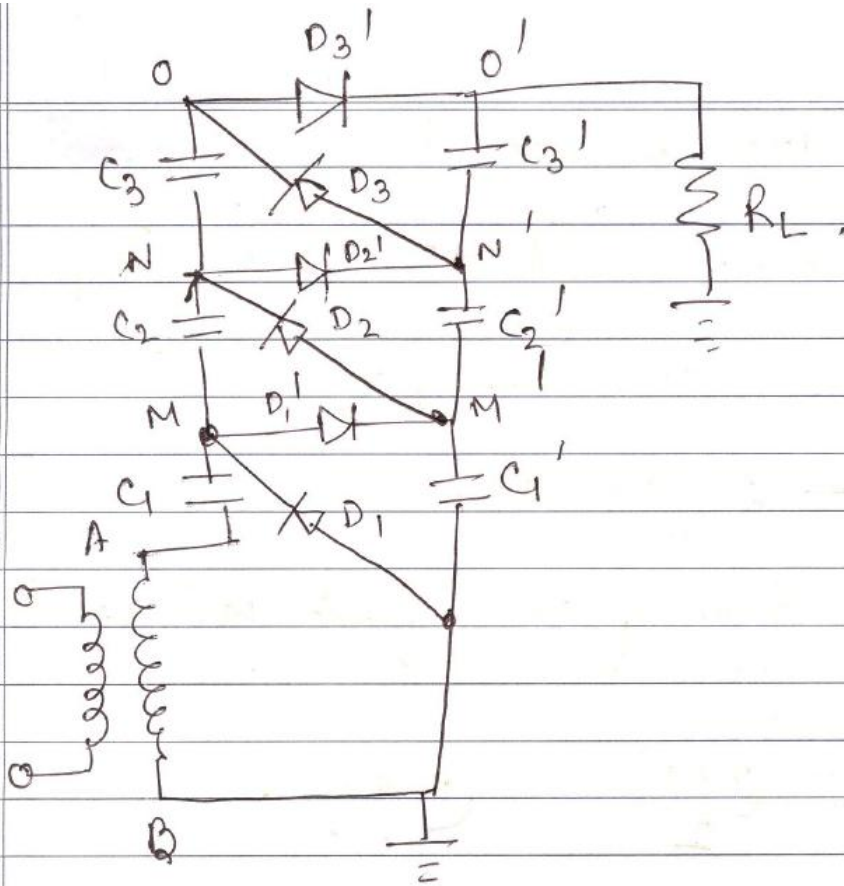
$\vdots$   
 $\vdots$   
 $\vdots$

$$+ [2n + 2(n-1) + 2(n-2) + \dots + 2 \times 2 + 2 \times 1] -$$

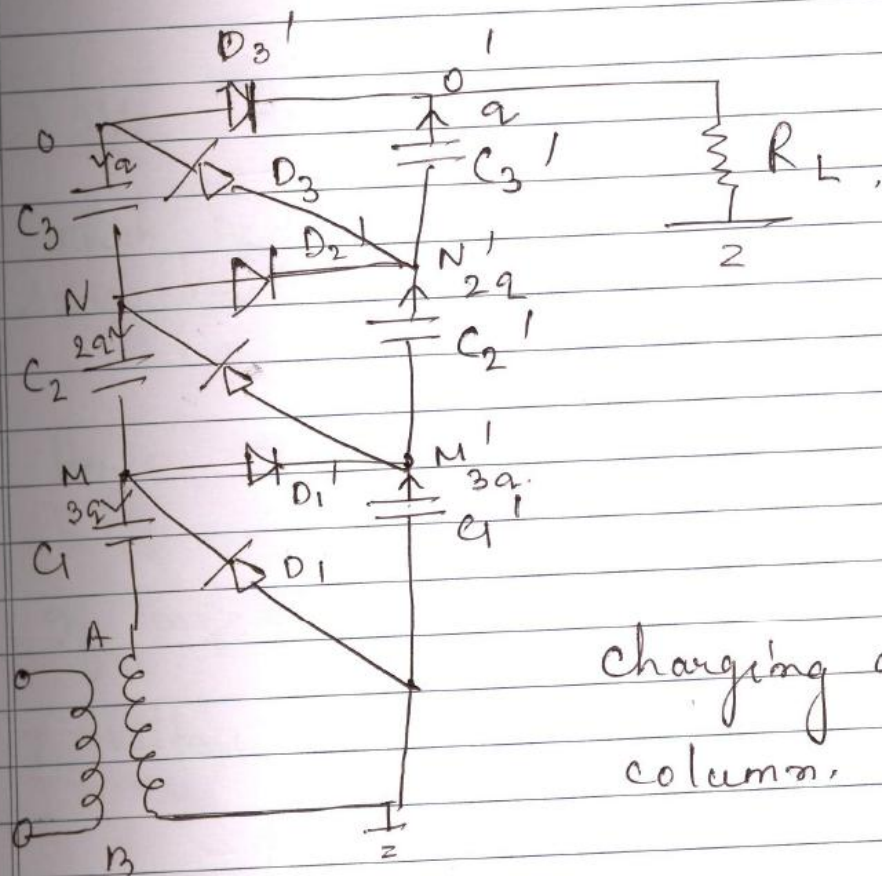
$$[2n + 2(n-1) + 2(n-2) + \dots + 2 \times 2]$$

or  $n + (n-1) + (n-2) + \dots + 2 + 1$

+  $(n-1)$  number of terms of  $[n + (n-1) + \dots + 2 + 1]$   
minus  $2 [1 + (1+2) + (1+2+3) + \dots + \{1+2+3+\dots+(n-1)\}]$



Charging of smoothing column.



charging of oscillating column.

$$= \frac{6n^3 + 6n^2 - 2n^3 - 3n^2 - n}{6}$$

$$= \frac{4n^3 + 3n^2 - n}{6} = \frac{2}{3}n^3 + \frac{n^2}{2} - \frac{n}{6}$$

Here again the lowest capacitor contributes most to the voltage drop  $\Delta V$ .

⇒ Advantageous to increase their capacitance suitable steps.

⇒ Only doubling of  $C$  is convenient as this cap has to withstand only half of voltage of other capacitors.

∴  $\Delta V$ , decreases by an amount  $\frac{nI}{fC}$  which reduces  $\Delta V$  of every stages by the same amount. i.e. by  $\frac{n \times nI}{2fC}$ .

$$\text{Hence } \Delta V = \frac{I}{fC} \left( \frac{2}{3} n^3 - \frac{n}{6} \right).$$

If  $n \geq 4$ , linear term can be neglected

∴ voltage drop can be approximated to

$$\Delta V \approx \frac{I}{fC} \times \frac{2}{3} n^3.$$

The maximum o/p voltage is given by

$$V_{o \max} = 2nV_{max} - \frac{I}{fC} \times \frac{2}{3} n^3.$$

From equation it is clear that for a given number of stages,  $f$  and  $C$  o/p voltage  $V_o$  linearly with load current  $I$ .

for a given load  $V_o = V_{max} - V$  may rise.

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(minimo term)

Date: / /

The last term is rewritten as.

$$2 \left[ 1 + (1+2) + \dots + \left\{ 1+2+3+\dots+(n-1) \right\} + \right. \\ \left. (1+2+3+\dots+n) \right] - 2 [1+2+3+\dots+n].$$

The  $n^{\text{th}}$  term of the first part of the above series is given as:

$$t_n = \frac{2n(n+1)}{2} = (n^2+n).$$

Therefore above terms are equal to.

$$\sum (n^2+n) - 2 \sum n \\ = \sum (n^2-n)$$

$\therefore$  Taking once again all the term.

$$T = \sum n + 2(n-1) \sum n - \sum (n^2-n) \\ = 2n \sum n - \sum n^2 \\ = 2n \cdot \frac{n(n+1)}{2} - \frac{n(n+1)(2n+1)}{6} \\ = \frac{6(n^3+n^2) - n(2n^2+3n+1)}{6}$$

Here again the lowest capacitors contribute most to the voltage drop  $\Delta V$ .

$\Rightarrow$  Advantageous to increase their cap'n suitable steps.

⇒ Only doubling of  $C$  is convenient as this cap has to withstand only half of voltage of other capacitors.

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for a given load  $V_o = V_{\max} - V$  may rise.



initially with number of stages  $n$ , and reaches a maximum value.

but decays beyond an optimum number of stages.

The optimum number of stages assuming a constant  $V_{max}$ ,  $I$ ,  $f$  and  $C$  can be obtained for maximum value of  $V_{omax}$  by differentiating above equation w.r.t  $n$ .

and equating it to zero.

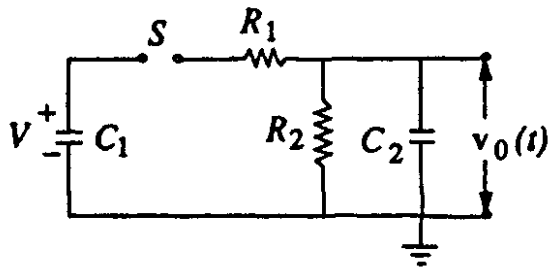
$$\frac{dV_{omax}}{dn} = 2V_{omax} - \frac{2}{\beta} \frac{I}{\delta C} \times \beta n^2 = 0$$

$$\therefore n_{opt} = \sqrt{\frac{V_{omax} f C}{I}}$$

Substituting  $n_{opt}$  in equation

$$\begin{aligned} (V_{omax})_{max} &= \sqrt{\frac{V_{omax} f C}{I}} \left( 2V_{omax} - \frac{2I}{2fC} \right) \\ &= \sqrt{\frac{V_{omax} f C}{I}} \times \frac{4}{3} V_{omax} \end{aligned}$$

It is to be noted that in general it is more economical to use high frequency and small value of capacitance to reduce the  $V$  supply voltage drop rather than low  $f$  and high



For the configuration shown in Fig. 6.15b, the output voltage across  $C_2$  is given

by,  $v_0(t) = \frac{1}{C_2} \int_0^t i_2 dt.$

Performing Laplace transformation,  $\frac{1}{C_2 s} I_2(s) = v_0(s)$

where  $i_2$  is the current through  $C_2$ .

Taking the current through  $C_1$  as  $i_1$  and its transformed value as  $I_1(s)$ ,

$$I_2(s) = \left[ \frac{R_2}{R_2 + \frac{1}{C_2 s}} \right] I_1(s)$$

$$I_1(s) = V(s) \frac{1}{\frac{1}{C_1 s} + R_1 + \frac{R_2 \cdot \frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s}}}$$

where,  $\frac{R_2 \cdot \frac{1}{C_2 s}}{R_2 + \frac{1}{C_2 s}}$  represents the impedance of the parallel combination of  $R_2$  and  $C_2$ .

Substitution of  $I_1(s)$  gives

$$v_0(s) = \frac{1}{C_2 s} \frac{R_2}{R_2 + \frac{1}{C_2 s}} \frac{V}{s} \frac{1}{\frac{1}{C_1 s} + R_1 + \frac{R_2(1/C_2 s)}{R_2 + (1/C_2 s)}}$$

After simplification and rearrangement,

$$v_0(s) = \frac{V}{R_1 C_2} \left[ \frac{1}{s^2 + \left( \frac{1}{C_1 R_1} + \frac{1}{C_2 R_2} + \frac{1}{C_2 R_1} \right) s + \frac{1}{C_1 C_2 R_1 R_2}} \right] \quad (6.18)$$

Hence, the roots of the equation

$$s^2 + \left[ \frac{1}{C_1 R_1} + \frac{1}{C_2 R_2} + \frac{1}{C_2 R_1} \right] s + \frac{1}{C_1 C_2 R_1 R_2}$$

are found from the relations,

$$\alpha + \beta = - \left[ \frac{1}{C_1 R_1} + \frac{1}{C_2 R_2} + \frac{1}{C_2 R_1} \right] \quad (6.19)$$

$$\alpha \beta = \frac{1}{C_1 C_2 R_1 R_2}$$

Taking inverse transform of  $v_0(s)$  gives

$$v_0(t) = \frac{V}{R_1 C_2 (\alpha - \beta)} [\exp(-\alpha t) - \exp(-\beta t)] \quad (6.20)$$

Usually,  $\frac{1}{C_1 R_1}$  and  $\frac{1}{C_2 R_2}$  will be much smaller compared to  $\frac{1}{R_1 C_2}$ .

Hence, the roots may be approximated as

$$\alpha \approx \frac{1}{R_1 C_2} \text{ and } \beta = \frac{1}{R_2 C_1} \quad (6.21)$$

An impulse generator has eight stages with each condenser rated for  $0.16 \mu\text{F}$  and  $125 \text{ kV}$ .

The load capacitor available is  $1000 \text{ pF}$ . Find the series resistance and damping resistance needed to produce  $1.2/50 \mu\text{s}$  impulse wave. What is the maximum o/p voltage of the generator if the charging voltage is  $120 \text{ kV}$ ?

Solution:

$$C_1 = \frac{0.16}{8} = 0.02 \mu\text{F}.$$

$$C_2 = \text{load cap} = 0.001 \mu\text{F}$$

$$t_1, \text{ time to front} = 1.2 \mu\text{s} = 3.0 R_1 \frac{C_1 C_2}{C_1 + C_2}.$$

$$\therefore R_1 = \frac{1.2 \times 10^{-6} \times (C_1 + C_2)}{C_1 C_2} \times \frac{1}{3}.$$

$$= \frac{1.2 \times 10^{-6} \times 0.021 \times 10^{-6}}{0.02 \times 0.001 \times 10^{-12}} \times \frac{1}{3}$$

$$R_1 = 420 \Omega.$$

$$t_2, \text{ time to tail} = 0.7 (R_1 + R_2) (C_1 + C_2).$$

$$50 \times 10^{-6} = 0.7 \times (R_1 + R_2) \times 0.021 \times 10^{-6}$$

$$R_1 + R_2 = \frac{50 \times 10^{-6}}{0.7 \times 0.021 \times 10^{-6}}.$$

$$R_2 = 2981 \Omega.$$

The d-c charging voltage for 8 days

$$V = 8 \times 120 = 960 \text{ kV}$$

max of p voltage

$$\frac{V}{R_1 C_2 (\alpha - \beta)} \left( e^{-\alpha t_1} - e^{-\beta t_1} \right)$$

$$\alpha = \frac{1}{R_1 C_2} = \frac{1}{420 \times 0.001 \times 10^{-6}} = 2.38 \times 10^6$$

$$\beta = \frac{1}{R_2 C_1} = 0.017 \times 10^6$$

→ max voltage

$$\frac{960 \times 2.38 \times 10^6}{(2.38 - 0.017) \times 10^6} \left( e^{-\alpha t_1} - e^{-\beta t_1} \right)$$

$$= 966 (0.057 - 0.979)$$
$$= 891 \text{ kV}$$

2a)

### THE CHUBB-FORTESCUE METHOD

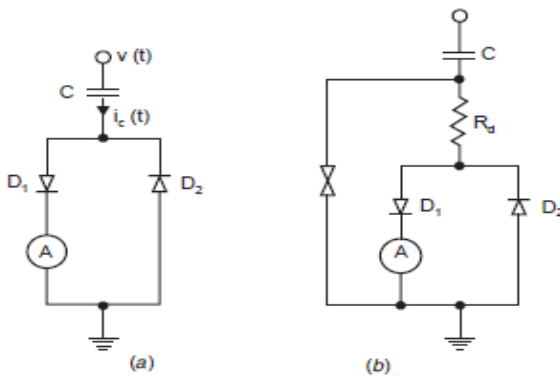
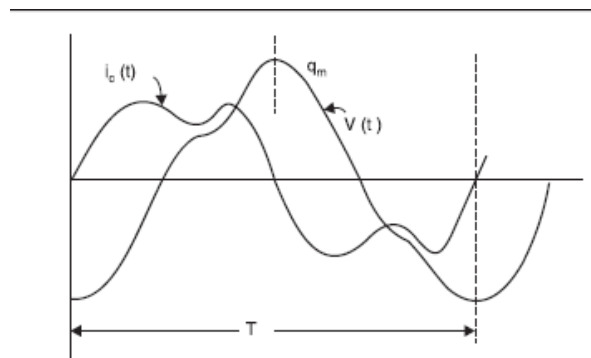


Fig. 4.11 (a) Basic circuit (b) Modified circuit

- Chubb and Fortescue suggested a simple and accurate method of measuring peak value of a.c. voltages.
- The basic circuit consists of a standard capacitor, two diodes and a current integrating ammeter (MC ammeter) as shown in Fig. 4.11 (a).
- The displacement current  $i_c(t)$ , Fig. 4.12 is given by the rate of change of the charge and hence the voltage  $V(t)$  to be measured flows through the high voltage capacitor  $C$  and is subdivided into positive and negative components by the back to back connected diodes.
- The voltage drop across these diodes can be neglected (1 V for Si Diodes) as compared with the voltage to be measured.
- The measuring instrument (M.C. ammeter) is included in one of the branches.
- The ammeter reads the mean value of the current.
- The relation is similar to the one obtained in case of generating voltmeters.

- An increased current would be obtained if the current reaches zero more than once during one half cycle.
- This means the wave shapes of the voltage would contain more than one maxima per half cycle.
- The standard a.c. voltages for testing should not contain any harmonics and, therefore, there could be very short and rapid voltages caused by the heavy pre-discharges, within the test circuit which could introduce errors in measurements.
- To eliminate this problem filtering of a.c. voltage is carried out by introducing a damping resistor in between the capacitor and the diode circuit, Fig.
- Also, if full wave rectifier is used instead of the half wave as shown in Fig. 4.11, the factor 2 in the denominator of the above equation should be replaced by 4.
- Since the frequency  $f$ , the capacitance  $C$  and current  $I$  can be measured accurately, the measurement of symmetrical a.c. voltages using Chubb and Fortescue method is quite accurate and it can be used for calibration of other peak voltage measuring devices.

$$I = \frac{1}{T} \int_{t_1}^{t_2} C \frac{dv(t)}{dt} \cdot dt = \frac{C}{T} \cdot 2V_m = 2V_m fC \text{ or } V_m = \frac{I}{2fC}$$



- Fig. 4.13 shows a digital peak voltage measuring circuit.
- the rectified current is not measured directly, instead a proportional analog voltage signal is derived which is then converted into a Proportional medium frequency for using a voltage to frequency convertor (Block A in Fig. 4.13).
- The frequency ratio  $f_m/f$  is measured with a gate circuit controlled by the a.c. power frequency (supply frequency  $f$ ) and a counter that opens for an adjustable number of period  $\Delta t = p/f$ .
- The number of cycles  $n$  counted during this interval is

$$n = \Delta t f_m = \frac{p}{f} f_m$$

where  $p$  is a constant of the instrument.

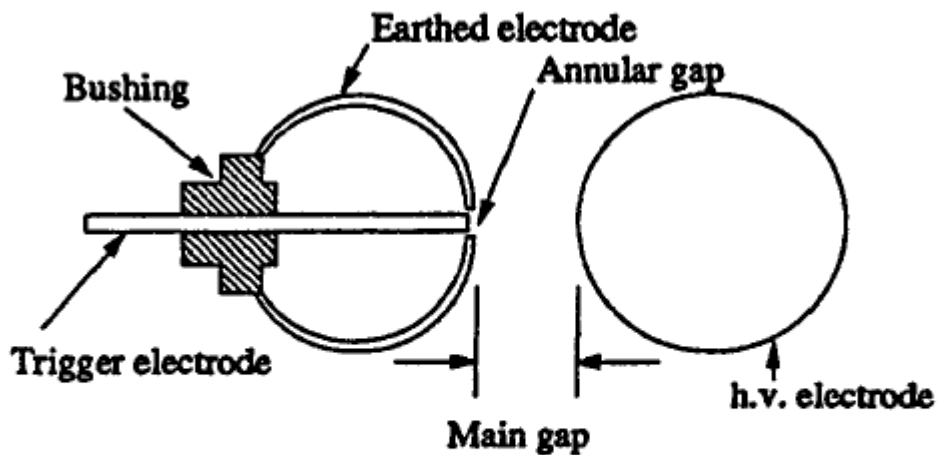
Let 
$$A = \frac{f_m}{Ri_c} = \frac{f_m}{R2V_m fC} = \frac{f_m}{f} \cdot \frac{1}{2RV_m C}$$

Therefore, 
$$n = p 2ARV_m C$$

where  $A$  represents the voltage to frequency conversion factor.

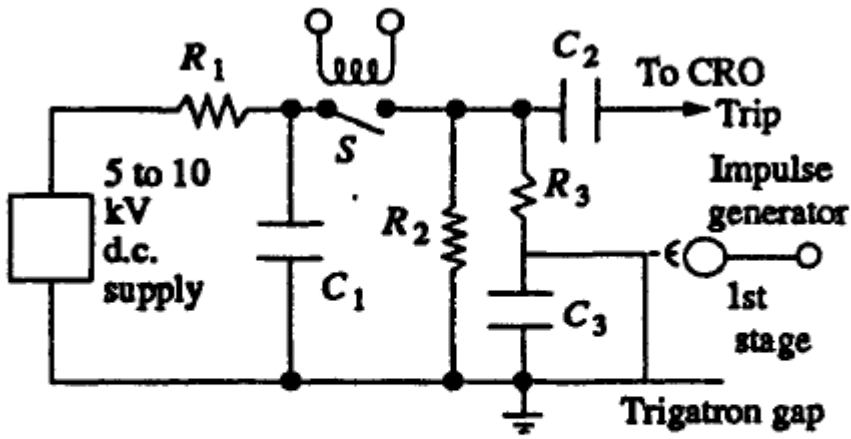
- Thus the indicator can be calibrated to read  $V_m$  directly by selecting suitable values of  $A$ ,  $p$  and  $R$ .
- The voltmeter is found to give an accuracy of 0.35%.

- The three electrode gap requires larger space and an elaborate construction.
- Now-a-days a **trigatron gap** shown in Fig. 6,24 is used, and this requires much smaller voltage for operation compared to the three electrode gap.
- A trigatron gap consists of a high voltage spherical electrode of suitable size, an earthed main electrode of spherical shape, and a trigger electrode through the main electrode.
- The trigger electrode is a metal rod with an annular clearance of about 1 mm fitted into the main electrode through a bushing.
- The trigatron is connected to a pulse circuit as shown in Fig. 6.24b.
- Tripping of the impulse generator is affected by a trip pulse which produces a spark between the trigger electrode and the earthed sphere.
- Due to space charge effects and distortion of the field in the main gap, sparkover of the main gap occurs.
- The trigatron gap is polarity sensitive and a proper polarity pulse should be applied for correct operation



**Fig. 6.24 (a) Trigatron gap**





(b) Tripping circuit using a trigatron

Fig. 6.24 Trigatron gap and tripping circuit

2c)

High Ohmic Series Resistance with Microammeter

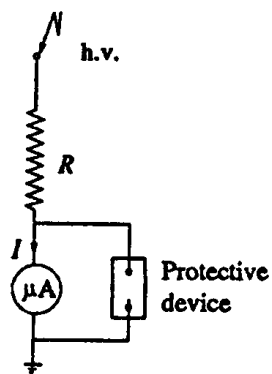


Fig. 7.1 Series resistance micrometer

- High d.c. voltages are usually measured by connecting a very high resistance (few hundreds of megaohms) in series with a microammeter as shown in Fig. 7. 1 .

- Only the current  $I$  flowing through the large calibrated resistance  $R$  is measured by the moving coil microammeter.
  - The voltage of the source is given by
  - The voltage drop in the meter is negligible, as the impedance of the meter is only few ohms compared to few hundred mega-ohms of the series resistance  $R$ .
  - A protective device like a paper gap, a neon glow tube, or a zener diode with a suitable series resistance is connected across the meter as a protection against high voltages in case the series resistance  $R$  fails or flashes over.
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  - A protective device like a paper gap, a neon glow tube, or a zener diode with a suitable series resistance is connected across the meter as a protection against high voltages in case the series resistance  $R$  fails or flashes over.
  - **The limitations** in the series resistance design are:
    - (i) power dissipation and source loading,
    - (ii) temperature effects and long time stability,
    - (iii) voltage dependence of resistive elements, and
    - (iv) sensitivity to mechanical stresses.
- Series resistance meters are built for 500 kV d.c. with an accuracy better than 0.2%.

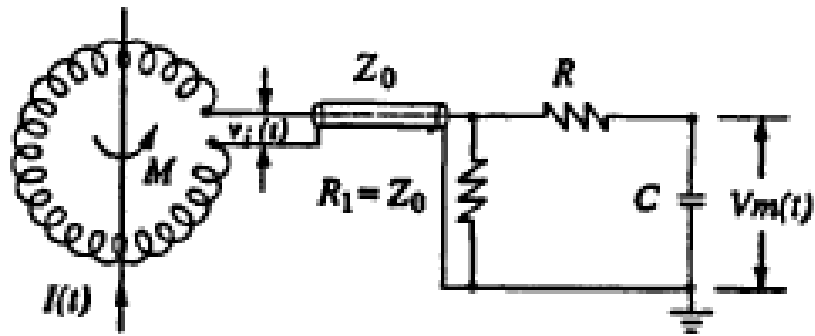
## 2d) Rogowski Colls:

- If a coil is placed surrounding a current carrying conductor, the voltage signal induced in the coil is

$$v_i(t) = M dI(t)/dt$$

- Where  $M$  is the mutual inductance between the conductor and the coil, and  $I$  is the current flowing in the conductor.
- Usually, the **coil is wound on a nonmagnetic former of toroidal shape** and is **coaxially placed surrounding the current carrying conductor**.
- The **number of turns** on the coil is chosen to be **large**, to get enough signal induced.
- The coil is wound cross-wise to reduce the leakage inductance.
- Usually an integrating circuit (see Fig. 7.52) is employed to get the output signal voltage proportional to the current to be measured.
- The output voltage is given by

$$V_m(t) = \frac{1}{CR} \int_0^t v_i(t) dt = \frac{M}{CR} I(t)$$



- $V_i(t)$  — Induced voltage in the coil =  $M \frac{d[i(t)]}{dt}$   
 $Z_0$  — Coaxial cable of surge impedance  $Z_0$   
 $R$ - $C$  — Integrating network

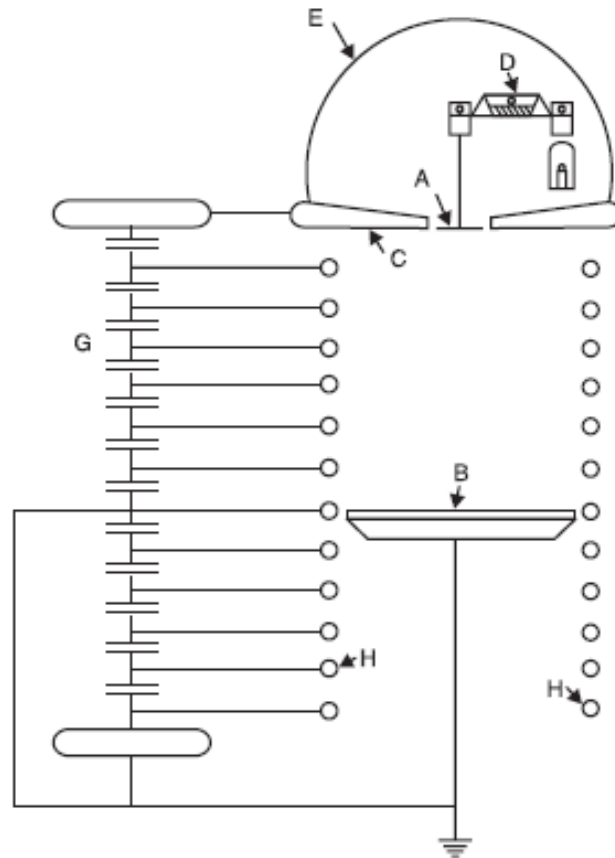
### Rogowski coil for high impulse current measurements

- Rogowski coils with electronic or active integrator circuits have large bandwidths (about 100 MHz).
- At frequencies greater than 100 MHz the response is affected by the skin effect, the capacitance distributed per unit length along the coil, and due to the electromagnetic interferences.

However, miniature probes having nanosecond response time are made using very few turns of copper strips for UHF measurements.

3.

- D/E –Metal Dome
- M/A-mounting Plate
- G/C –guard plate
- P/B –Fixed Plate
- H- Guard loop or ring
- B/D-Balance
- C/G-Capacitance Divider
- W Balancing Weight



**Schematic diagram of electrostatic voltmeter**

- Fig. shows a schematic diagram of an absolute electrostatic voltmeter.
  - The hemispherical metal dome *D* encloses a sensitive balance *B* which measures the force of attraction between the movable disc which hangs from one of its arms and the lower plate *P*.
  - The movable electrode *M* hangs with a clearance of above 0.01 cm, in a central opening in the upper plate which serves as a guard ring.
  - The diameter of each of the plates is 1 metre.
  - Light reflected from a mirror carried by the balance beam serves to magnify its motion and to indicate to the operator at a safe distance when a condition of equilibrium is reached.
  - As the spacing between the two electrodes is large (about 100 cms for a voltage of about 300 kV), the uniformity of the electric field is maintained by the guard rings *G* which surround the space between the discs *M* and *P*.
  - The guard rings *G* are maintained at a constant potential in space by a capacitance divider ensuring a uniform spatial potential distribution.
  - When voltages in the range 10 to 100 kV are measured, the accuracy is of the order of 0.01 per cent.
- 
- **Principle:** The electric field according to Coulomb is the field of forces. The electric field is produced by voltage and, therefore, if the **field force** could be measured, the **voltage** can also be measured.
  - The voltmeters are used for the measurement of high a.c. and d.c. voltages. The measurement of voltages lower than about 50 volt is, however, not possible, as the forces become too small.
  - When a voltage is applied to a parallel plate electrode arrangement, an electric field is set up between the plates.
  - It is possible to have uniform electric field between the plates with suitable arrangement of the plates.
  - The field is uniform, normal to the two plates and directed towards the negative plate.

- If  $A$  is the area of the plate and  $E$  is the electric field intensity between the plates  $\epsilon$  the permittivity of the medium between the plates, we know that the energy density of the electric field between the plates is given as,

$$W_d = \frac{1}{2} \epsilon E^2$$

Consider a differential volume between the plates and parallel to the plates with area  $A$  and thickness  $dx$ , the energy content in this differential volume  $A dx$  is

$$dW = W_d A dx = \frac{1}{2} \epsilon E^2 A dx$$

Now force  $F$  between the plates is defined as the derivative of stored electric energy along the field direction i.e.,

$$F = \frac{dW}{dx} = \frac{1}{2} \epsilon E^2 A$$

Now  $E = V/d$  where  $V$  is the voltage to be measured and  $d$  the distance of separation between the plates. Therefore, the expression for force

$$F = \frac{1}{2} \epsilon \frac{V^2 A}{d^2}$$

Since the two plates are oppositely charged, there is always force of attraction between the plates. If the voltage is time dependant, the force developed is also time dependant. In such a case the mean value of force is used to measure the voltage. Thus

$$F = \frac{1}{T} \int_0^T F(t) dt = \frac{1}{T} \int \frac{1}{2} \epsilon \frac{V^2(t)}{d^2} A dt = \frac{1}{2} \frac{\epsilon A}{d^2} \cdot \frac{1}{T} \int V^2(t) dt = \frac{1}{2} \epsilon A \frac{V_{rms}^2}{d^2}$$

- Electrostatic voltmeters measure the force based on the above equations and are arranged such that one of the plates is rigidly fixed whereas the other is allowed to move.
- With this the electric field gets disturbed.
- For this reason, the movable electrode is allowed to move by not more than a fraction of a millimetre to a few millimetres even for high voltages so that the change in electric field is negligibly small.
- As the force is proportional to square of  $V_{rms}$ , the meter can be used both for a.c. and d.c. voltage measurement
- The force developed between the plates is sufficient to be used to measure the voltage.

- Various designs of the voltmeter have been developed which differ in the construction of electrode arrangement and in the use of different methods of restoring forces required to balance the electrostatic force of attraction.
- Some of the methods are
  - (i) *Suspension of moving electrode on one arm of a balance.*
  - (ii) *Suspension of the moving electrode on a spring.*
  - (iii) *Pendulous suspension of the moving electrode.*
  - (iv) *Torsional suspension of moving electrode.*
- The small movement is generally transmitted and amplified by electrical or optical methods.
- If the electrode movement is minimised and the field distribution can exactly be calculated, the meter can be used for absolute voltage measurement as the calibration can be made in terms of the fundamental quantities of length and force
- for a given voltage to be measured, the higher the force, the greater is the precision that can be obtained with the meter.
- to achieve higher force for a given voltage, the area of the plates should be large, the spacing between the plates ( $d$ ) should be small and some dielectric medium other than air should be used in between the plates.
- If uniformity of electric field is to be maintained an increase in area  $A$  *must be accompanied by an increase* in the area of the surrounding guard ring and of the opposing plate and the electrode may, therefore, become unduly large specially for higher voltages.
- Similarly the gap length cannot be made very small as this is limited by the breakdown strength of the dielectric medium between the plates.
- If air is used as the medium, gradients upto 5 kV/cm have been found satisfactory.
- 

For higher gradients vacuum or SF<sub>6</sub> gas has been used

- **advantage** of the electrostatic voltmeter is its extremely low loading effect as only electric fields are required to be set up.
- Because of high resistance of the medium between the plates, the active power loss is negligibly small.
- The voltage source loading is, therefore, limited only to the reactive power required to charge the instrument capacitance which can be as low as a few picofarads for low voltage voltmeters.
- The measuring system as such does not put any upper limit on the frequency of supply to be measured.
- However, as the load inductance and the measuring system capacitance form a series resonance circuit, a limit is imposed on the frequency range.
- For low range voltmeters, the upper frequency is generally limited to a few MHz

3or

### GENERATING VOLTMETER

- When the source loading is not permitted or when direct connection to the high voltage source is to be avoided, the generating principle is employed for the measurement of high voltages
- **Principle:** A generating voltmeter is a variable capacitor electrostatic voltage generator which generates current proportional to the voltage to be measured.
- Similar to electrostatic voltmeter the generating voltmeter provides loss free measurement of d.c. and a.c. voltages.

- The device is driven by an external constant speed motor and does not absorb power or energy from the voltage measuring source.
- The principle of operation explained with the help of Fig.
- **Construction:**
- H is a high voltage electrode
- the earthed electrode is subdivided into a sensing or pick up electrode P, a guard electrode G and a movable electrode M, all of which are at the same potential.
- The high voltage electrode H develops an electric field between itself and the electrodes P, G and M.
- The field lines are shown in Fig. 4.8.
- The electric field density  $\sigma$  is also shown.
- If electrode M is fixed and the voltage  $V$  is changed, the field density  $\sigma$  would change and thus a current  $i(t)$  would flow between P and the ground.

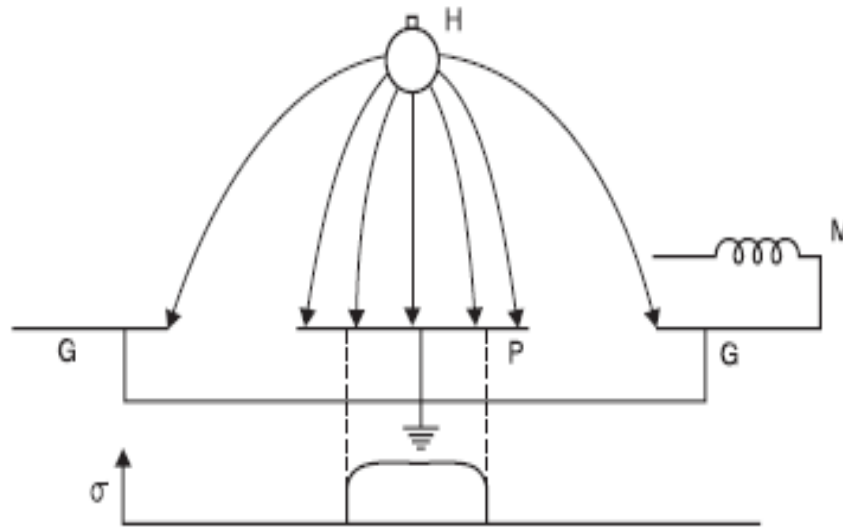


Fig. 4.8 Principle of generating voltmeter

$$i(t) = \frac{dq(t)}{dt} = \frac{d}{dt} \left[ \int \sigma(a) da \right]$$

- Where  $\sigma(a)$  is the electric field density or charge density along some path and is assumed constant over the differential area  $da$  of the pick up electrode.
- In this case  $\sigma(a)$  is a function of time also and  $\int da$  the area of the pick up electrode P exposed to the electric field.
- If the voltage  $V$  to be measured is constant (d.c voltage), a current  $i(t)$  will flow only if it is moved i.e. now  $\sigma(a)$  will not be function of time but the charge  $q$  is changing because the area of the pick up electrode exposed to the electric field is changing.
- The current  $i(t)$  is given by

$$i(t) = \frac{d}{dt} \int_{A(t)} \sigma(a) da = \epsilon \frac{d}{dt} \int_{A(t)} E(a) da$$

where  $\sigma(a) = \epsilon E(a)$  and  $\epsilon$  is the permittivity of the medium between the high voltage electrode and the grounded electrode.

The integral boundary denotes the time varying exposed area.

The high voltage electrode and the grounded electrode in fact constitute a capacitance system.

The capacitance is, however, a function of time as the area  $A$  varies with time and, therefore, the charge  $q(t)$  is given as

$$q(t) = C(t)V(t)$$

$$i(t) = \frac{dq}{dt} = C(t) \frac{dV(t)}{dt} + V(t) \frac{dC(t)}{dt}$$

For d.c. voltages  $\frac{dV(t)}{dt} = 0$

Hence  $i(t) = V \frac{dC(t)}{dt}$

If the capacitance varies linearly with time and reaches its peak value  $C_m$  in time  $T_c/2$  and again reduces to zero linearly in time  $T_c/2$ , the capacitance is given as

$$C(t) = 2 \frac{C_m}{T_c} t$$

For a constant speed of  $n$  rpm of synchronous motor which is varying the capacitance, time  $T_c$  is given by  $T_c = 60/n$ .

Therefore  $I = 2C_m V \frac{n}{60} = \frac{n}{30} C_m V$

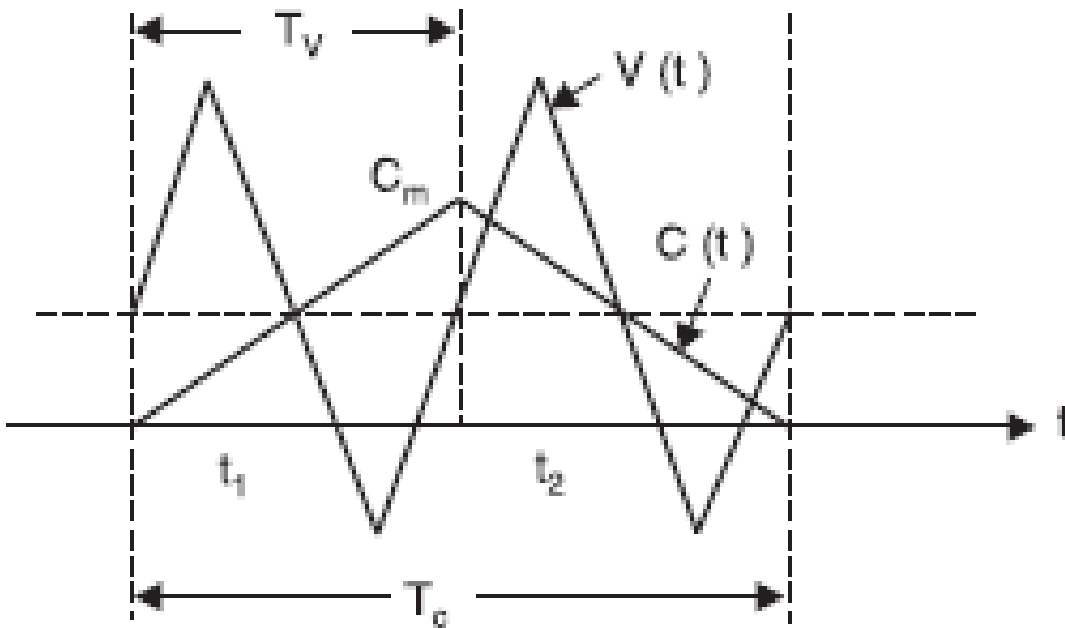
If the capacitance  $C$  varies sinusoidally between the limits  $C_0$  and  $(C_0 + C_m)$  then

$$C = C_0 + C_m \sin \omega t$$

and the current  $i$  is then given as

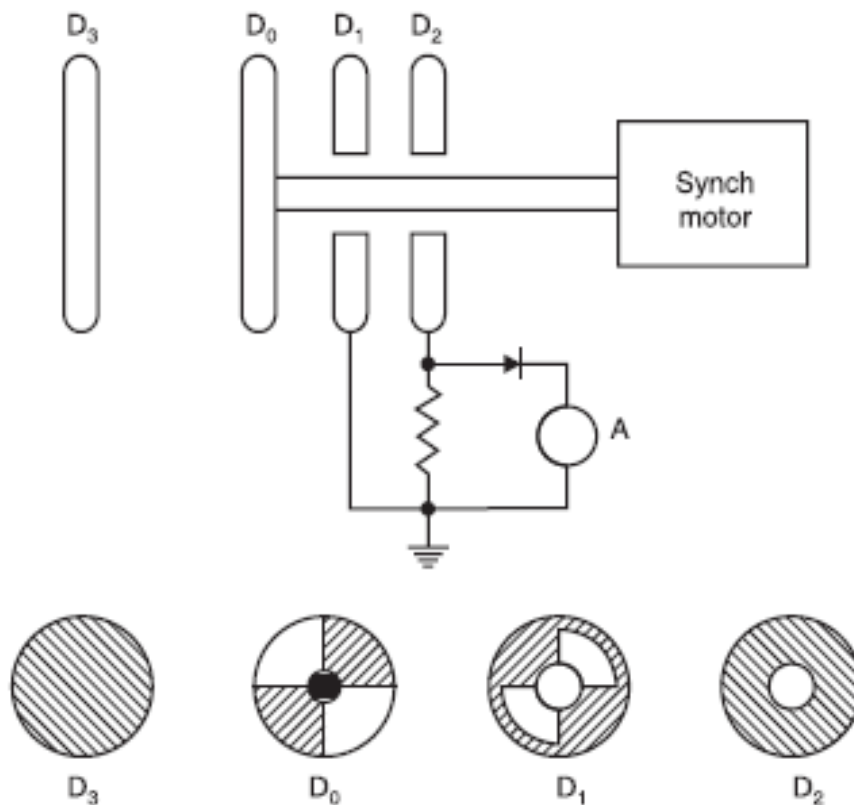
$$i(t) = i_m \cos \omega t \text{ where } i_m = VC_m \omega$$





## Capacitance and voltage variation

- $\omega$  is the angular frequency of variation of the capacitance.
- If  $\omega$  is constant the current measured is proportional to the voltage being measured.
- Generally the current is rectified and measured by a moving coil meter.
- Generating voltmeters can be used for a.c. voltage measurement also provided the angular frequency  $\omega$  is the same or equal to half that of the voltage being measured.
- Fig 4.9 shows the variations of  $C$  as a function of time together with a.c. voltage, the frequency of which is twice the frequency of  $C(t)$ .
- whatever be the phase relation between voltage and the capacitance, over one cycle variation of the voltage is same (e.g.  $V(t_1) - V(t_2)$ )
- the rate of change of capacitance over the period  $T_v$  is equal to  $C_m/T_v$ . Therefore, the instantaneous value of current  $i(t) = C_m f_v V(t)$
- where  $f_v = 1/T_v$  the frequency of voltage.  
 Since  $f_v = 2f_c$  and  $f_c = 60/n$  we obtain  
 $I(t) = n/30 C_m V(t)$



**Schematic diagram of generating voltmeter**

- Fig. shows a schematic diagram of a generating voltmeter which employs rotating vanes for variation of capacitance.
- The high voltage electrode is connected to a disc electrode  $D_3$  which is kept at a fixed distance on the axis of the other low voltage electrodes  $D_2$ ,  $D_1$ , and  $D_0$ .
- The rotor  $D_0$  is driven at a constant speed by a synchronous motor at a suitable speed.
- The rotor vanes of  $D_0$  cause periodic change in capacitance between the insulated disc  $D_2$  and the high voltage electrode  $D_3$ .
- The number and shape of vanes are so designed that a suitable variation of capacitance (sinusoidal or linear) is achieved.
- The a.c. current is rectified and is measured using moving coil meters.

If the current is small an amplifier may be used before the current is measured

- Generating voltmeters are linear scale instruments and applicable over a wide range of voltages.
- The sensitivity can be increased by increasing the area of the pick up electrode and by using amplifier circuits.
- **Advantages**
- (i) scale is linear and can be extrapolated
- (ii) source loading is practically zero
- (iii) no direct connection to the high voltage electrode.

### **Limitations of Generating Voltmeters**

- They require calibration,
- careful construction is needed and is a cumbersome instrument requiring an auxiliary drive, and
- disturbance in position and mounting of the electrodes make the calibration invalid.
- The breakdown of insulating materials depends upon the magnitude of voltage applied and the time of application of voltage.

if the peak value of voltage is large as compared to breakdown strength of the insulating material, the disruptive discharge phenomenon is in general caused by the instantaneous maximum field gradient stressing the material.

Various methods discussed so far can measure peak voltage a more convenient though less accurate method: the use of a testing transformer wherein the output voltage is measured and recorded and the input voltage is obtained by multiplying the output voltage by the transformation ratio.

output voltage depends upon the loading of the secondary winding and wave shape variation is caused by the transformer impedances and hence this method is unacceptable for peak voltage measurements.

#### 4. MEASUREMENT OF HVAC AND HVDC AND IMPULSE VOLTAGE USING SPHERE GAP

- Sphere gap is by now considered as one of the standard methods for the measurement of peak value of d.c., a.c. and impulse voltages and is used for checking the voltmeters and other voltage measuring devices used in high voltage test circuits.
- Two identical metallic spheres separated by certain distance form a sphere gap.
- The sphere gap can be used for measurement of impulse voltage of either polarity provided that the impulse is of a standard wave form and has wave front time at least 1 micro sec. and wave tail time of 5 micro sec.
- **the gap length between the sphere should not exceed a sphere radius.**
- If these conditions are satisfied and the specifications regarding the shape, mounting, clearances of the spheres are met, the results obtained by the use of sphere gaps are reliable to within  $\pm 3\%$ .

in standard specification that in places where the availability of ultraviolet radiation is low, irradiation of the gap by radioactive or other ionizing media should be used when voltages of magnitude less than 50 kV are being measured or where higher voltages with accurate results are to be obtained

##### Specifications on Spheres and Associated Accessories

- The spheres should be so made that their surfaces are smooth and their curvatures as uniform as possible.
- For smaller size, the spheres are placed in horizontal configuration whereas large sizes (diameters), the spheres are mounted with the axis of the sphere gaps vertical and the lower sphere is grounded.
- In either case, it is important that the spheres should be so placed that the space between spheres is free from external electric fields and from bodies which may affect the field between the spheres

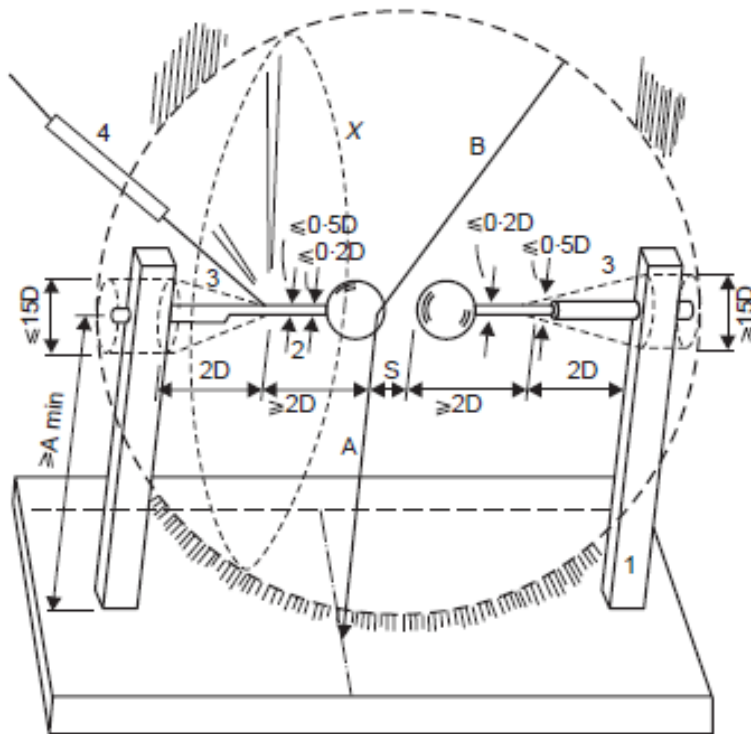


Fig. 4.1

Fig. 4.1

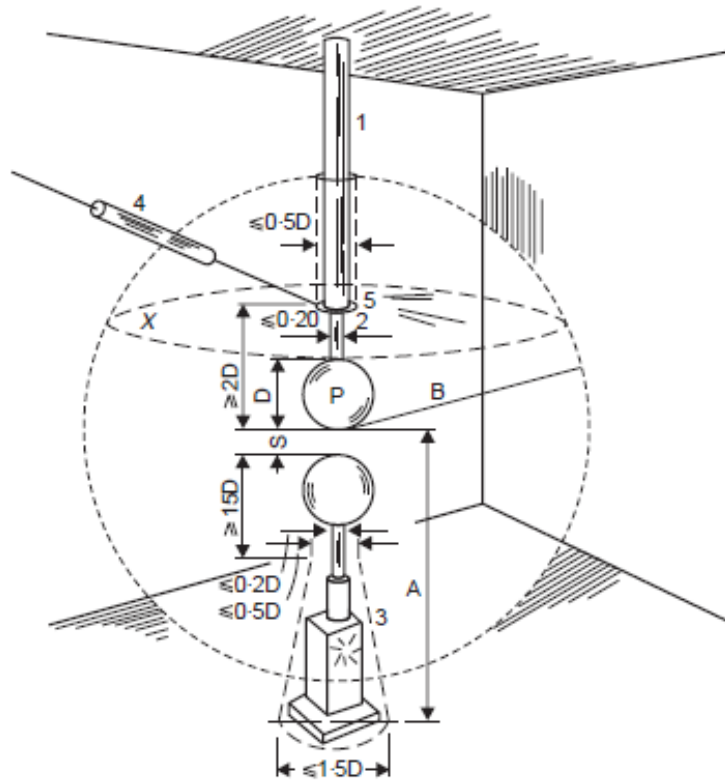


Fig. 4.2

- For the measurement of a.c. or d.c. voltage, a reduced voltage is applied to begin with so that the switching transient does not flash over the sphere gap and then the voltage is increased gradually till the gap breaks down.
- **Alternatively** the voltage is applied across a relatively large gap and the spacing is then gradually decreased till the gap breaks down. Corresponding to this gap the value of peak voltage can be read out from the calibration tables.
- The calibration tables values correspond to 760 mm Hg pressure and 20°C temperature.
- Any deviation from the value, a correction factor will have to be used to get the correct value of the voltage being measured.
- For the measurement of 50% impulse disruptive discharge voltages, the spacing of the sphere gap or the charging voltage of the impulse generator is adjusted in steps of 3% of the expected disruptive voltage.
- Six applications of the impulse should be made at each step and the interval between two applications is 5 seconds.
- The value giving 50% probability to disruptive discharge is preferably obtained by interpolation between at least two gap or voltage settings, one resulting in two disruptive discharges or less out of six applications and the other in four disruptive discharges or more out of again six applications
- The breakdown voltage of a sphere gap increases with increase in pressure and decreases with increase in temperature.
- The value of disruptive voltages as given in Tables 4.2 and 4.3 correspond to 760 mm Hg pressure and 20°C.
- For small variation in temperatures and pressures, the disruptive voltage
- is closely proportional to the relative air density. The relative air density  $\delta$  is given by

$$\delta = \frac{293b}{760(273 + t)}$$

- where b and t are the atmospheric conditions (pressure in mm of Hg and temperature in °C respectively) during measurement.
- The disruptive voltage V is given  $V = KdV0$

- Where  $V_0$  is the disruptive voltage as given in the Tables 4.2 and 4.3 and  $K_d$  is a correction factor  $K_d$  is a slightly non-linear function of  $\delta$  a result explained by Paschen's law.

Various factors that affect the sparkover voltage of a sphere gap are:

- nearby earthed objects,
- atmospheric conditions and humidity,
- irradiation, and
- polarity and rise time of voltage waveforms.

- Effect of nearby earthed objects**

- The effect of nearby earthed objects was investigated by Kuffel(14) by enclosing the earthed sphere inside an earthed cylinder.
- It was observed that the sparkover voltage is reduced.
- The reduction was observed to be

$$\Delta V = m \log (B/D) + C$$

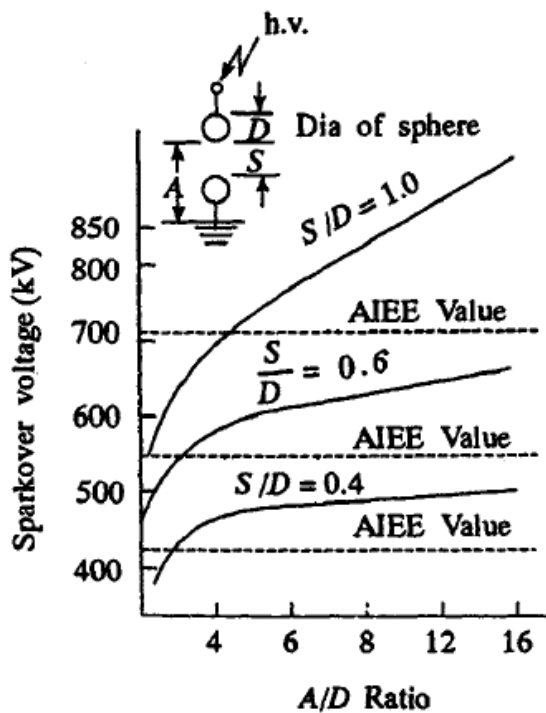
where,

$\Delta V =$  percentage reduction,

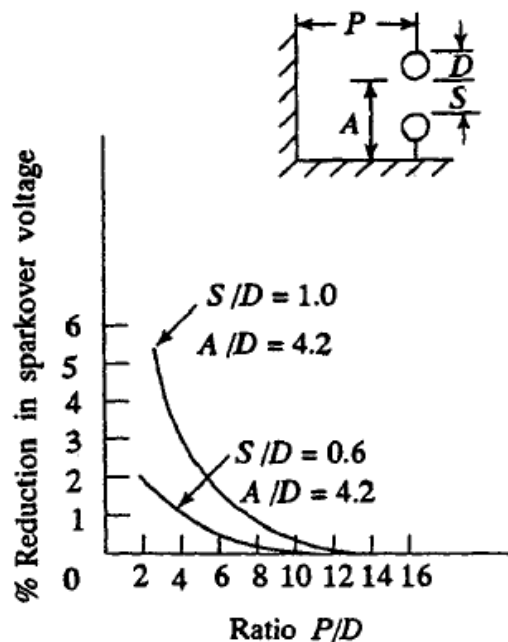
$B =$  diameter of earthed enclosing cylinder,

$D =$  diameter of the spheres,

$S =$  spacing, and  $m$  and  $C$  are constants.



(a)



(b)

## (ii) Effect of atmospheric conditions

- The sparkover voltage of a spark gap depends on the air density which varies with the changes in both temperature and pressure.
- Let sparkover voltage =  $V$  under test conditions of temperature  $T$  and pressure  $p$  torr
- sparkover voltage =  $V_0$  under standard conditions of temperature  $T = 20^\circ\text{C}$  and pressure  $p = 760$  torr, then  $V = kV_0$
- where  $k$  is a function of the air density factor  $d$ , given by

$$d = \frac{p}{760} \left( \frac{293}{273+T} \right)$$

### Influence of Humidity

- Kuffel has studied the effect of the humidity on the breakdown voltage by using spheres of 2 cms to 25 cms diameters and uniform field electrodes.
- The effect was found to be maximum in the region 0.4 mm Hg. and thereafter the change was decreased.
- Between 4–17 mm Hg. the relation between breakdown voltage and humidity was practically linear for spacing less than that which gave the maximum humidity effect.
- Fig. 4.4 shows the effect of humidity on the breakdown voltage of a 25 cm diameter sphere with spacing of 1 cm when a.c. and d.c. voltages are applied.
- It can be seen that
- (i) The a.c. breakdown voltage is slightly less than d.c. voltage.
- (ii) The breakdown voltage increases with the partial pressure of water vapour.
- It has also been observed that
- (i) The humidity effect increases with the size of spheres and is largest for uniform field electrodes.
- (ii) The voltage change for a given humidity change increase with gap length.

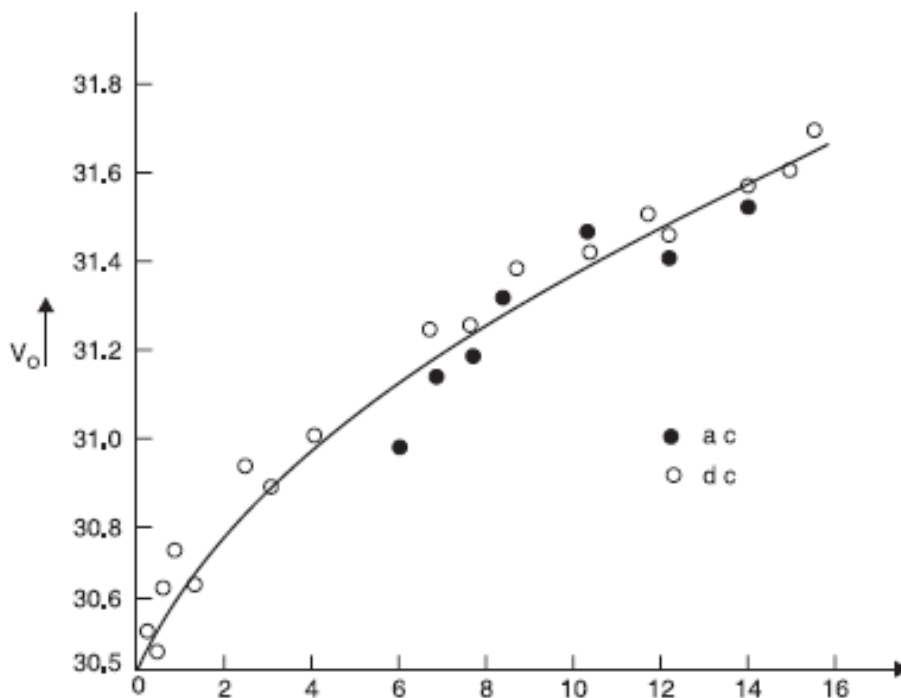


Fig. 4.4 Breakdown voltage humidity relation for a.c. and d.c. for 1.0 cm gap between 25 cms diameter spheres

- The increase in breakdown voltage with increase in partial pressure of water vapour and this increase in voltage with increase in gap length is due to the relative values of ionisation and attachment coefficients in air.
- The water particles readily attach free electrons, forming negative ions.
- These ions therefore slow down and are unable to ionise neutral molecules under field conditions in which electrons will readily ionise.
- It has been observed that within the humidity range of 4 to 17 g/m<sup>3</sup> (relative humidity of 25 to 95% for 20°C temperature) the relative increase of breakdown voltage is found to be between 0.2 to 0.35% per gm/m<sup>3</sup> for the largest sphere of diameter 100 cms and gap length upto 50 cms.
- **Influence of Dust Particles**
- When a dust particle is floating between the gap this results into erratic breakdown in homogeneous or slightly in homogenous electrode configurations.
- When the dust particle comes in contact with one electrode under the application of d.c. voltage, it gets charged to the polarity of the electrode and gets attracted by the opposite electrode due to the field forces and the breakdown is triggered shortly before arrival.
- Gaps subjected to a.c. voltages are also sensitive to dust particles but the probability of erratic breakdown is less.
- Under d.c. voltages erratic breakdowns occur within a few minutes even for voltages as low as 80% of the nominal breakdown voltages.
- This is a major problem, with high d.c. voltage measurements with sphere gaps.
- **Effect of Irradiation**
- Illumination of sphere gaps with ultra-violet or x-rays aids easy ionization in gaps.
- **The effect of irradiation is pronounced for small gap spacings.**
- A reduction of about 20% in sparkover voltage was observed for spacings of 0.1 *D* to 0.3 *D* for a 1.3 cm sphere gap with d.c. voltages.
- The reduction in sparkover voltage is less than 5% for gap spacings more than 1 cm, and for gap spacings of 2 cm or more it is about 1.5%.
- Hence, irradiation is necessary for smaller sphere gaps of gap spacing less than 1 cm for obtaining consistent values.
- **(Iv) Effect of polarity and waveform**
- It has been observed that the sparkover voltages for positive and negative polarity impulses are different.
- Experimental investigation showed that for sphere gaps of 6.25 to 25 cm diameter, the difference between positive and negative d.c. voltages is not more than 1%.
- **For smaller sphere gaps (2 cm diameter and less) the difference was about 8% between negative and positive impulses of 1/50 μs waveform.**
- the wave front and wave tail durations also influence the breakdown voltage.
- For wave fronts of less than 0.5 μs and wave tails less than 5 μs the breakdown voltages are not consistent and hence the use of sphere gap is not recommended for voltage measurement in such cases.

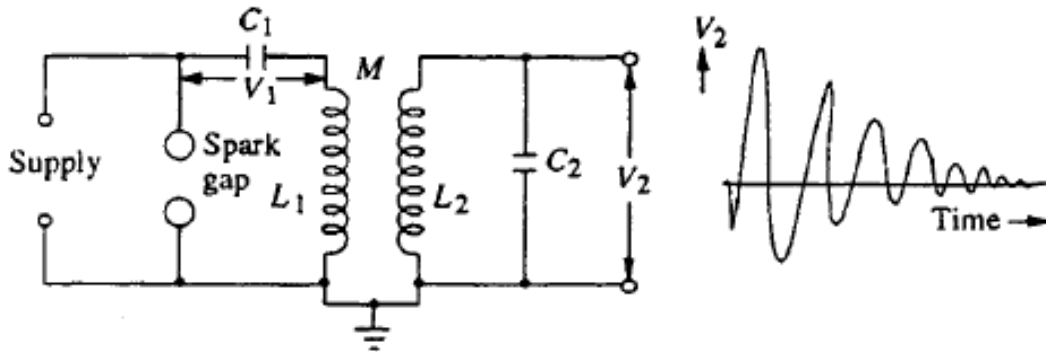
## 5. Generation of High Frequency a.c. High Voltages:

- High frequency high voltages are required for
  - rectifier d.c. power supplies
- testing electrical apparatus for switching surges, high frequency high voltage damped oscillations are needed which need high voltage high frequency transformers.

The advantages of these high frequency transformers are:

- the absence of iron core in transformers and hence saving in cost and size,
- pure sine wave output,
- slow build-up of voltage over a few cycles and hence no damage due to switching surges, and
- uniform distribution of voltage across the winding coils due to subdivision of coil stack into a number of units.

- The commonly used high frequency resonant transformer is the Tesla coil
- Doubly tuned resonant circuit
- The primary voltage rating is 10 kV
- The secondary may be rated to as high as 500 to 1000 kV.
- The primary is fed from a d.c. or a.c. supply through the condenser C<sub>1</sub>.
- A spark gap G connected across the primary is triggered at the desired voltage V<sub>1</sub> which induces a high self-excitation in the secondary.
- The primary and the secondary windings (L<sub>1</sub>) and (L<sub>2</sub>) are wound on an insulated former with no core (air-cored) and are immersed in oil.
- The windings are tuned to a frequency of 10 to 100 kHz by means of the condensers C<sub>1</sub> and C<sub>2</sub>



(a) Equivalent circuit

(b) Output waveform

- output voltage V<sub>2</sub> is a function of L<sub>1</sub>, L<sub>2</sub>, C<sub>1</sub>, C<sub>2</sub>, M.
- winding resistances will be small, contribute only for damping of the oscillations.
- neglecting the winding resistances.
- Let the condenser C<sub>1</sub> be charged to a voltage V<sub>1</sub> when the spark gap is triggered.
- Let a current i<sub>1</sub> flow through the primary winding L<sub>1</sub> and produce a current i<sub>2</sub> through L<sub>2</sub> and C<sub>2</sub>

Laplace transformed equation

$$V_1 = \frac{1}{C_1} \int_0^t i_1 dt + L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

$$\frac{V_1}{s} = \left[ L_1 s + \frac{1}{C_1 s} \right] I_1 + M s I_2$$

$$0 = \frac{1}{C_2} \int_0^t i_2 dt + L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$0 = [M s] I_1 + \left[ L_2 s + \frac{1}{[C_2 s]} \right] I_2$$



$$V_2 = \frac{1}{C_2} \int_0^t i_2 dt; \text{ or its transformed equation is } V_2(s) = \frac{I_2}{C_2 s}$$

$$V_2 = \frac{MV_1}{\sigma L_1 L_2 C_1} \frac{1}{\gamma_2^2 - \gamma_1^2} [\cos \gamma_1 t - \cos \gamma_2 t]$$

where,

$$\sigma^2 = 1 - \frac{M^2}{L_1 L_2} = 1 - K^2$$

$K$  = coefficient of coupling between the windings  $L_1$  and  $L_2$

$$\gamma_{1,2} = \frac{\omega_1^2 + \omega_2^2}{2} \pm \sqrt{\left(\frac{\omega_1^2 + \omega_2^2}{2}\right)^2 - \omega_1^2 \omega_2^2 (1 - K^2)}$$

$$\omega_1 = \frac{1}{\sqrt{L_1 C_1}} \text{ and } \omega_2 = \frac{1}{\sqrt{L_2 C_2}}$$

The peak amplitude of the secondary voltage  $V_2$  can be expressed as,

$$V_{2\max} = V_1 e \sqrt{\frac{L_2}{L_1}}$$

$$e = \frac{2\sqrt{(1-\sigma)}}{\sqrt{(1+a)^2 - 4\sigma a}}$$

$$a = \frac{L_2 C_2}{L_1 C_1} = \frac{W_1^2}{W_2^2}$$

Simplified analysis of Tesla coil from energy consideration

$$W_1 = \frac{1}{2} \eta C_1 V_1^2 = \left(\frac{1}{2} C_2 V_2^2\right)$$

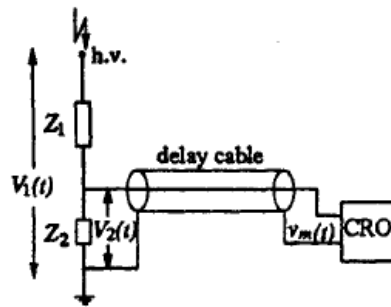
$$V_2 = V_1 \sqrt{\eta \frac{C_1}{C_2}}$$

For high value of  $K$  and winding resistance the waveform may become unidirectional

## 6. Potential Dividers for Impulse Voltage Measurements

- Potential or voltage dividers for high voltage impulse measurements, high frequency a.c. measurements, or for fast rising transient voltage measurements are usually either **resistive** or **capacitive** or **mixed element type**.
- *The low voltage arm of the divider is usually connected to a fast recording oscillograph or a peak reading instrument through a delay cable.*
- A schematic diagram of a potential divider with its terminating equipment is given in Fig. *is usually a resistor or a series of resistors in case of a resistance potential divider, or a single or a number of capacitors in case of a capacitance divider.*
- It can also be a combination of both resistors and capacitors.
- $Z_2$  will be a resistor or a capacitor or an  $R-C$  impedance *depending upon the type of the divider.*

• Each element in the divider, in case of high voltage dividers, has a **selfresistance** or **capacitance**. In addition, the **resistive elements** have **residual inductances**, a **terminal stray capacitance** to ground, and **terminal to terminal capacitances**.



**Fig. 7.23** Schematic diagram of a potential divider with a delay cable and oscilloscope

- 
- The following elements mainly constitute the different errors in the measurement:
- (i) residual inductance in the elements;
- (ii) stray capacitance occurring
  - (a) *between the elements,*
  - (b) *from sections and terminals of the elements to ground, and*
  - (c) *from the high voltage lead to the elements or sections;*
- (iii) the impedance errors due to
  - (a) *connecting leads between the divider and the test objects, and*
  - (b) *ground return leads and extraneous current in ground leads; and*
- (iv) parasitic oscillations due to lead and cable inductances and capacitance of high voltage terminal to ground.

- The effect to residual and lead inductances becomes pronounced when fast rising impulses of less than one microsecond are to be measured.
- The residual inductances damp and slow down the fast rising pulses.
- Secondly, the layout of the test objects, the impulse generator, and the ground leads also require special attention to minimize recording errors

- **Resistance Potential Divider for Very Low Impulse Voltages and Fast Rising Pulses**

- A simple resistance potential divider consists of two resistances  $R_1$  and  $R_2$  in series ( $R_1 \gg R_2$ ).
- The attenuation factor of the divider or the voltage ratio is given by

$$a = \frac{V_1(t)}{V_2(t)} = 1 + \frac{R_1}{R_2}.$$

- The divider element  $R_2$ , in practice, is connected through the coaxial cable to the oscilloscope.
- The cable will generally have a surge impedance  $Z_0$  and this will come in parallel with the oscilloscope input impedance ( $R_m, C_m$ ).
- $R_m$  will generally be greater than one megaohm and  $C_m$  may be 10 to 50 picofarads.
- For high frequency and impulse voltages (since they also contain high frequency fundamental and harmonics), the ratio in the frequency domain will be given by

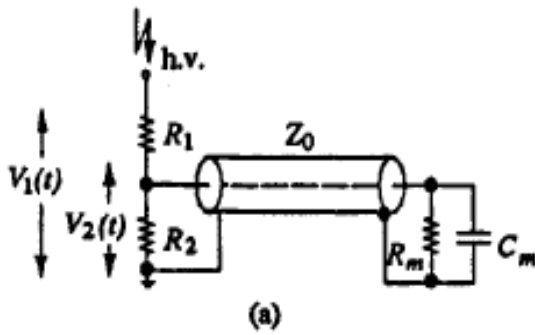
$$a = \frac{V_1}{V_2} = 1 + \frac{R_1}{(R_2/1 + j\omega R_2 C_m)}$$

- the ratio is a function of the frequency.
- To avoid the frequency dependance of the voltage ratio  $a$ , the divider is compensated by adding an additional capacitance  $C_1$  across  $R_1$ .
- The value of  $C_1$ , to make the divider independent of the frequency, may be obtained from the relation,

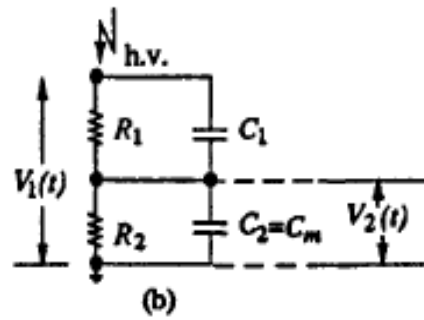
$$\frac{R_1}{R_2} = \frac{C_m}{C_1}$$

$$R_1 C_1 = R_2 C_m$$

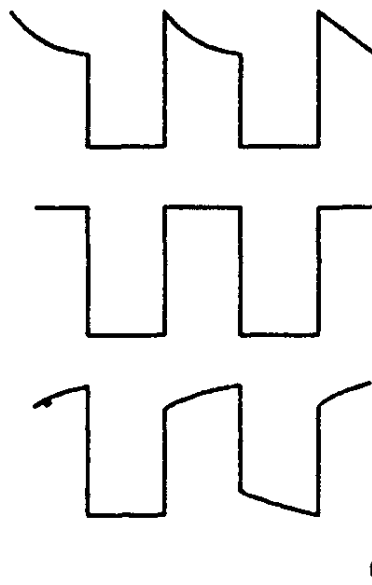
meaning that the time constant of both the arms should be the same. This compensation is used for the construction of high voltage dividers and probes used with oscilloscopes



(a) Resistance potential divider with surge cable and oscilloscope terminations



(b) Compensated resistance potential divider



(i) Over compensated

(ii) Correctly compensated  
 $C_1 R_1 = C_2 R_m$

(iii) Under compensated

(c)

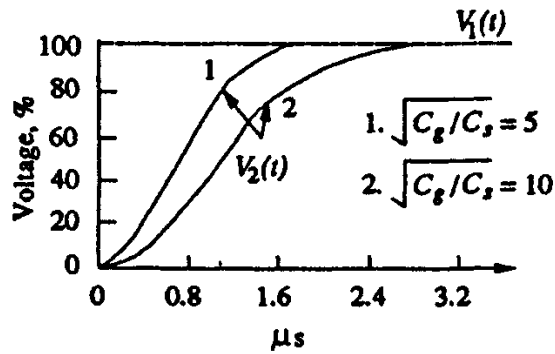
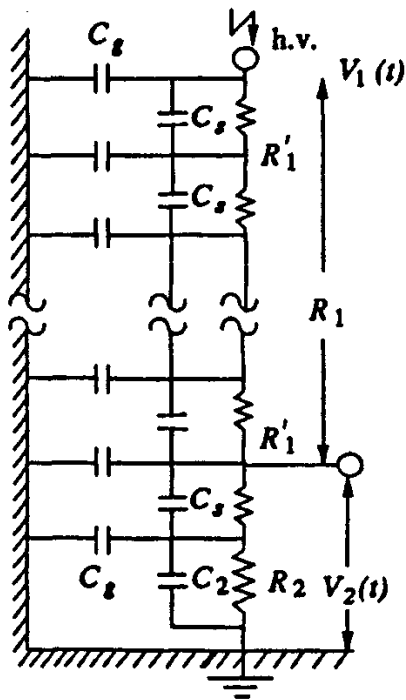
Output of compensated resistance voltage divider for different degrees of compensation

### Potential Dividers Used for High Voltage Impulse Measurements

In a resistance potential divider,  $R_1$  and  $R_2$  are considered as resistors of small dimensions. For voltages above 100 kV,  $R_1$  is no longer small in dimension and is usually made of a number of sections.

Hence the divider is no longer a small resistor of lumped parameters, but has to be considered as an equivalent distributed network with its terminal to ground capacitances and inter-sectional series capacitances as shown in Fig. 7.26.

The total series resistance  $R_1$  is made of  $n$  resistors of value  $R_1'$  and  $R = nR_1'$ .  $C_g$  is the terminal to ground capacitance of each of the



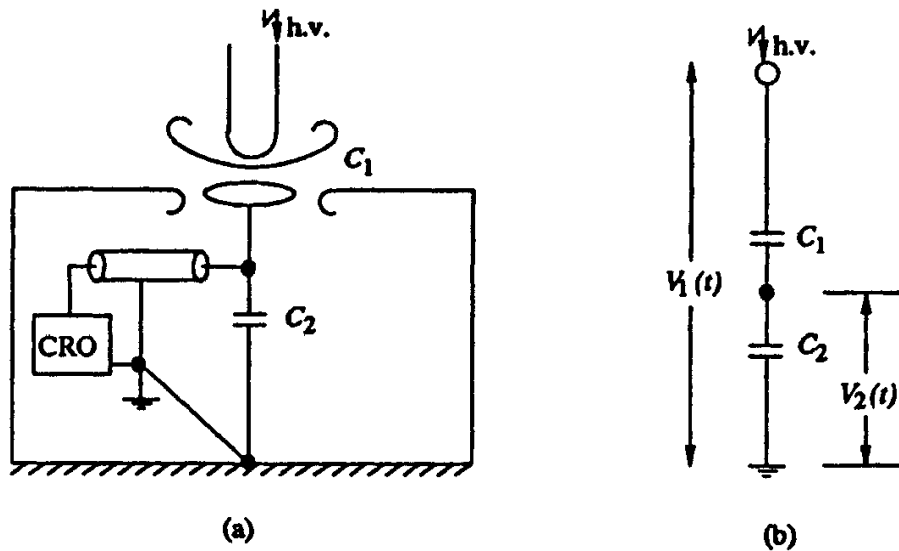
**Resistance potential divider with inter-sectional and ground capacitances**

Output of the divider shown in Fig. 7.26 for a step input

### Capacitance Voltage Dividers

- Capacitance voltage dividers are ideal for measurement of fast rising voltages and pulses.
- The capacitance ratio is independent of the frequency, if their leakage resistance is high enough to be neglected.
- But usually the dividers are connected to the source voltage through long leads which introduce lead inductances and residual resistances.
- The capacitance used for very high voltage work is not small in dimension and hence cannot be considered as a lumped element.
- The output of the divider for high frequencies and impulses is distorted as in the case of resistance dividers
- **Pure Capacitance Dividers**
- A pure capacitance divider for high voltage measurements and its electrical equivalent network without stray elements is shown in Fig. 7.29.
- The ratio of the divider

$$a = \frac{V_1(t)}{V_2(t)} = 1 + \frac{C_2}{C_1}$$

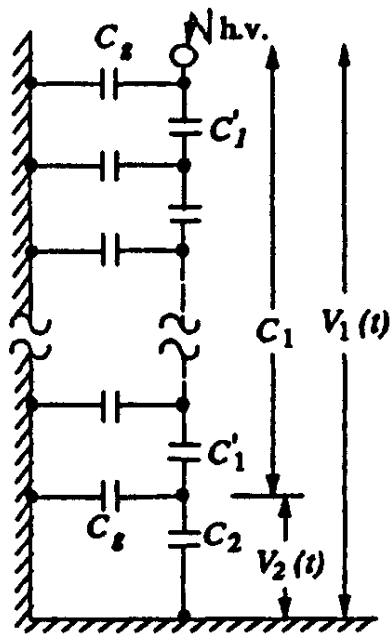


### Capacitance voltage divider for very high voltages and its electrical equivalent circuit

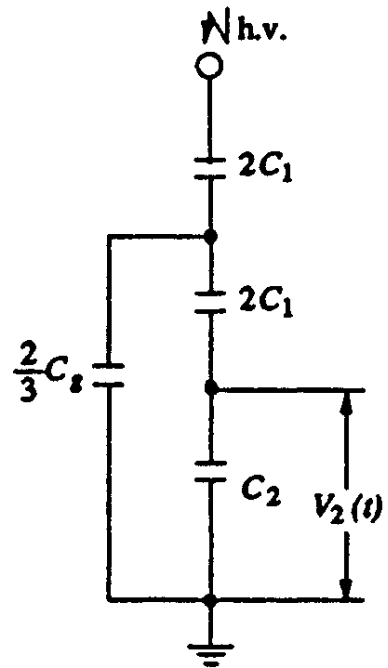
- Capacitance  $C_1$  is formed between the h.v. terminal of the source (impulse generator) and that of the test object or any other point of measurement
- The CRO is located within the shielded screen surrounding capacitance  $C_2$ .
- $C_2$  includes the capacitance used, the lead capacitance, input capacitance of the CRO, and other ground capacitances. The advantage of this connection is that the loading on the source is negligible; but a small disturbance in the location of  $C_1$  or h.v. electrode or the presence of any stray object nearby changes the capacitance  $C_1$ , and hence the divider ratio is affected.
- capacitors  $C_1$  in series for the given voltage  $V_1$ . In such cases the equivalent circuit is similar to that of a string insulator unit used in transmission lines (Fig. 7.30).
- The voltage distribution along the capacitor chain is non-linear and hence causes distribution of the output wave. But the ratio error is constant and is independent on frequency as compared to resistance dividers. A simplified equivalent circuit is shown in Fig. 7.30 b, which can be used if  $C_1 \ll C_2$  and  $C_g \ll C_1$ . The voltage ratio is

$$a = \frac{V_1(t)}{V_2(t)} = \left[ 1 + \frac{C_2}{C_1} \right] \left[ 1 + \frac{C_g}{6C_1} \right]$$

- This ratio is constant and gives an error of less than 5% when  $C_1 = 3C_g$ . This equivalent circuit is quite satisfactory up to 1 MHz.



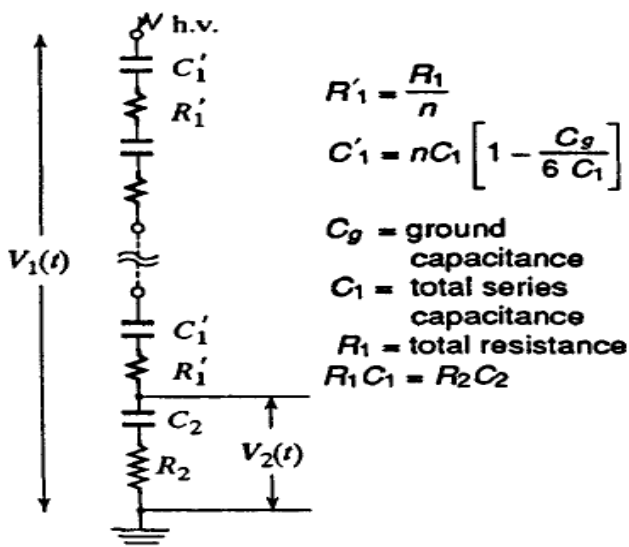
(a) Distributed network



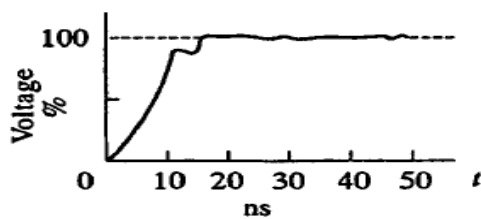
(b) Approximate equivalent circuit

Capacitance voltage divider with distributed network and its equivalent circuit

Mixed R-C Potential Dividers



(a) Equivalent circuit



(b) Step response determined with low voltage step pulse

Equivalent circuit of a series R-C voltage divider and its step response