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CMRIT — CMR INSTITUTE OF TECHNOLOGY, BENGALURU. ACCREDITED WITH A+ GRADE BY NAAC

INTERNAL ASSESSMENT TEST – I

Sub:	DIGITAL SIGNAL PROCESSING							Code:	15EC52
Date:	18 / 09 / 2017	Duration:	90 mins	Max Marks:	50	Sem:	V	Branch:	ECE(D),TCE(B)

Answer any 5 full questions

		Marks	CO	RBT
1.	Derive an expression for DFT of a discrete time signal $x[n], 0 \le n \le N-1$. Explain clearly how $x[n]$ can be reconstructed from its DFT.	[10]	CO1	L2
2.	Find the 6-point DFT of the sequence $x[n] = [1,2,3,1]$. Plot the magnitude and the phase spectra.	[10]	CO1	L2
3.	Without explicitly determining the DFT $X[k]$ of the sequence $x[n] = [2,1,1,0,3,2,0,3,4,6]$, evaluate the following expressions. a) $X(0)$ b) $X(5)$ c) $\sum_{k=0}^{9} X[k]$ d) $\sum_{k=0}^{9} \lvert X[k]\rvert^2$ e) $\sum_{k=0}^{9} e^{j\frac{4\pi k}{5}} X[k]$	[10]	CO1	L3
4(a).	Derive the relationship between DFT and Z-transform of a finite length sequence $x[n], 0 \le n \le N-1$.	[05]	CO1	L2
4(b).	Find the Z-transform of the sequence $x[n] = [0.5, 0, 0.5, 0]$. Using Z-transform find the DFT of $x[n]$.	[05]	CO1	L2
5(a).	Prove the following properties of DFT a) Circular Time Shift Property b) Circular Frequency Shift Property	[06]	CO1	L2
5(b).	Let $x[n]$ be a 4-point sequence with 4-point DFT $X[k]$. Express the DFT of the following signals in terms of $X[k]$. a) $y[n] = x[(n-1)_4]$ b) $z[n] = \cos\left(\frac{\pi n}{2}\right) x[n]$	[04]	CO1	L2

6 (a).	Prove that *a)* $DFT[DFT[x[n]]] = NX[-n]$ *b)* $DFT[DFT[DFT[DFT[x[n]]]]] = N^2 x[n]$	[06]	CO1	L3
6(b).	Find the DFT of the sequence $x[n] = 0.5^n, 0 \leq n \leq 3$ by evaluating the DFT of $x[n] = a^n, 0 \leq n \leq N-1$.	[04]	CO1	L2
7.	Find the 4-point circular convolution of $x[n] = [2,1,2,1]$ and $h[n] = [1,2,3,4]$ using Stockham's method (DFT-IDFT method).	[10]	CO1	L2
8.	Consider an FIR filter with impulse response $h[n] = [3,2,1,1]$. If the input is $x[n] = [1,2,3,3,2,1,-1,-2,-3,5,6,-1,2,0,2,1]$, find the output using overlap-add method. Use 7-point circular convolution.	[10]	CO1	L2

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Solutions and Scheme of Evaluation

1 $X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$, ... (1) ---- (1)

$X(\omega + 2\pi) = X(\omega)$ ... (2)
$\forall \omega$.

$$X\left(\frac{2\pi}{N}k\right) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\frac{2\pi}{N}kn}.$$

$k = 0$ to $N-1$

$$= \sum_{l=-\infty}^{\infty} \sum_{n=lN}^{lN+N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{l=-\infty}^{\infty} \sum_{n=0}^{N-1} x(n+lN) e^{-j\frac{2\pi}{N}k(n+lN)}$$

$$= \sum_{n=0}^{N-1} \sum_{l=-\infty}^{\infty} x(n+lN) e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{n=0}^{N-1} x_p(n) e^{-j\frac{2\pi}{N}kn} \quad \text{... (2)} \quad \text{--- (4M)}$$

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x_p(n) e^{-j\frac{2\pi}{N}kn}. \quad \text{... (3)}$$

$\therefore X\left(\frac{2\pi}{N}k\right) = N a_k$.

$$a_k = \frac{1}{N} X\left(\frac{2\pi}{N}k\right) \quad \text{... (4)}$$

$$x_p(n) = \sum_{k=0}^{N-1} a_k e^{j\frac{2\pi}{N}kn}.$$

$n = 0$ to $N-1$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X\left(\frac{2\pi}{N}k\right) e^{j\frac{2\pi}{N}kn}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N}kn}. \quad \text{... (5)} \quad \text{--- (2M)}$$

To avoid aliasing:

Number of DFT points $\geq$ length of $x(n)$. ---(1M)

Under no aliasing.

$x(n) = x_p(n), \quad 0 \leq n \leq N-1$.

$$\therefore x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi}{N}kn}, \quad n = 0 \text{ to } N-1$$

where
$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}, \quad k = 0 \text{ to } N-1$$ ---(2M)

2 $x(n) = (1, 2, 3, 1, 0, 0)$

$$X(k) = \sum_{n=0}^{5} x(n) e^{-j\frac{2\pi}{N}kn}, \quad k = 0 \text{ to } N-1$$

$X(0) = 1+2+3+1 = 7$

$X(3) = 1-2+3-1 = 1$.

$$X(1) = \sum_{n=0}^{5} x(n) e^{-j\frac{2\pi}{6}n}$$

$= -0.5 - 4.33j$

$$X(2) = \sum_{n=0}^{5} x(n) e^{-j\frac{2\pi}{6}2n}.$$

$= -0.5 + 0.866j$

$X(4) = X^*(2) = -0.5 - 0.866j$

$X(5) = X^*(1) = -0.5 + 4.33j$ (8M)

k	0	1	2	3	4	5
$\lvert X(k) \rvert$	7	4.359	1	1	1	4.359
$\angle X(k)$	0	-1.6858	~~4.19~~ 2.0944	0	-2.0944	1.6858

(2M)

3 a) $X(0) = \sum_{n=0}^{9} x(n)$

$= 22$ (2M)

b) $X(5) = 2-1+1-0+3-2+0-3+4-6$

$= -2$ (2M)

c) $\sum_{k=0}^{9} X(k) = 10\,x(0)$ (2M)

$= 20$

d) $\sum_{k=0}^{9} |X(k)|^2 = 10\left[2^2+1^2+1^2+3^2+2^2+3^2+4^2+6^2\right].$ (2M)

$= 800$

e) $\sum_{k=0}^{9} e^{j\frac{2\pi}{10}4k} X(k) = 10\left(x(4)\right)$ (2M)

$= 30$

4a $X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}, \quad k = 0 \text{ to } N-1$

$X(z) = \sum_{n=0}^{N-1} x(n) z^{-n}, \quad \text{ROC}: |z| \neq 0$

$X(k) = X(z)\Big|_{z = e^{j\frac{2\pi}{N}k}}, \quad k = 0 \text{ to } N-1$ (2M)

$X(z) = \frac{1-z^{-N}}{N} \sum_{k=0}^{N-1} \frac{X(k)}{1-e^{j\frac{2\pi}{N}k} z^{-1}}$ (3M)

4b $X(z) = 0.5 + 0.5 z^{-2}, \quad |z| \neq 0$ (2M)

$$X(k) = 0.5 + 0.5\,e^{-j\pi k}$$

$$= (1, 0, 1, 0) \quad \text{(3M)}$$

5(a) a) $x(n) \xleftrightarrow[N]{DFT} X(k)$

$$x((n-l)_N) \xleftrightarrow[N]{DFT} e^{-j\frac{2\pi}{N}lk} X(k) \quad \text{(3M)}$$

b) $x(n) \xleftrightarrow[N]{DFT} X(k)$

$$x(n)e^{j\frac{2\pi}{N}ln} \xleftrightarrow[N]{DFT} X((k-l)_N) \quad \text{(3M)}$$

5b a) $Y(k) = e^{-j\frac{2\pi}{4}k} X(k)$ (2M)

b) $Z(k) = \frac{1}{2}\left[X(k-1)_4 + X(k+1)_4\right]$ (2M)

6a DFT of $X(k) = \sum_{k=0}^{N-1} X(k)\, e^{-j\frac{2\pi}{N}kn}$

$$= \sum_{k=0}^{N-1} \sum_{m=0}^{N-1} x(m) e^{-j\frac{2\pi}{N}km} e^{-j\frac{2\pi}{N}kn}$$

$$= \sum_{m=0}^{N-1} x(m) \sum_{k=0}^{N-1} e^{-j\frac{2\pi}{N}k(m+n)}$$

$$= N x(-n) \quad \text{(4M)}$$

$$DFT(DFT(DFT(DFT(x(n))))) = N^2 x(n) \quad \text{(2M)}$$

(5)

6b $X(k) = \sum_{n=0}^{N-1} a^n e^{-j\frac{2\pi}{N}kn}$.

$k = 0 \text{ to } N-1$

$= \sum_{n=0}^{N-1} \left(a e^{-j\frac{2\pi}{N}k}\right)^n$

$= \dfrac{1 - a^N}{1 - a e^{-j\frac{2\pi}{N}k}}$, $a \neq 1$, $k \neq 0$ (2M)

If $x(n) = 0.5^n$ $0 \le n \le 3$.

$X(k) = \dfrac{1 - 0.5^4}{1 - a e^{-j\frac{2\pi}{4}k}}$ (2M)

$k = 0 \text{ to } 3$

(3M)

7. $X_1(k) = (6, 0, 2, 0)$

$X_2(k) = (10, -2+2j, -2, -2-2j)$ (3M)

$X(k) = X_1(k)\, X_2(k)$

$= (60, 0, -4, 0)$ (1M)

(3M)

$x(n) = (14, 16, 14, 16)$

8 $h(n) = (3, 2, 1, 1, 0, 0, 0)$

$x_1(n) = (1, 2, 3, 3, 0, 0, 0)$

$x_2(n) = (2, 1, -1, -2, 0, 0, 0)$

$x_3(n) = (-3, 5, 6, -1, 0, 0, 0)$

$x_4(n) = (2, 0, 2, 1, 0, 0, 0)$

$y_1(n) = (3, 8, 14, 18, 11, 6, 3)$ (2M)

$y_2(n) = (6, 7, 1, -5, -4, -3, -2)$ (2M)

$y_3(n) = (-9, 9, 25, 11, 9, 5, -1)$ (2M)

$y_4(n) = (6, 4, 8, 9, 4, 3, 1)$ (2M)

Final output:

$y(n) = (3, 8, 14, 18, 17, 13, 4, -5, -13, 6, 23, 11,$
$15, 9, 7, 9, 4, 3, 1)$ (2M)