Engineering Electromagnetics
Scheme of Evaluation - IAT 1

1.(a) Statement : 2 m
Vector form : 2 m
Cartesian co-ordinates : 2 m

1.(b) Approach : 3 m
Result : 1 m

2.(a) Flux density definition : 2 m
Relation of $\vec{D}$ and $\vec{E}$: 3 m.

2.(b) Approach : 4 m
Result : 1 m

3.(a) Definition of $\vec{E}$: 2 m
$\vec{E}$ due to infinite line charge : 8 m.

4.(a) $\vec{E}_1$ due to line charge : 3 m.
$\vec{E}_2$ due to first point charge : 3 m.
$\vec{E}_3$ due to second point charge : 3 m
Result : 1 m

5.(a) Poisson's and Laplace's eqn. : 5 m.
Representation in all co-ordinate systems : 2 m.

5.(b) Approach : 10 m
Result : 2 m.

6.(a) Diagram : 1 m
Solution of Laplace's eqn. 5 m
Potential : 1 m
E : 2 m.
Capacitance : 1 m

7.(a) Uniqueness theorem statement: 2 m

proof: 5 m.

7.(b) Approach: 1m.

Result: 2m.

8.(a) Approach: 1 m.

Calculation of V: 3 m

calculation of $\vec{E}$: 2 m.

8.(b) Approach: 1m.

solution of V: 1 m

solution of ρ_v: 2m.

9.(a) statement: 2m

Explanation: 3 m.

9.(b) Approach: 2m

Solution and final result: 3 m

10. Diagram: 1 m

Derivation: 8 m

Final result: 1 m.

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Internal Assesment Test - I									
Sub:	ENGINEERING ELECTROMAGNETICS							Code:	15EC36
Date:	21 / 09 / 2017	Duration:	90 mins	Max Marks:	50	Sem:	3rd	Branch:	ECE
Answer FIVE FULL Questions									

		Marks	OBE CO	OBE RBT
1.(a)	State and explain Coulomb's law in vector form. Express the result in Cartesian coordinates.	[6]	CO1	L1

Soln The force b/w two very small charged objects separated in vacuum or free space by a distance which is large compared to their size is proportional to the charge on each and inversely proportional to the square of the dist. b/w them.

Q_1 —— R —— Q_2

i.e. $F = \frac{k Q_1 Q_2}{R^2}$

Q_1 & Q_2 → +ve or −ve quantities of charge
unit coulomb (C)

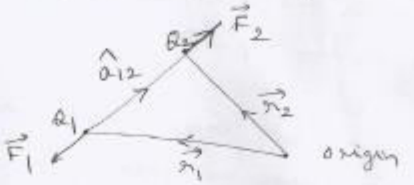
R → separation in m.
k → constant of proportionality.

$k = \frac{1}{4\pi\epsilon_0}$
where, ϵ_0 → permittivity of free space.

$\epsilon_0 = 8.854 \times 10^{-12}$ F/m
$= \frac{1}{36\pi} \times 10^{-9}$ F/m.

F → Force in Newton.

1.(a) Vector form of Coulomb's law



$\vec{r}_1 \to$ locates Q_1

$\vec{r}_2 \to$ locates Q_2

Q_1, Q_2 of same sign, F_2 in the direction as indicated.

$F_2 \to$ force ~~ext~~ exerted on Q_2 by Q_1.

$\hat{a}_{12} \to$ unit vector along $\vec{R}_{12}$.

Then, the vector form of Coulomb's law is

$$\vec{F}_2 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{12}^2}\,\hat{a}_{12}$$

where, $$\hat{a}_{12} = \frac{\vec{R}_{12}}{|\vec{R}_{12}|} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|}$$

$|\vec{R}_{12}| = R =$ Distance b/w the ~~tw~~ two charges.

let, $\vec{F}_1$ be the force exerted by Q_1 on Q_2.

$\vec{r}_1 = \vec{r}_2 + \vec{R}_{21}$

$\Rightarrow \vec{R}_{21} = \vec{r}_1 - \vec{r}_2 = -(\vec{r}_2 - \vec{r}_1)$

$\therefore \hat{a}_{12} = -\hat{a}_{21}$.

1.(a) $\therefore F_1 = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{21}^2}(-\hat{a}_{21}) = -\vec{F_2}$

$\Rightarrow$ Coulomb's law is a mutual force.

Important observations:

i) charges should be point charges and stationary in nature.

ii) Should consider the signs of the charges to decide whether the force will be attractive or repulsive.

iii) Coulomb's law is linear.

i.e. if $\vec{F_2} = -\vec{F_1}$

then, $n\vec{F_2} = -n\vec{F_1}$

where n is a scalar.

iv) Force on a charge in the presence of several other charges is the sum of the forces on that charge due to each of the other charges acting alone.

(b) Two point charges of magnitude 2mC and -7mC are located at places $P_1(4,7,-5)$ and $P_2(3,2,-9)$ respectively in free space. Evaluate the vector force **F** on the charge at P_2 due to the charge placed at P_1. [4] CO1 L3

$$F = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{|R|^2}\hat{a}_R$$

$$P_1\ 2mC\ (4,7,-5) \longrightarrow P_2\ -7mC\ (3,2,-9)$$

$$= \frac{1}{4\pi\epsilon_0}\frac{2\times(-7)\times 10^{-6}\ \hat{a}_R}{\left(\sqrt{(3-4)^2+(2-7)^2+(-9+5)^2}\right)^2}$$

$$\bar{R} = (3-4)\hat{a}_x + (2-7)\hat{a}_y + (-9+5)\hat{a}_z$$

$$\bar{R} = -\hat{a}_x - 5\hat{a}_y - 4\hat{a}_z$$

$$= \frac{9\times10^9\ (-14)\times10^{-6}\ (-\hat{a}_x - 5\hat{a}_y - 4\hat{a}_z)}{\left(\sqrt{1+25+16}\right)^2 \times \sqrt{1+25+16}}$$

$$= \frac{9\times(-14)\times10^3\ (-\hat{a}_x - 5\hat{a}_y - 4\hat{a}_z)}{42\sqrt{42}}$$

$$\boxed{\bar{F} = 462.91(\hat{a}_x + 5\hat{a}_y + 4\hat{a}_z)}$$

OR

2. (a) Define electric flux density. Derive the relation between electric flux density and electric field intensity. [5] CO1 L2

The direction of D at a pt. is the direction of the flux lines at that point, and the mag. is given by the no. of flux lines crossing a surface normal to the lines divided by the surface area

$$\therefore \vec{D}\Big|_{r=a} = \frac{Q}{4\pi a^2}\,\hat{a}_r \quad \text{(inner sphere)}$$

$$\vec{D}\Big|_{r=b} = \frac{-Q}{4\pi b^2}\,\hat{a}_r \quad \text{(outer sphere)}.$$

$a \le r \le b$

$$\vec{D} = \frac{Q}{4\pi r^2}\,\hat{a}_r$$

Shrink inner sphere, smaller and smaller, we reach point charge

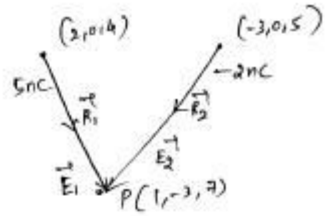
$$\vec{D} = \frac{Q}{4\pi r^2}\,\hat{a}_r .$$

Q lines of flux are symmetrically directed outward from the pt. and pass through an imaginary spherical surface of area $4\pi r^2$.

$$\vec{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\hat{a}_r$$

$\therefore \boxed{D = \epsilon_0 E} \rightarrow$ free space

(b) Two point charges 5nC and -2nC are located at (2, 0, 4) and (-3, 0, 5), respectively. Find the electric field intensity **E** at (1,-3, 7). [5] CO1 L3



$$\vec{R_1} = -\vec{a_x} - 3\vec{a_y} + 3\vec{a_z}$$

$$|\vec{R_1}| = \sqrt{1^2+9+9} = \sqrt{19}$$

$$\vec{E} = \vec{E_1} + \vec{E_2}$$

$$\vec{R_2} = 4\vec{a_x} - 3\vec{a_y} + 2\vec{a_z}$$

$$= \sqrt{16+9+4}$$

$$|\vec{R_2}| = \sqrt{29}$$

$$\frac{2}{(\sqrt{29})^3} = 0.0128$$

$$\vec{E} = \frac{Q_1}{4\pi\epsilon_0 |\vec{R_1}|^3} \cdot \vec{R_1} + \frac{Q_2}{4\pi\epsilon_0 |\vec{R_2}|^3} \cdot \vec{R_2}$$

$$= 9\times10^{9} \times 10^{-9} \left[\frac{5}{(\sqrt{19})^3} \cdot (-\vec{a_x} - 3\vec{a_y} + 3\vec{a_z}) - \frac{2(4\vec{a_x} - 3\vec{a_y} + 2\vec{a_z})}{(\sqrt{29})^3} \right]$$

$$= 9\left[-0.0603\,\vec{a_x} - 0.1811\,\vec{a_y} + 0.1811\,\vec{a_z} - 0.05122\,\vec{a_x} + 0.0384\,\vec{a_y} - 0.0256\,\vec{a_z} \right]$$

$$\boxed{\vec{E} = -1.00368\,\vec{a_x} - 1.2843\,\vec{a_y} + 1.3995\,\vec{a_z}} \quad V/m$$

3.(a) State and explain electric field intensity. Obtain an expression for electric field intensity due to an infinitely long uniform line charge distribution. [2+8] CO1 L2

9) <u>Electric - field Intensity</u>

- consider ~~the~~ one charge fixed in position say Q_1.
- we move a charge slowly around.

There exists a force everywhere on this 2nd charge. i.e. force field.

- let the 2nd charge be Q_t.

Then the force on it is:

$$\vec{F}_t = \frac{Q_1 Q_2}{4\pi\epsilon_0 R_{1t}^2} \hat{a}_{1t}$$

$\therefore$ Force per unit charge:

$$\frac{F_t}{Q_t} = \frac{Q_1}{4\pi\epsilon_0 R_{1t}^2} \hat{a}_{1t}.$$

R.H.S. f^n of Q_1 and ~~Q~~ the directed line segment from Q_1 to the position of the test charge.

- This describes a vector field and is called the electric field intensity.

<u>Def^n</u> Electric field intensity is the vector force on a unit +ve test charge when the charge placed in a electric field.

$\boxed{\text{Unit: } N/C}$

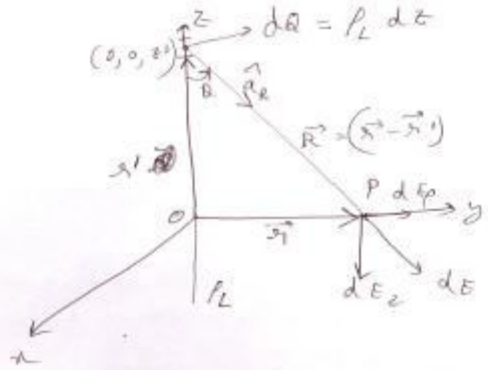
But $V = \frac{J}{C} = \frac{N.m}{C}$.

$$\Rightarrow \frac{V}{m} = \frac{N}{C}$$

$\therefore$ E practically expressed in $\frac{V}{m}$.

Finally, $\boxed{\vec{E} = \frac{\vec{F}_t}{Q_t}}$

$$\boxed{\vec{E} = \frac{Q_1}{4\pi\epsilon_0 R_{1t}^2} \hat{a}_{1t}}$$



- consider a general point $P(0, y, 0)$ on the y-axis to determine the field.
- We consider incremental charge,

$$dQ = \rho_L \, dz'$$

- Aim to find incremental field.

$$\therefore d\vec{E} = \frac{\rho_L \, dz' \, (\vec{r} - \vec{r}\,')}{4\pi\epsilon_0 |r - r'|^3}$$

where, $\vec{r} = y\,\hat{a}_y = \rho\,\hat{a}_\rho$

$\vec{r}\,' = z'\,\hat{a}_z$

$$\therefore \vec{r} - r' = (\rho\,\hat{a}_\rho - z'\,\hat{a}_z)$$

$$\therefore d\vec{E} = \frac{\rho_L \, dz' \, (\rho\,\hat{a}_\rho - z'\,\hat{a}_z)}{4\pi\epsilon_0 \, (\rho^2 + z'^2)^{3/2}}$$

We know only ρ component is present.

$$dE_\rho = \int_{-\infty}^{\infty} \frac{\rho_L \rho \, dz'}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}$$

$$E_\rho = \int_{-\infty}^{\infty} \frac{\rho_L \rho \, dz'}{4\pi\epsilon_0 (\rho^2 + z'^2)^{3/2}}.$$

Let, $z' = \rho \cot\theta$

$\therefore dz' = -\rho \operatorname{cosec}^2\theta \, d\theta$

$z' = \rho \frac{\cos\theta}{\sin\theta}$.

$\infty = \frac{\cos 0}{\sin 0}$. $\Rightarrow$ for $z' = \infty$, $\theta = 0°$.

$z' = -\frac{\cos(\pi)}{\sin(\pi)} \Rightarrow z' = -\infty$, $\theta = \pi$.

$$E_\rho = \frac{\rho_L}{4\pi\epsilon_0} \int_{\pi}^{0} \frac{\rho . \, (-\rho \operatorname{cosec}^2\theta \, d\theta)}{(\rho^2 + \rho^2 \cot^2\theta)^{3/2}}$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \int_{\pi}^{0} \frac{-\rho^2 \operatorname{cosec}^2\theta \, d\theta}{(\rho^2)^{3/2} (1+\cot^2\theta)^{3/2}}.$$

$$= \frac{\rho_L}{4\pi\epsilon_0 \rho} \cdot \int_{\pi}^{0} \frac{-\operatorname{cosec}^2\theta \, d\theta}{\operatorname{cosec}^3\theta}$$

$$= \frac{\rho_L}{4\pi\epsilon_0 \rho} \cdot \int_{\pi}^{0} -\sin\theta \, d\theta = \frac{\rho_L}{4\pi\epsilon_0 \rho} \left[+\cos\theta\right]_{\pi}^{0}$$

$$= \frac{\rho_L}{4\pi\epsilon_0 \rho} \cdot [1+1] = \frac{2\rho_L}{4\pi\epsilon_0 \rho}$$

$$= \frac{\rho_L}{2\pi\epsilon_0 \rho}$$

OR

4. (a) An infinite line charge with ρ_L = 2 nC/m lies along the x-axis in free space, while point charges of 8nC each are located at(0,0,1) and (0,0,-1). Find **E** at (2,3,-4). [10] CO1 L3

Soln. $\vec{E}$ at $(2,3,-4)$ due to line charge

$$\vec{E}_1 = \frac{2\times10^{-9}}{2\pi\epsilon_0 (\sqrt{9+16})^2} (3\hat{a}_y - 4\hat{a}_z) \text{ V/m}$$

$\vec{E}$ at $(2,3,-4)$ due to charge at $(0,0,1)$ is.

$$\vec{E}_2 = \frac{8\times10^{-9}}{4\pi\epsilon_0} \frac{(2\hat{a}_x + 3\hat{a}_y - 5\hat{a}_z)}{(4+9+25)^{3/2}}$$

$\vec{E}$ at $(2,3,-4)$ due to charge at $(0,0,-1)$ is,

$$\vec{E}_3 = \frac{8\times10^{-9}(2\hat{a}_x + 3\hat{a}_y - 3\hat{a}_z)}{4\pi\epsilon_0 (4+9+9)^{3/2}}$$

$\therefore$ The total field, $\vec{E} = \vec{E}_1 + \vec{E}_2 + E_3$

$= 2.0\hat{a}_x + 7.3\hat{a}_y - 9.4\hat{a}_z$ V/m

5.(a) Starting from Gauss's law deduce Poisson's and Laplace's equation and write down the equation in all the three coordinate systems. [7] CO3 L2

Soln. From Gauss's law,

$\vec{\nabla}\cdot\vec{D} = \rho_v$

$\vec{D} = \epsilon\vec{E}$

and $\vec{E} = -\vec{\nabla}V$

$\therefore \vec{\nabla}\cdot\vec{D} = \vec{\nabla}\cdot(\epsilon\vec{E}) = -\vec{\nabla}(\epsilon\vec{\nabla}V) = \rho_v$

$\Rightarrow -\vec{\nabla}\cdot(\vec{\nabla}V) = \frac{\rho_v}{\epsilon}$

$\Rightarrow \vec{\nabla}\cdot(\vec{\nabla}V) = \frac{-\rho_v}{\epsilon}$

$\Rightarrow \nabla^2 V = -\frac{\rho_v}{\epsilon}$

$\boxed{\nabla^2 V = \frac{-\rho_v}{\epsilon}}$ → Poisson's equation

Now, $\vec{\nabla}\cdot\vec{A} = \left(\hat{a}_x \frac{\partial}{\partial x} + \hat{a}_y \frac{\partial}{\partial y} + \hat{a}_z \frac{\partial}{\partial z}\right)\cdot(A_x\hat{a}_x + A_y\hat{a}_y + A_z\hat{a}_z)$

$= \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

$\vec{\nabla}V = \frac{\partial V}{\partial x}\hat{a}_x + \frac{\partial V}{\partial y}\hat{a}_y + \frac{\partial V}{\partial z}\hat{a}_z$

$\vec{\nabla}\cdot\vec{\nabla}V = \frac{\partial}{\partial x}\left(\frac{\partial V}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial V}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial V}{\partial z}\right)$

$= \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

$\boxed{\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}}$ → rectangular co-ordinates

If $\rho_v = 0$ then $\nabla^2 V = 0$

In cylindrical co-ordinates,

$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial V}{\partial\rho}\right) + \frac{1}{\rho^2}\left(\frac{\partial^2 V}{\partial\phi^2}\right) + \frac{\partial^2 V}{\partial z^2}$

$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial V}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial\phi^2}$

(b) Verify if the field given by $V = 2x^2 - 3y^2 + z^2$ satisfies Laplace's equation or not. [3] CO3 L3

Soln. $\frac{\partial V}{\partial x} = 4x$, $\frac{\partial^2 V}{\partial x^2} = 4$ | $\frac{\partial V}{\partial y} = -6y$, $\frac{\partial^2 V}{\partial y^2} = -6$ | $\frac{\partial V}{\partial z} = 2z$, $\frac{\partial^2 V}{\partial z^2} = 2$

$\therefore \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 4 - 6 + 2 = 0 = \nabla^2 V$

$\therefore \boxed{\nabla^2 V = 0}$ → given V satisfies Laplace's eqn.

OR

6.(a) Derive the expression for capacitance of coaxial cable using Laplace's equation. Consider radius of inner conductor 'a' and outer conductor 'b'. Also consider the potential $V = V_0$ at $r = a$ and $V = 0$ at $r = b$. [10] CO3 L2

$$\frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\rho\frac{\partial V}{\partial \rho}\right] = 0$$

$$\rho\frac{\partial V}{\partial \rho} = C_1$$

$$\frac{\partial V}{\partial \rho} = \frac{C_1}{\rho}$$

$$V = C_1 \ln\rho + C_2$$

$V = 0$ at $\rho = b$

$$0 = C_1 \ln b + C_2$$

$$V_0 = C_1 \ln a + C_2$$

$$-V_0 = C_1[\ln b - \ln a]$$

$$V_0 = C_1 \ln(a/b)$$

$$\Rightarrow C_1 = \frac{V_0}{\ln(a/b)}$$

$$C_2 = \frac{-V_0 \ln b}{\ln(a/b)}$$

$$V = \frac{V_0 \ln\rho}{\ln(a/b)} - \frac{V_0 \ln b}{\ln(a/b)}$$

$$\vec{E} = -\vec{\nabla} V$$

$$= \frac{-\partial V}{\partial r}\hat{a}_r$$

$$= \frac{\partial}{\partial r}\left[\frac{V_0 \ln\rho}{\ln(a/b)} - \frac{V_0 \ln(b)}{\ln(a/b)}\right]\hat{a}_\rho$$

$$\vec{E} = \frac{-V_0}{\rho \ln(a/b)}\hat{a}_\rho$$

$$\vec{D} = \epsilon_0 \vec{E}$$

$$\vec{D} = \frac{-V_0\epsilon_0}{\rho \ln(a/b)}\hat{a}_\rho$$

$$\vec{D} = \frac{V_0\epsilon_0}{\rho \ln(b/a)}\hat{a}_\rho$$

$$Q = \vec{D}\cdot d\vec{s}$$

$$Q = \left[\frac{V_0\epsilon_0}{\rho \ln(b/a)}\hat{a}_\rho\right]\left[2\pi\rho L\hat{a}_\rho\right]$$

$$Q = \frac{2\pi L V_0 \epsilon_0}{\ln(b/a)}$$

$$C = \frac{Q}{V_0}$$

$$\boxed{C = \frac{2\pi L\epsilon_0}{\ln(b/a)}}$$

7.(a) State and prove uniqueness theorem for solution of Laplace's equation. [7] CO3 L2

"If the solution of Laplace's equation satisfies the boundary condition then that solution is unique by whatever method it is obtained"

"the solution of Laplace's equation gives the field which is unique, satisfying the same boundary conditions, in a given region

Proof :- $\nabla^2 V = 0$

we take two solution of Laplace's equation V_1 and V_2

$\nabla^2 V_1 = 0 \qquad \nabla^2 V_2 = 0 \quad \Rightarrow \nabla^2(V_1 - V_2) = 0$

we assume V_1 and V_2 are solutions of Laplace equation. of V_1 on boundary V_{1b}

— " — V_2 — " — V_{2b}.

$V_b \rightarrow$ given potential value on boundary

$\therefore V_{1b} = V_{2b} = V_b$

$\Rightarrow V_{1b} - V_{2b} = 0$

considering the vector identity

$\vec{\nabla} \cdot (V\vec{D}) = V(\vec{\nabla} \cdot \vec{D}) + \vec{D} \cdot (\vec{\nabla} V)$

considering a scalar to be $(V_1 - V_2)$

considering the vector to be $\vec{\nabla}(V_1 - V_2)$

$\vec{\nabla} \cdot [(V_1 - V_2)\vec{\nabla}(V_1 - V_2)] = (V_1 - V_2)[\vec{\nabla} \cdot \vec{\nabla}(V_1 - V_2)] + [\vec{\nabla}(V_1 - V_2) \cdot \vec{\nabla}(V_1 - V_2)]$

taking volume integral on both sides

$\int_{vol} \vec{\nabla} \cdot [(V_1 - V_2)\vec{\nabla}(V_1 - V_2)] \, dv = \int_{vol} \underbrace{(V_1 - V_2)\nabla^2(V_1 - V_2)}_{0 \text{ [by hypothesis]}} \, dv + \int_{vol} [\vec{\nabla}(V_1 - V_2)]^2 \, dv \quad (1)$

$LHS = \oint_S [(V_1 - V_2)\vec{\nabla}(V_1 - V_2)] \cdot d\vec{S}$

$= \oint_S [(V_{1b} - V_{2b})\,\vec{\nabla}(V_{1b} - V_{2b})] \cdot d\vec{S}$

$= 0$

$\therefore \int_{vol} [\vec{\nabla}(V_1 - V_2)]^2 \, dv = 0$ [Note: an integral may be zero if either the integrand is everywhere zero, or the integrand is +ve in some regions and −ve in other regions, and the contributions cancel algebraically. In this case the 1st reason holds good as $[\vec{\nabla}(V_1 - V_2)]^2$ can not be −ve.

$\Rightarrow \vec{\nabla}(V_1 - V_2) = 0$

$V_1 - V_2 = \text{Constant}$

on the boundary

$V_1 - V_2 = V_{1b} - V_{2b} = 0$

$\Rightarrow V_1 - V_2 = 0$

$\boxed{V_1 = V_2}$

(b) Given the potential function $V(\rho, \phi) = \frac{V_0 \rho}{d} \cos\phi$. Check if $\nabla^2 V = 0$ using cylindrical co-ordinates. [3] CO3 L3

Soln: (a) $\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2}$

$= \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial}{\partial \rho}\frac{V_0 \rho \cos\phi}{d}\right) + \frac{1}{\rho^2}\frac{\partial^2}{\partial \phi^2}\left(\frac{V_0 \rho}{d}\cos\phi\right)$

$= \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{V_0}{d}\cos\phi\right) - \frac{1}{\rho^2}\frac{\partial}{\partial \phi}\left(\frac{V_0 \rho}{d}\sin\phi\right)$

$= \frac{1}{\rho}\frac{V_0 \cos\phi}{d} - \frac{1}{\rho^2}\frac{V_0 \rho}{d}\cos\phi = 0$

(b) Constant potential ...

OR

8.(a) Semi-infinite conducting planes at $\phi = 0$ and $\phi = \frac{\pi}{6}$ are separated by an infinitesimal insulating gap along z-axis. If $V(\phi = 0) = 0$ and $V(\phi = \frac{\pi}{6}) = 100V$, calculate V and **E** in the region between the plates. [6] CO3 L3

$\frac{d^2V}{d\phi^2} = 0$

$\therefore \frac{dV}{d\phi} = c_1$

here, $c_1 \rightarrow$ constant of integration.

$\therefore dV = c_1 \, d\phi$

Integrating, $V = c_1\phi + c_2$

$c_2 \rightarrow$ constant of integration

$0 = c_1 \cdot 0 + c_2$

$\therefore c_2 = 0$.

$\therefore V = c_1 \phi$

and $100 = c_1 \cdot \pi/6$

$\therefore c_1 = \frac{600}{\pi}$

$\therefore V = \frac{600}{\pi}\phi$ V

$\vec{E} = -\vec{\nabla} V = -\frac{1}{\rho}\frac{\partial V}{\partial \phi}\hat{a}_\phi = -\frac{600}{\pi\rho}\hat{a}_\phi$ V/m

(b) Calculate the numerical values for V and ρ_v in free space of $V = \frac{4yz}{x^2+1}$ at P (1, 2, 3). [4] CO3 L3

Soln. $V = \frac{4yz}{x^2+1}$ at $P(1,2,3)$ is obtained as,

$V_P = \frac{4.2.3}{1^2+1} = 12V$

$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$

$\frac{\partial V}{\partial x} = \frac{\partial}{\partial x}\left[4yz(x^2+1)^{-1}\right] = -4yz(x^2+1)^{-2}.2x$

$\frac{\partial^2 V}{\partial x^2} = \frac{\partial}{\partial x}\left[-4yz(x^2+1)^{-2}.2x\right]$

$= 8yz.2x(x^2+1)^{-3}.2x - 4yz(x^2+1)^{-2}.2x$

$= \frac{4yz}{(x^2+1)^2}\left[\frac{8x^2}{(x^2+1)} - 2\right] = \frac{-4yz}{(x^2+1)^3}(x^2+1-4x^2)$

$= \frac{-8yz(1-3x^2)}{(x^2+1)^3}$

$\frac{\partial^2 V}{\partial y^2} = 0$

$\frac{\partial^2 V}{\partial z^2} = 0$

Now $\nabla^2 V = -\frac{\rho_v}{\epsilon}$

$\Rightarrow \rho_v = -\epsilon \nabla^2 V = \frac{\epsilon . 8yz(1-3x^2)}{(x^2+1)^3}$

$\therefore$ At $(1,2,3)$, $\rho_v = \frac{8.854 \times 10^{-12} \times 8 \times 2 \times 3(1-3)}{(1^2+1)^3}$

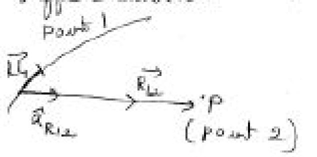
$= -106.248$ pC.

9.(a) State and explain Biot-Savart's law. [5] CO1 L1

(a) Biot-Savart's law :-

Let us consider a differential current element as a vanishingly small section of a current-carrying filamentary conductor, where a filamentary conductor is the limiting case of a cylindrical conductor of circular cross-section as the radius approaches zero.

We assume a current I flowing in a different vector length of the filament $\vec{dL}$. The Biot-Savart's law then states that, at any point P the mag. of the mag. field intensity produced by the differential element is proportional to the product of the current, the mag. of the differential length and the sine of the angle lying b/w the filament and a line connecting the filament to the point P at which the field is desired. Also, the magnitude of the mag. field intensity is inversely proportional to the square of the distance from the differential element to point P.

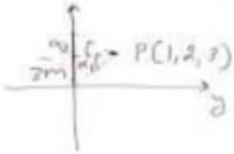


$$d\vec{H_2} = \frac{I_1 d\vec{L_1} \times \hat{a}_{R_{12}}}{4\pi R_{12}^2}$$

where $d\vec{H_2} \rightarrow$ mag. field intensity produced by a differential current element $I_1 \vec{dL_1}$. The direction of $d\vec{H_2}$ is into the page.

(b) A current filament carries a current of 10A in the $\mathbf{a_z}$ direction on the z axis. Find the magnetic field intensity $\mathbf{H}$ at point P(1,2,3) due to this filament if it extends from $z = 0$ to 5m. [5] CO1 L3

(b)



$\rho = \sqrt{x^2+y^2} = \sqrt{5}$

$\alpha_1 = -\tan^{-1}\frac{3}{\sqrt{5}} = -53.3^\circ$

$\alpha_2 = \tan^{-1}\left(\frac{2}{\sqrt{5}}\right) = 41.81^\circ$

$\therefore \vec{H} = \frac{I}{4\pi\rho}\left[\sin\alpha_2 - \sin\alpha_1\right]\hat{a}_\phi$

$= \frac{10}{4\pi\sqrt{5}}\left[\sin 41.81^\circ - \sin(-53.3^\circ)\right]\hat{a}_\phi$

$= 0.5225\,\hat{a}_\phi$

$H_x = \vec{H}\cdot\hat{a}_x = 0.5225(\hat{a}_\phi\cdot\hat{a}_x) = -.5225\sin\phi$

$H_y = \vec{H}\cdot\hat{a}_y = 0.5225(\hat{a}_\phi\cdot\hat{a}_y) = 0.5225\cos\phi$

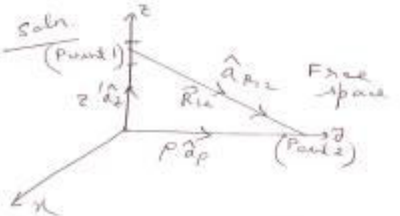
$\phi = \tan^{-1}\left(\frac{2}{1}\right) = 63.43^\circ$

$\therefore \vec{H} = -0.4673\,\hat{a}_x + 0.2337\,\hat{a}_y$ A/m

OR

10. Derive an expression for magnetic field intensity at a point P due to an infinitely long straight filament carrying a direct current I. [10] CO1 L2

Soln.



No variation with z or ϕ.

Point 2 chosen at z = 0 plane.

$\therefore \vec{r} = \rho\hat{a}_\rho$

$\vec{r'} = z'\hat{a}_z$

$\vec{R}_{12} = \vec{r} - \vec{r'}$

$\vec{R}_{12} + z'\hat{a}_z = \rho\hat{a}_\rho$

$\Rightarrow \vec{R}_{12} = (\rho\hat{a}_\rho - z'\hat{a}_z)$

$\therefore \hat{a}_{R_{12}} = \frac{(\rho\hat{a}_\rho - z'\hat{a}_z)}{\sqrt{\rho^2+z'^2}}$

we take $\vec{dl} = dz'\hat{a}_z$

$\therefore \vec{dH_2} = \frac{I\,dz'\hat{a}_z \times (\rho\hat{a}_\rho - z'\hat{a}_z)}{4\pi(\rho^2+z'^2)^{3/2}}$

$\therefore \vec{H_2} = \int_{-\infty}^{\infty} \frac{I\,dz'\hat{a}_z \times (\rho\hat{a}_\rho - z'\hat{a}_z)}{4\pi(\rho^2+z'^2)^{3/2}}$

$= \frac{I}{4\pi}\int_{-\infty}^{\infty}\frac{\rho\,dz'\,\hat{a}_\phi}{(\rho^2+z'^2)^{3/2}} = \frac{I\rho\hat{a}_\phi}{4\pi}\int_{-\infty}^{\infty}\frac{dz'}{(\rho^2+z'^2)^{3/2}}$

let $z' = \rho \tan\theta$

$\Rightarrow dz' = \rho \sec^2\theta \, d\theta$

at, $z' = -\infty$, $\theta = -\pi/2$

$z' = +\infty$, $\theta = \pi/2$

$$\vec{H}_2 = \frac{I \rho \hat{a}_\phi}{4\pi} \int_{-\pi/2}^{\pi/2} \frac{\rho \sec^2\theta \, d\theta}{(\rho^2 + \rho^2 \tan^2\theta)^{3/2}}$$

$$= \frac{I \rho^2 \hat{a}_\phi}{4\pi \rho^3} \int_{-\pi/2}^{\pi/2} \frac{\sec^2\theta}{\sec^3\theta} d\theta = \frac{I \hat{a}_\phi}{4\pi\rho} \int_{-\pi/2}^{\pi/2} \cos\theta \, d\theta$$

$$= \frac{I \hat{a}_\phi}{4\pi\rho} \left[\sin\theta\right]_{-\pi/2}^{\pi/2} = \frac{I \hat{a}_\phi \, 2}{{}_2\cancel{4}\pi\rho} = \frac{I}{2\pi\rho} \hat{a}_\phi \ A/m$$