1. Define the following with diagrams:

(a) Critical angle: Def+Dig

1(a)

Critical Angle:-

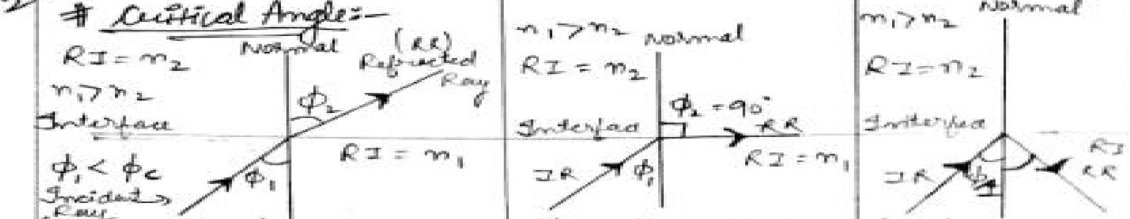


Fig 4:- Critical angle

→ When the angle of incidence ϕ_1, is progressively increased, there will be progressive increase of refractive angle ϕ_2.

→ At some condition (at some angle of incidence ϕ_1), the refractive angle ϕ_2 becomes 90° to the normal.

→ When this happens the refracted light ray travels along the interface as shown in Fig 4(b)

→ The angle of incidence (ϕ_1) at the point at which the refractive angle (ϕ_2) becomes 90° to the normal, is called as critical angle. It is denoted by ϕ_c.

The critical angle is defined as the minimum angle of incidence (ϕ_1) at which the ray strikes the interface of two media and causes an angle of refraction (ϕ_2) equal to 90°.

• Derivation:- From fig 4(b)

At critical angle, $\phi_1 = \phi_c$

and $\phi_2 = 90°$

Applying Snell's law at the interface

$$n_1 \sin\phi_1 = n_2 \sin\phi_2$$

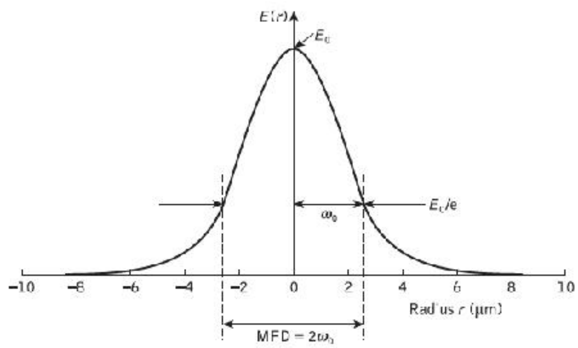
$$\Rightarrow \sin\phi_c = \frac{n_2}{n_1}\sin 90°$$

$$\because \sin 90° = 1$$

$$\therefore \sin\phi_c = \frac{n_2}{n_1}$$

critical angle $\phi_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$

(b) Mode field diameter:



1(b) # Mode feild Diameter :- MFD & Spot size :-

→ It is a fundamental parameter of a single mode fiber.

→ Can be determined from the mode-feild distribution of the fundamental LP_{01} mode.

→ It is analogous to the core dia in multimode fibers, except that in single mode fibers not all the light that propogates through the fiber is carried in the core.

→ For measuring MFD, the $\vec{E}$ feild distribution is assumed to be gaussian

$$E(r) = E_0 \exp(-r^2/w_0)$$

where r = radius

E_0 - feild at zero radius

→ MFD is the distance b/w the opposite $\frac{1}{e}$ (= 0.37) feild amplitude points and the power $\frac{1}{e^2}$ (= 0.135) points in relation to the corresponding values on the fiber axis as shown in fig.

MFD = $2w_0$, where w_0 = nominal half width of the I/P excitation.

(c) V-number: Def+Dig

V- Number :-

1(c) or Normalized freq :-

$$V = \frac{2\pi a}{\lambda}(NA)$$

$$= \frac{2\pi a}{\lambda} n_1 (2\Delta)^{1/2}$$

$$= \frac{2\pi a}{\lambda} (n_1^2 - n_2^2)^{1/2}$$

where, a = core radius

λ = operating wavelength

n_1 = core RI

n_2 = cladding RI

NA = Numerical Aperture

Δ = Relative RI index

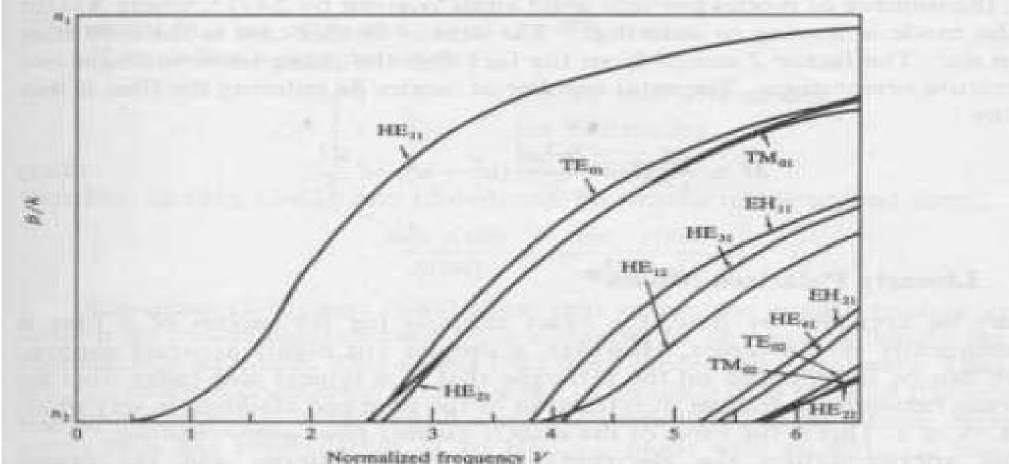
→ determines how many modes a fiber can support.

→ The no of modes that can exist in a waveguide as a functn of V in terms of normalized propogatn const b

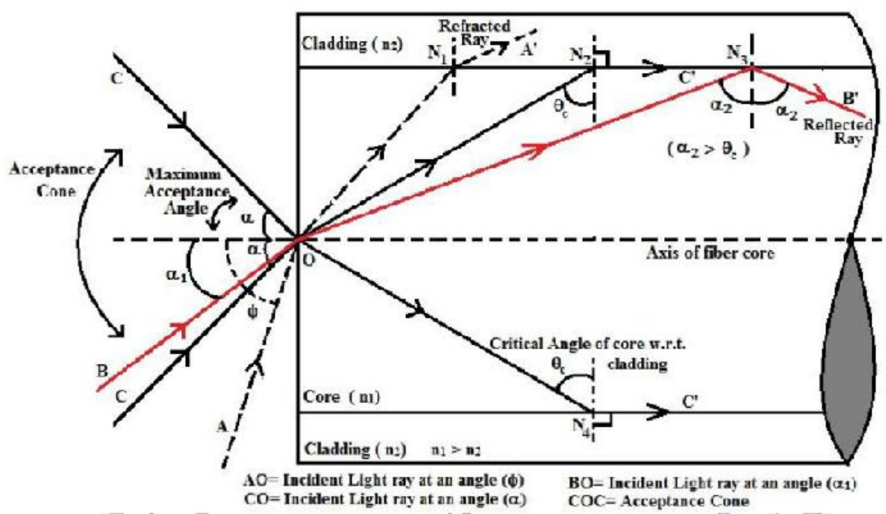
$$b = \frac{a^2 w^2}{V^2} = \frac{(\beta/k)^2 - n_2^2}{n_1^2 - n_2^2}$$

A plot of b (in terms of β/k) as a function of V for a few lower order modes is shown in fig.

$$V = \frac{2\pi a}{\lambda}(n_1^2 - n_2^2)^{1/2} \leq 2.405$$



(d) Acceptance angle and acceptance cone: Def+Dig



Assuming that light is launched as meridional rays into the optical fiber, let us now carry out a simple analysis. For that let us concentrate on figure 3.7 below. The figure shows a cross-section of an optical fiber with a core of refractive index n_1 and a cladding of refractive index n_2. The incident ray AO (shown by dotted line) is incident at an angle ϕ with the axis of the fibre. The refracted ray for AO in the core (dotted line ON_1) fails to be incident on the core-cladding interface at angle greater or equal to the critical angle of the core w.r.t. cladding and hence refracts out of the core and is lost to the cladding. In other words, the angle of incidence of a refracted ray at the core-cladding interface in turn depends on the initial angle at which the incoming ray was launched into the fiber. If this launching angle (with the fiber axis) is decreased, the angle of incidence which the refracted ray makes at the core-cladding interface increases. If this increase is such, as to exceed the critical angle of the core-cladding interface, then total internal reflection of the refracted ray takes place and the light remains in the core and is guided along the fiber. The ray CO is launched into the fiber at such an angle ‘α’ that its refracted ray is incident at the core-cladding boundary at its critical angle ‘θ_c’. If any light ray is launched at an angle more than α then the refracted ray just refracts out to the cladding because the angle of incidence of its refracted ray at the core-cladding interface is less than the critical angle. Thus the angle α is indicative of the maximum possible angle of launching of a light ray that is accepted by the fiber. Consequently, the angle α is called the angle of acceptance of the fiber core. Since the optical fiber is symmetrical about its axis, it is very clear that all the launched rays, which make an angle α with the axis, considered together, form a sort of a cone. This cone is called the acceptance cone of the fiber as shown in the above figure. Any launched ray that lies within this cone is accepted by the fiber and the light of this ray is guided along the fiber by virtue of multiple TIRs as shown by the red ray BO in the figure 3.7.

(e) Cut-off freq (for single mode fiber):

It may be noted by rearrangement of Eq. (2.70) that single-mode operation only occurs above a theoretical cutoff wavelength λ_c given by:

$$\lambda_c = \frac{2\pi a n_1}{V_c}(2\Delta)^{\frac{1}{2}} \tag{2.98}$$

where V_c is the cutoff normalized frequency. Hence λ_c is the wavelength above which a particular fiber becomes single-moded. Dividing Eq. (2.98) by Eq. (2.70) for the same fiber we obtain the inverse relationship:

$$\frac{\lambda_c}{\lambda} = \frac{V}{V_c} \tag{2.99}$$

Thus for step index fiber where $V_c = 2.405$, the cutoff wavelength is given by [Ref. 43]:

$$\lambda_c = \frac{V\lambda}{2.405} \tag{2.100}$$

2. (a) Estimate the maximum core diameter for an optical fiber with refractive index of core 1.48 and relative index difference of 1.6%, in order that it may be suitable for single mode operation for an operating wavelength of 0.9 micrometer. Further estimate the maximum core diameter for a single mode operation when the relative index difference is reduced by a factor of 10..

Solu 2 (a)

Part A :- when $\Delta = 1.6\%$

dia = ?

raidus = ?

$\Delta = 1.6\%$

$n_1 = 1.48$

$\lambda = 0.9\ \mu m$, $V = 2.405$

$$V = \frac{2\pi a}{\lambda} n_1 (2\Delta)^{1/2}$$

$$a = \frac{V\lambda}{2\pi n_1 (2\Delta)^{1/2}}$$

$$= \frac{2.405 \times 0.9 \times 10^{-6}}{2\pi \times 1.48\ (2 \times 0.016)^{1/2}}$$

$$= \frac{2.405 \times 0.9 \times 10^{-6}}{2\pi \times 1.48 \times 0.17888}$$

$a = \underline{1.30\ \mu m}$

dia $= 1.30 \times 2 = \underline{2.60}\ \mu m.$

Part B :- when $\Delta = \frac{1.6}{10}\% = 0.016$

$$a = \frac{V\lambda}{2\pi n_1 (2\Delta)^{1/2}}$$

$$= \frac{2.405 \times 10^{-6} \times 0.9}{2\pi \times 1.48 \times \sqrt{0.0032}}$$

$a = \underline{4.1168\ \mu m}$

dia $= 2 \times 4.1168 = \underline{8.2336}\ \mu m.$

(b) Explain different types of absorption losses.

⇒ A number of mechanisms are responsible for signal attenuat^n within fibers

① Material Absorption:—

→ It is a loss due to material composition and the fabricat^n process for the for the fiber due to which some of the Xmitted optical power dissipated in terms of heat in the fiber

⇒ The absorpt^n of the light signal may be

① intrinsic	② extrinsic
interact^n with Caused by one or more components of the glass	Caused by impurities within the glass

⇒ Basically, energy of the signal get absorbed and the signal amplitude loses.

(A) Intrinsic Absorpt^n :—

⇒ It occurs when the material is in perfect state with no density variat^n, impurities, material inhomogeneities etc.

⇒ It sets the fundamental lower limit of absorpt^n for any particular material.

→ It results from electronic absorpt^n bands in UV region and from

- atomic vibration bands in the near-Infra red region
- The electronic absorpt bands are associated with the band gaps of the amorphous glass materials.
- Absorptn occurs when a photon interacts with an e^- in the valence band and excites it to higher energy level
- The UV edge of absorptn bands of both amorphous & crystalline materials follow the empirical relationship

$$\boxed{\alpha_{UV} = C e^{E/E_0}}$$

i.e Urbach's rule

C = empirical const

E = photon energy & $E = h\nu$

$$\left[\text{or } E(eV) = \frac{1.2406}{\lambda(\mu m)}\right]$$

$$\therefore E \propto \frac{1}{\lambda}$$

$\therefore$ UV absorption decays exponentially with rising λ.

~~In the near infrared~~

$$\alpha_{UV} = \frac{154.2x}{46.6x + 60} \times 10^{-2} \exp\left(\frac{4.63}{\lambda}\right)$$

UV loss contribution in dB/km at any λ as a function of the mole fraction x of GeO_2.

2) → ~~In the near Infra-red region the loss is determined by~~

- In near IR region the it is associated with the characteristic vibratn freq of particular chemical bond b/w atoms of which the fiber is composed.
- An interactn b/w vibrating bond & the em field of optical signal results in X-fer of energy from the field to the bond which results in absorption

$$\boxed{\alpha_{IR} = 7.81 \times 10^{11} \exp\left(-\frac{48.48}{\lambda}\right)}$$

loss in IR region in dB/km for $GeO_2 - SiO_2$ glass.

- Acc to absorptn curve, GeO_2 doped fiber material are the most desirable.

Extrinsic Absorption (Part III)

(1) when fibers are prepared by melting techniques, extrinsic absorption from transition metal element impurities takes place which are the major source of signal attenuation due to extrinsic absorption.

→ eg Cr^{3+}, C^{2+}, Cu^{2+}, Fe^{2+}, Ni^{2+}
Fe^{3+}, Mn^{3+}, V^{4+}

Common metallic impurities found in a glass

→ mainly Cu & Cr in their worst valence state can cause attenuation ≃ 1 dB/km in near IR region

→ These impurities can be reduced to acceptable levels by glass refining techniques eg VPO

(2) due to hydroxy ions (OH) water dissolved in the glass

- these ions are bonded into the glass structure & have vibrations which occur at λ ≃ 2.7 to 4.2 μm.

- ~~due to~~ these vibrations will give rise to overtones appearing almost harmonically at 1.38, 0.95 & 0.72 μm.

(3) due to direct melt method losses ≃ 1 to dB/km

(4) & due to VAP ≃ 1 or 2 order of lower magnitude.

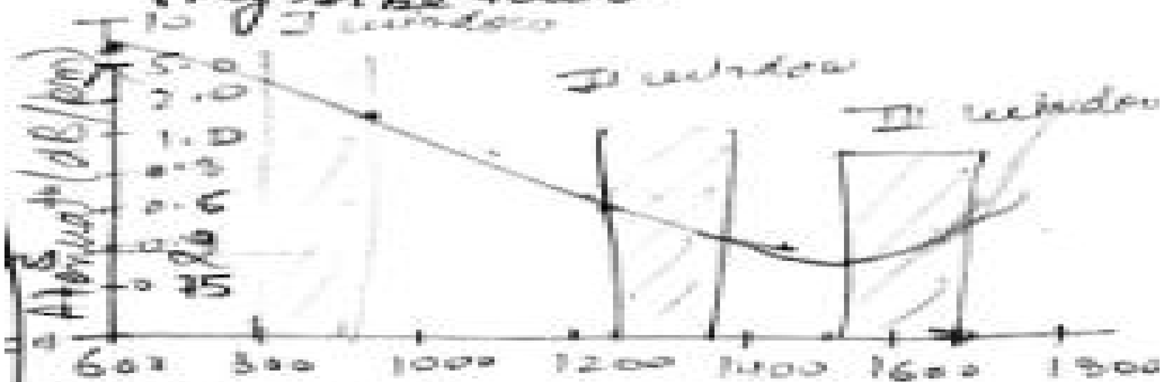
(4) Impurity absorptⁿ losses occurs due to

(i) electronic x-sitⁿ b/w the energy levels associated with incompletely filled inner subshell of these ions or &

(ii) b/c of charge x-sitⁿ from one ion to another

(b) Presence of OH^- ions impurities in mainly due to the use of oxyhydrogen flame used for hydrolysis reactⁿ of $SiCl_4$, $GeCl_4$ etc.

eg → If the loss due to attenuation have to be maintained to less than 20 dB/km then less than a few ppb impurity only sl be there.



(5) Atomic defects

→ imperfectⁿ in the atomic structure of the fiber material.

Eg — missing molecules, high-density clusters of atom groups or oxygen defects in the glass structure

→ Usually these losses are negligible compared to intrinsic & extrinsic

→ but can be significant if the

→ fiber is exposed to ionizing radiation
eg which occur in nuclear
reactor environment / medical —
radiation therapies, space
missions etc.

— Radiation damages a material
by changing its internal structure

— The damage effect d/on the
energy of the ionizing particles
or rays (eg. gamma rays) &
radiation flux & the fluence (ie
particles per square cm)

Total dose a material receives
(flux)
is expressed in rad (Si), which
is a measure of radiation absorb
in bulk Si. defined as:

1 rad(Si) = 100 erg/g = 0.01 J/kg

→ Main response of a fiber to the
ionizing radiation is an ↑se in
attenuation due to creation of
atomic defects or attenuation cent
that absorb optical energy.

→ higher the radiation level
larger the attenuation.

Note → LWPF
low water peak fiber
or dry fiber

LWPF permits the transmission of
optical signals over ~~the~~ range of
1.260 to 1.675 μm with losses
less than 0.4 dB/km.

3.(a) Determine the cut-off wavelength for a step index fiber to exhibit single mode operation when core refractive index and radius are 1.46 and 4.5 micrometer respectively, with relative index difference being 0.25%.

$$\lambda_c = \frac{2\pi a n_1 (2\Delta)^{\frac{1}{2}}}{2.405} = \frac{2\pi 4.5 \times 1.46 (0.005)^{\frac{1}{2}}}{2.405} \, \mu m$$

$$= 1.214 \, \mu m$$

$$= 1214 \, nm$$

Hence the fiber is single-moded to a wavelength of 1214 nm.

(b) Explain different types of scattering losses.

Scattering losses :-

1a

→ arise from (1) microscopic variations in material density → glass is composed of a randomly connected n/w of molecules due to which naturally the structure contains regions in which the molecular density is higher or lower than the average density of glass

(2) Compositional fluctuations :- ∵ glass is made-up of several oxides SiO_2, GeO_2 and P_2O_5 . ∴ compositional fluctuations can occur

→ Because of these two ~~variations~~ variation will arise in the RI.

Linear Scattering :- ①

→ Cause transfer of some or all of the optical power contained within one propagating mode to be transferred linearly (proportionally to the mode power) into diff mode.

→ Hence due the which loss of power will be there.

→ This process tends to result in attenuation of the signal as the transfer may be to a leaky or radiation mode which do not continue to propagate within fiber core, but is radiated from the fiber.

→ with all linear processes there will be no change ~~in~~ of frequency on scattering.

→ Types :- 1. Rayleigh Scattering
2. Mie Scattering

→ Both are the result of non ~~ideal~~ ideal physical properties of the manufactured fiber.

1. Rayleigh Scattering :-

→ ~~Scattering losses in glass arise from microscopic variations in the material density~~

→ Scattering losses in glass arise due to :
(a) microscopic variations in the material density
(b) compositional fluctuations

→ Both the factors give rise to variation in the refractive index.

(2)

→ Both the inhomogeneities are frozen into glass lattice on cooling.

→ The compositional variations may be reduced by improved fabricat^n, but the refractive index fluctuations cannot be avoided.

→ All the inhomogeneities are of random nature and are occuring on a small scale compared with the wavelength of light. α (loss)

→ The attenuation loss^n due to density fluctuations ~~are~~ ~~is~~

α loss directly proportional to $\frac{1}{\lambda^4}$.

→ Scattering of signal will be almost in all directions.

→ For single component glass, the Rayleigh scattering coefficient is γ_R.

$$\gamma_R = \frac{8\pi^3}{3\lambda^4} n^8 p^2 \beta_c k T_F$$

γ_R = Rayleigh Scattering Coefficient

λ = optical wavelength

n = R.I. of the medium

p = average photoelastic coefficient

β_c = Isothermal compressibility at a fictive temp T_F.

k = Boltzman's Constant.

T_F : - fictive temp: defined as the temperature at which the glass can reach a state of thermal equilibrium and is closely related to the anneal temp.

$$\mathcal{L} = \exp(-\gamma_R L)$$

$\mathcal{L}$ = Transmission loss factor (transmissivity)

& L = length of the fiber.

→ $\because \gamma_R \propto \frac{1}{\lambda^4}$

$\therefore$, Rayleigh scattering is strongly reduced by operating at the longest possible λ.

2. <u>Mie Scattering</u> :-

→ This scattering occur at inhomogeneities which are comparable in size with the guided wavelength.

→ ~~This These results from the non perfect cylindrical structure of the fiber~~

→ Mie scattering losses arise due to:-

ⓐ non perfect cylindrical structure of the fiber

ⓑ fiber imperfections such as:-

↳ Irregularities in the core-cladding interface

↳ core-cladding refractive index differences along the fiber length.

↳ diameter fluctuations

↳ strains and bubbles.

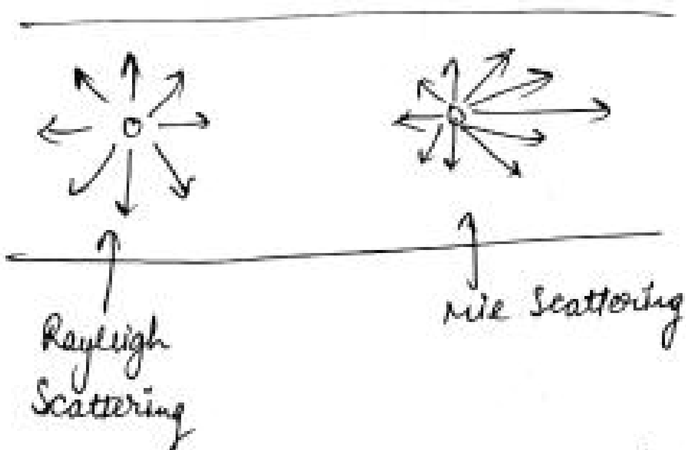
→ When the scattering inhomogeneity size is greater than $\lambda/10$, the scattered intensity can be very large which has an angular dependence.

→ The scattering created by the inhomogeneities (with size larger than the λ) is mainly in the forward direction c/a Mie scattering.

(4)

→ The inhomogeneities Can be reduced by:-

- Removing imperfections due to glass manufacturing process
- carefully controlled extrusion and coating of the fiber
- rising the fiber guidance by rising the relative refractive index diff



Concept:- Because of variation in the RI (diff than n_2 & n_1) when the light is launched within fiber incidents on the diff RI inhomogeneity will get scattered in all direction.

↓

And as we know that for a sustained guiding of light, the light ~~rays~~ must be confined within NA cone OR it should propogate in a particular mode

↓

But the scattered light will propogate through leaky mode due to which there will be loss of power

Eg

Nonlinear Scattering :-

→ ~~Due to this type of scattering~~

→ It causes the optical power from one mode to be transferred in ~~either~~
- ↳ to the forward direction
- ↳ or to the backward direction
- ↳ or to the same direction

} to the same mode or the other mode

→ Depend on the optical power ~~den~~ density, so become significant only above threshold power levels.

→ Types :- 1. Stimulated Brillouin Scattering (SBS)
2. Stimulated Raman Scattering (SRS)

→ These give an optical gain but with a shift in frequency & hence it contributes to attenuation for light transmission at a specific λ.

1. SBS :-

⇒ In SBS, modulation of light take place through thermal molecular vibrations within the fiber. → happens due to adiabatic compression & decompression

→ The scattered light appears as upper and lower sidebands which are separated from the incident light by the modulation freq

⊛→ In this process the incident photon produces a phonon of acoustic freq and a scattered photon

* phonon → It is a quantum of elastic wave in a crystal lattice

* Acoustic waves ⇒ are a type of ~~lon~~ longitudinal waves that propogate by means of adiabatic compression & decompression.

* <u>Longitudinal waves</u> :- waves that have the same direction of vibration as their direction of travel.

→ In SBS, there is an optical freq shift which varies with the scattering angle, b/c freq of sound waves varies with acoustic λ.

* Types of quasiparticles:-
  1) Phonon → mass oscillation (acoustic) modes
  2) Polaron → charge displacement modes (in dielectrics)
  3) Magnon → magnetic spin oscillation modes (in magnetic materials.

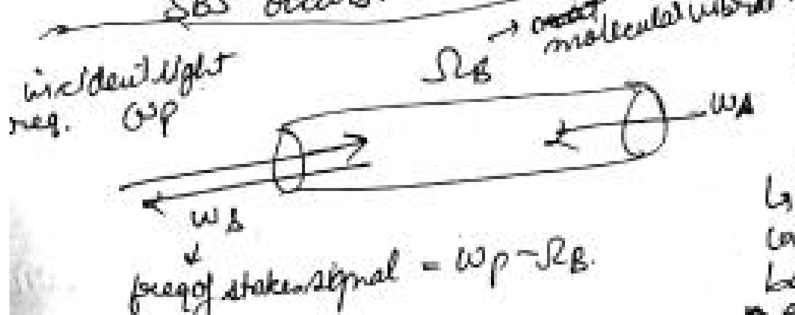
→ The freq shift is maximum in the backward direction, and reduces to almost zero in the forward direction. Hence SBS is mainly a backward process.

→ Threshold power density for SBS is:

$$P_B = 4.4 \times 10^{-3} d^2 \lambda^2 \alpha_{dB} \nu \text{ watts.}$$

d = fiber core dia (μm)
λ = operating λ. (μm)
α_{dB} = fiber attenuation (dB/km)
ν = Source bandwidth (laser) * in GHz

→ P_B is the threshold optical power which must be launched into single-mode optical fiber before SBS occurs.



Summary of SBS
↳ SBS arises when lightwaves scatter from acoustic waves.
↳ Resultant scattered wave propagates in the backward direction which experiences gain from the incoming signal which leads to depletion of i/p power

2. <u>SRS:-</u> (7)

→ SRS is interaction between light waves and the vibrational modes of silica molecules

→ If a photon with energy $h\nu_1$ incident on a molecule ~~to~~ having ~~for~~ vibrational freq. ν_m, the molecule can absorb some energy from the photon

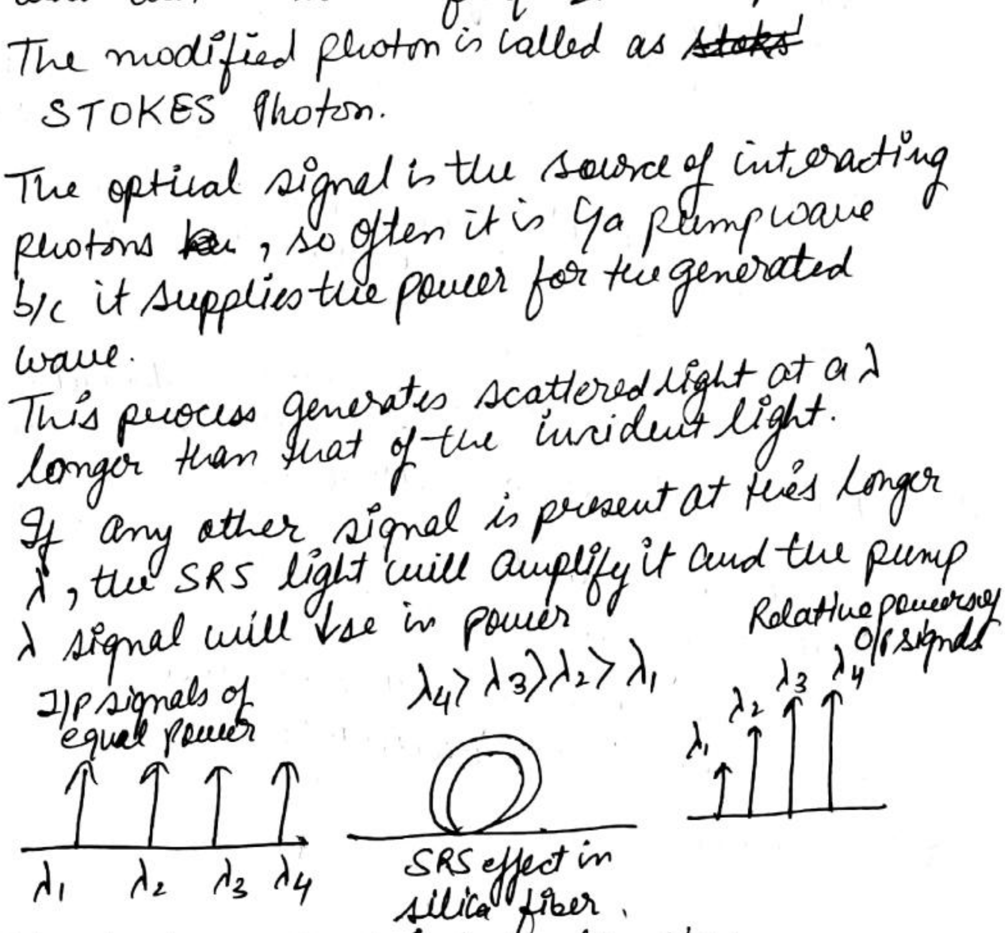
→ In this interaction, the photon is scattered ~~with~~ with a lower freq ν_2 & energy $h\nu_2$.

→ The modified photon is called as ~~stoks~~ STOKES Photon.

→ The optical signal is the source of interacting photons ~~ka~~, so often it is ya pump wave b/c it supplies the power for the generated wave.

→ This process generates scattered light at a λ longer than that of the incident light.

→ If any other signal is present at this longer λ, the SRS light will amplify it and the pump λ signal will ↓se in power



→ This effect can occur in both directions

→ $\boxed{P_R = 5.9 \times 10^{-2} d^2 \lambda \alpha_{dB} \text{ watts.}}$

P_R = Threshold power

4 (a) A multimode step index fiber with core diameter of 80μm and a relative index difference of 1.5% is operating at a wavelength of 0.85μm. If the core refractive index is 1.48, estimate the normalized frequency for the fiber and number of guided modes.

$$V \simeq \frac{2\pi}{\lambda} a n_1 (2\Delta)^{\frac{1}{2}} = \frac{2\pi \times 40 \times 10^{-6} \times 1.48}{0.85 \times 10^{-6}} (2 \times 0.015)^{\frac{1}{2}} = 75.8$$

(b) The total number of guided modes is given by Eq. (2.74) as:

$$M_s \simeq \frac{V^2}{2} = \frac{5745.6}{2}$$

$$= 2873$$

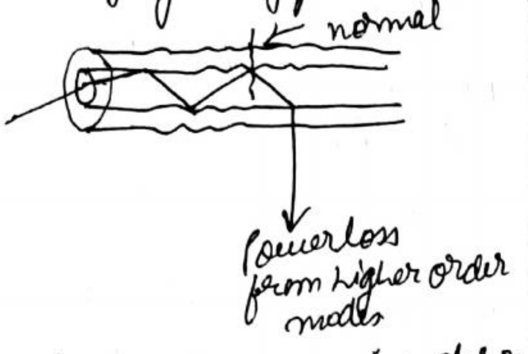
Hence this fiber has a V number of approximately 76, giving nearly 3000 guided modes.

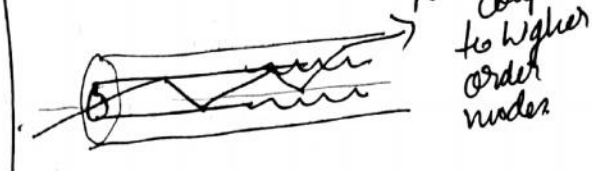
(b) Explain different types of bending losses.

Microbending losses

(2)

→ fiber is deformed over microscales then locally some leakage of energy takes place





→ b/c of pressure microdeformatn can occur at the core & cladding

↓

due to which the normal is changed

↓

so the ray which was satisfying the ~~criti~~ critical angle conditn for straight normal, will no more statisfy the critical angle conditn

↓

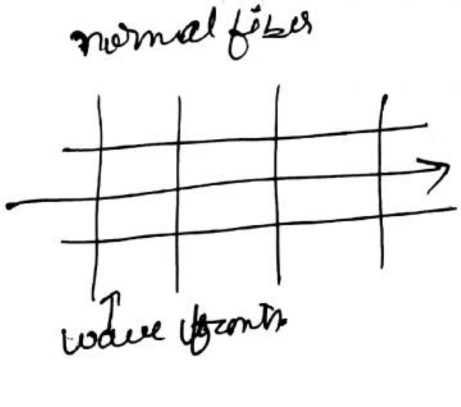
Hence the energy will ~~be~~ leaks out.

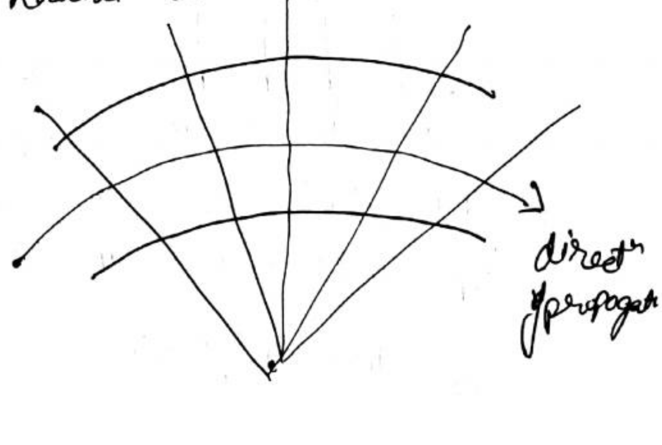
Macrobending OR Radiatn Loss :

→ If the fiber is gently bent over a large arc and the radius of the arc 'R', $R >> \lambda$.

↓

leakage of energy will take place & this phenomenon is a Radiatn Loss





→ When fiber is bent

↓

Wave fronts are not ||el to each other

↓

WFs will move like a fan which are pivoted to the center of curvature of the arc.

↓

So every point on the WF will not move with the same speed.

↓

Points closer to the center of curvature will move with lower velocity

&

and as we go farther velocity will ↑se

↓

And a region will come where velocity of ≈ c intrinsically (and after that velocity cant ↑se)

↓

so energy beyond that cannot propagate along the mode.

↓

it will start leaking the radiatn hence c/a Radiatn loss.

at x_c, velocity of WF points = c (in ~~the~~ cladd medium)

After x_c, energy cant propagate in the medium and will be lost.

(10)

⇒ energy lost (radiated) will d/on
↳ how close x_c is from the core
↳ how feild is ~~separated~~ spreaded inside fiber

As fiber bent more
↓
R ↓se
↓
x_c will come close
↓
Radiatⁿ loss (↑se)

→ Material Absorpⁿ
- Extrinsic → X-sth metal impurities, OH^- ions
- Intrinsic → due to intrinsic mat properties
- Atomic defects → due to imperfectⁿ in the atomic structure of fiber

$$\alpha_r = c_1 \exp(-c_2 R)$$

R = Radius of curvature of the fiber bend
c_1 & c_2 = Constants which are independent of R

$$R_{cM} = \frac{3 n_1^2 \lambda}{4\pi (n_1^2 - n_2^2)^{3/2}}$$

R_c = critical bending loss tend to occur in multimode fiber at a critical radius of curvature R_c

* Can be reduced (Bending loss) by :-
↳ designing fibers with large relative refractive index diff
↳ operating at the shortest λ possible

$$R_{cs} = \frac{20\lambda}{(n_1 - n_2)^{3/2}} \left(2.748 - 0.996\frac{\lambda}{\lambda_c}\right)^{-3} \rightarrow \text{for SMF}$$

λ_c → cut-off λ for SMF

5 (a) With the help of neat diagrams discuss the structure of single mode and multimode step index fiber with appropriate mathematical equations.

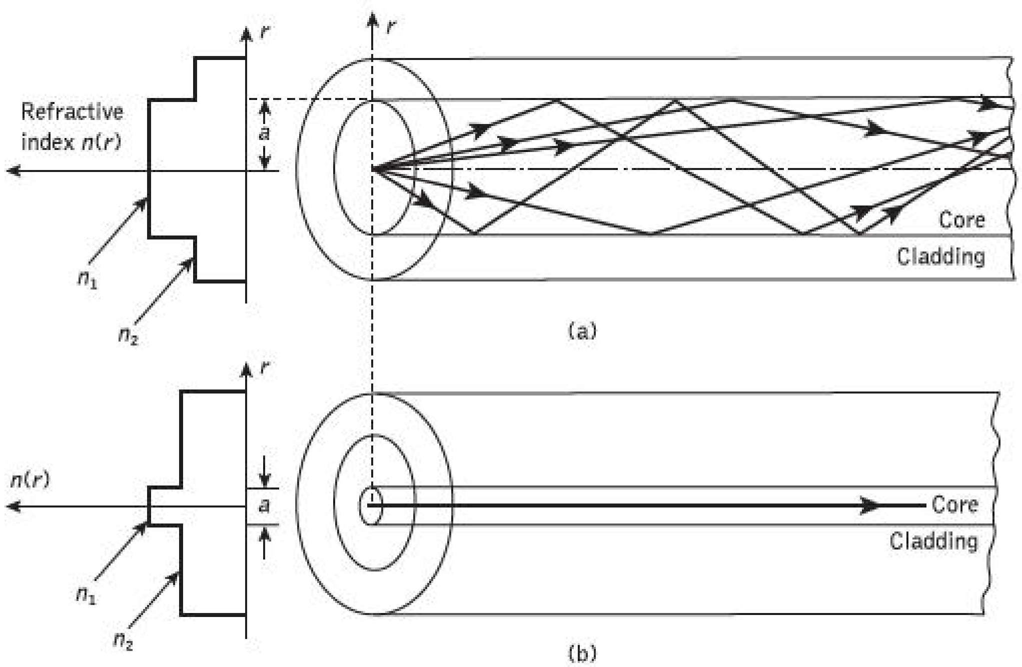


Figure 2.21 The refractive index profile and ray transmission in step index fibers: (a) multimode step index fiber; (b) single-mode step index fiber

The refractive index profile may be defined as:

$$n(r) = \begin{cases} n_1 & r < a \quad \text{(core)} \\ n_2 & r \geq a \quad \text{(cladding)} \end{cases}$$

(b) Derive an expression for pulse broadening due to material dispersion which is a function of wavelength and time delay.

Pulse broadening due to material dispersion results from the different group velocities of the various spectral components launched into the fiber from the optical source. It occurs when the phase velocity of a plane wave propagating in the dielectric medium varies non-linearly with wavelength, and a material is said to exhibit material dispersion when the second differential of the refractive index with respect to wavelength is not zero (i.e. $d^2n/d\lambda^2 \neq 0$). The pulse spread due to material dispersion may be obtained by considering the group delay τ_g in the optical fiber which is the reciprocal of the group velocity υ_g defined by Eqs (2.37) and (2.40). Hence the group delay is given by:

$$\tau_g = \frac{d\beta}{d\omega} = \frac{1}{c}\left(n_1 - \lambda\frac{dn_1}{d\lambda}\right) \tag{3.13}$$

where n_1 is the refractive index of the core material. The pulse delay τ_m due to material dispersion in a fiber of length L is therefore:

$$\tau_m = \frac{L}{c}\left(n_1 - \lambda\frac{dn_1}{d\lambda}\right) \tag{3.14}$$

For a source with rms spectral width σ_λ and a mean wavelength λ, the rms pulse broadening due to material dispersion σ_m may be obtained from the expansion of Eq. (3.14) in a Taylor series about λ where:

$$\sigma_m = \sigma_\lambda\frac{d\tau_m}{d\lambda} + \sigma_\lambda\frac{2d^2\tau_m}{d\lambda^2} + \ldots \tag{3.15}$$

As the first term in Eq. (3.15) usually dominates, especially for sources operating over the 0.8 to 0.9 μm wavelength range, then:

$$\sigma_m \simeq \sigma_\lambda\frac{d\tau_m}{d\lambda} \tag{3.16}$$

Hence the pulse spread may be evaluated by considering the dependence of τ_m on λ, where from Eq. (3.14):

$$\begin{aligned}\frac{d\tau_m}{d\lambda} &= \frac{L\lambda}{c}\left[\frac{dn_1}{d\lambda} - \frac{d^2n_1}{d\lambda^2} - \frac{dn_1}{d\lambda}\right] \\ &= \frac{-L\lambda}{c}\frac{d^2n_1}{d\lambda^2}\end{aligned} \tag{3.17}$$

Therefore, substituting the expression obtained in Eq. (3.17) into Eq. (3.16), the rms pulse broadening due to material dispersion is given by:

$$\sigma_m \simeq \frac{\sigma_\lambda L}{c}\left|\lambda\frac{d^2n_1}{d\lambda^2}\right| \tag{3.18}$$

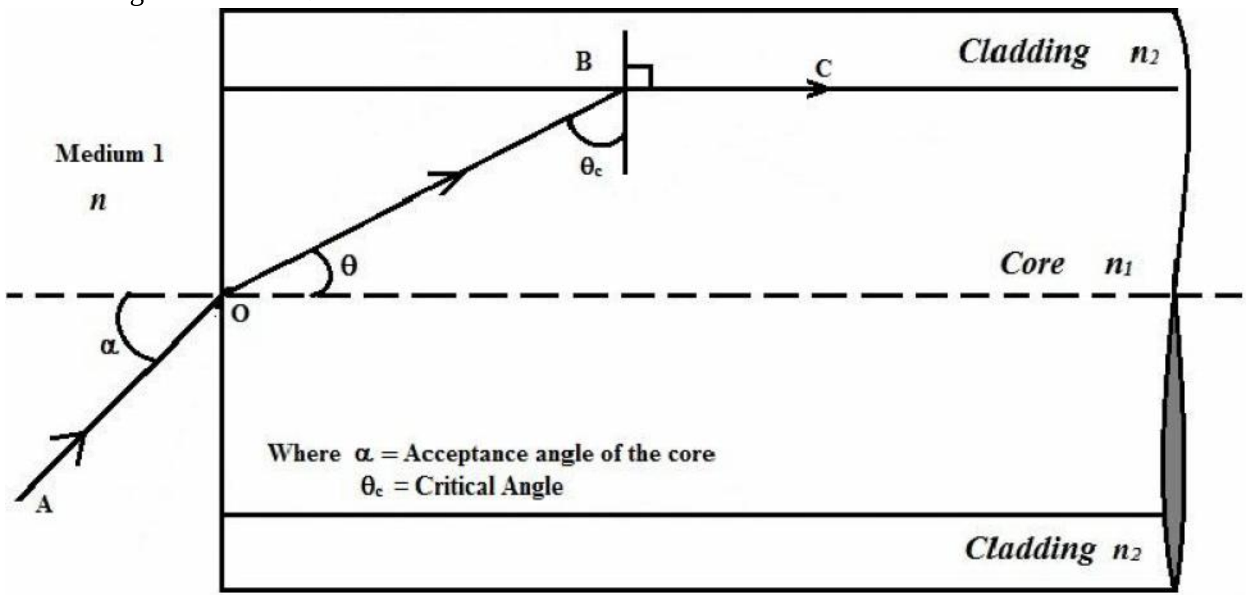
The material dispersion for optical fibers is sometimes quoted as a value for $|\lambda^2(d^2n_1/d\lambda^2)|$ or simply $|d^2n_1/d\lambda^2|$.

However, it may be given in terms of a material dispersion parameter M which is defined as:

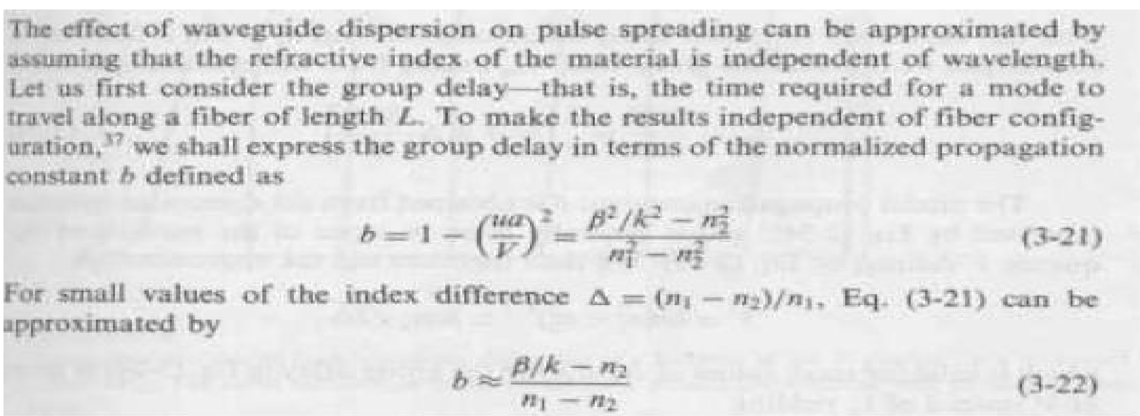
$$M = \frac{1}{L}\frac{d\tau_m}{d\lambda} = \frac{\lambda}{c}\left|\frac{d^2n_1}{d\lambda^2}\right| \tag{3.19}$$

and which is often expressed in units of ps nm^{-1} km^{-1}.

6.(a) Define numerical aperture, derive the expression for NA with light launched in the meridional plane. State the significance also.



(b) Derive an expression for pulse broadening due to waveguide dispersion.



The effect of waveguide dispersion on pulse spreading can be approximated by assuming that the refractive index of the material is independent of wavelength. Let us first consider the group delay—that is, the time required for a mode to travel along a fiber of length L. To make the results independent of fiber configuration,[37] we shall express the group delay in terms of the normalized propagation constant b defined as

$$b = 1 - \left(\frac{ua}{V}\right)^2 = \frac{\beta^2/k^2 - n_2^2}{n_1^2 - n_2^2} \tag{3-21}$$

For small values of the index difference $\Delta = (n_1 - n_2)/n_1$, Eq. (3-21) can be approximated by

$$b \approx \frac{\beta/k - n_2}{n_1 - n_2} \tag{3-22}$$

Solving Eq. (3-22) for β, we have

$$\beta \approx n_2 k(b\Delta + 1) \tag{3-23}$$

With this expression for β and using the assumption that n_2 is not a function of wavelength, we find that the group delay τ_{wg} arising from waveguide dispersion is

$$\tau_{wg} = \frac{L}{c}\frac{d\beta}{dk} = \frac{L}{c}\left[n_2 + n_2\Delta\frac{d(kb)}{dk}\right] \tag{3-24}$$

The modal propagation constant β is obtained from the eigenvalue equation expressed by Eq. (2-54), and is generally given in terms of the normalized frequency V defined by Eq. (2-57). We shall therefore use the approximation

$$V = ka(n_1^2 - n_2^2)^{1/2} \simeq kan_2\sqrt{2\Delta}$$

which is valid for small values of Δ, to write the group delay in Eq. (3-24) in terms of V instead of k, yielding

$$\tau_{wg} = \frac{L}{c}\left[n_2 + n_2\Delta\frac{d(Vb)}{dV}\right]$$

7. Explain the following:
   (a) Photonic crystal fibers:

2.6 Photonic crystal fibers

The previous discussion in this chapter has concentrated on optical fibers comprising solid silica core and cladding regions in which the light is guided by a small increase in refractive index in the core facilitated through doping the silicon with germanium. More recently, however, a new class of microstructured optical fiber containing a fine array of air holes running longitudinally down the fiber cladding [Ref. 69] has been developed. Since the microstructure within the fiber is often highly periodic due to the fabrication process, these fibers are usually referred to as photonic crystal fibers (PCFs), or sometimes just as holey fibers [Ref. 70]. Whereas in conventional optical fibers electromagnetic modes are guided by total internal reflection in the core region, which has a slightly raised refractive index, in PCFs two distinct guidance mechanisms arise.

Although the guided modes can be trapped in a fiber core which exhibits a higher average index than the cladding containing the air holes by an effect similar to total internal reflection, alternatively they may be trapped in a core of either higher, or indeed lower, average index by a photonic bandgap effect. In the former case the effect is often termed modified total internal reflection and the fibers are referred to as index guided, while in the latter they are called photonic bandgap fibers. Furthermore, the existence of two different guidance mechanisms makes PCFs versatile in their range of potential applications. For example, PCFs have been used to realize various optical components and devices including long period gratings [Ref. 71], multimode interference power splitters [Ref. 72], tunable coupled cavity fiber lasers [Ref. 73], fiber amplifiers [Ref. 74], multichannel add/drop filters [Ref. 75], wavelength converters [Ref. 76] and wavelength demultiplexers [Ref. 77]. As with conventional optical fibers, however, a crucial issue with PCFs has been the reduction in overall transmission losses which were initially several hundred decibels per kilometer even with the most straightforward designs. Increased control over the homogeneity of the fiber structures together with the use of highly purified silicon as the base material has now lowered these losses to a level of a very few decibels per kilometer for most PCF types, with a loss of just 0.3 dB km^{-1} at 1.55 μm for a 100 km span being recently reported [Ref. 78].

2.6.1 Index-guided microstructures

Although the principles of guidance and the characteristics of index-guided PCFs are similar to those of conventional fiber, there is greater index contrast since the cladding contains air holes with a refractive index of 1 in comparison with the normal silica cladding index of 1.457 which is close to the germanium-doped core index of 1.462. A fundamental physical difference, however, between index-guided PCFs and conventional fibers arises from the manner in which the guided mode interacts with the cladding region. Whereas in a conventional fiber this interaction is largely first order and independent of wavelength, the large index contrast combined with the small structure dimensions cause the effective cladding index to be a strong function of wavelength. For short wavelengths the effective cladding index is only slightly lower than the core index and hence they remain tightly confined to the core. At longer wavelengths, however, the mode samples more of the cladding and the effective index contrast is larger. This wavelength dependence results in

(a) (b)

Figure 2.37 Two index-guided photonic crystal fiber structures. The dark areas are air holes while the white areas are silica

a large number of unusual optical properties which can be tailored. For example, the high index contrast enables the PCF core to be reduced from around 8 μm in conventional fiber to less than 1 μm, which increases the intensity of the light in the core and enhances the nonlinear effects.

Two common index-guided PCF designs are shown diagrammatically in Figure 2.37. In both cases a solid-core region is surrounded by a cladding region containing air holes. The cladding region in Figure 2.37(a) comprises a hexagonal array of air holes while in Figure 2.37(b) the cladding air holes are not uniform in size and do not extend too far from the core. It should be noted that the hole diameter d and hole to hole spacing or pitch Λ are critical design parameters used to specify the structure of the PCF. For example, in a silica PCF with the structure depicted in Figure 2.37(a) when the air fill fraction is low (i.e. $d/\Lambda < 0.4$), then the fiber can be single-moded at all wavelengths [Ref. 79]. This property, which cannot be attained in conventional fibers, is particularly significant for broadband applications such as wavelength division multiplexed transmission [Ref. 80].

As PCFs have a wider range of optical properties in comparison with standard optical fibers, they provide for the possibility of new and technologically important fiber devices. When the holey region covers more than 20% of the fiber cross-section, for instance, index-guided PCFs display an interesting range of dispersive properties which could find application as dispersion-compensating or dispersion-controlling fiber components [Ref. 81]. In such fibers it is possible to produce very high optical nonlinearity per unit length in which modest light intensities can induce substantial nonlinear effects. For example, while several kilometers of conventional fiber are normally required to achieve 2R data regeneration (see Section 10.6), it was obtained with just 3.3 m of large air-filling fraction PCF [Ref. 82]. In addition, filling the cladding holes with polymers or liquid crystals allows external fields to be used to dynamically vary the fiber properties. The temperature sensitivity of a polymer within the cladding holes may be employed to tune a Bragg grating written into the core [Ref. 83]. By contrast, index-guided PCFs with small holes and large hole spacings provide very large mode area (and hence low optical nonlinearities) and have potential applications in high-power delivery (e.g. laser welding and machining) as well as high-power fiber lasers and amplifiers [Ref. 74]. Furthermore, the large index contrast between silica and air enables production of such PCFs with large multimoded cores which also have very high numerical aperture values (greater than 0.7). Hence these fibers are useful for the collection and transmission of high optical powers in situations where signal distortion is not an issue. Finally, it is apparent that PCFs can be readily spliced to conventional fibers, thus enabling their integration with existing components and subsystems.

2.6.2 Photonic bandgap fibers

Photonic bandgap (PBG) fibers are a class of microstructured fiber in which a periodic arrangement of air holes is required to ensure guidance. This periodic arrangement of cladding air holes provides for the formation of a photonic bandgap in the transverse plane of the fiber. As a PBG fiber exhibits a two-dimensional bandgap, then wavelengths within this bandgap cannot propagate perpendicular to the fiber axis (i.e. in the cladding) and they can therefore be confined to propagate within a region in which the refractive index is lower than the surrounding material. Hence utilizing the photonic bandgap effect light can, for example, be guided within a low-index, air-filled core region creating fiber properties quite different from those obtained without the bandgap. Although, as with index-guided PCFs, PBG fibers can also guide light in regions with higher refractive index, it is the lower index region guidance feature which is of particular interest. In addition, a further distinctive feature is that while index-guiding fibers usually have a guided mode at all wavelengths, PBG fibers only guide in certain wavelength bands, and furthermore it is possible to have wavelengths at which higher order modes are guided while the fundamental mode is not.

Two important PBG fiber structures are displayed in Figure 2.38. The honeycomb fiber design shown in Figure 2.38(a) was the first PBG fiber to be experimentally realized in 1998 [Ref. 84] and adaptations of this structure continue to be pursued [Ref. 85]. A triangular array of air holes of sufficient size as displayed in Figure 2.38(b), however, provides for the possibility, unique to PBG fibers, of guiding electromagnetic modes in air. In this case a large hollow core has been defined by removing the silica around seven air holes in the center of the structure. These fibers, which are termed air-guiding or hollow-core PBG fibers, enable more than 98% of the guided mode field energy to propagate in the air regions [Ref. 81]. Such air-guiding fibers have attracted attention because they potentially provide an environment in which optical propagation can take place with little attenuation as the localization of light in the air core removes the limitations caused by material absorption losses. The fabrication of hollow-core fiber with low propagation losses, however, has proved to be quite difficult, with losses of the order of 13 dB km^{-1} [Ref. 86]. Moreover, the fibers tend to be highly dispersive with narrow transmission windows and

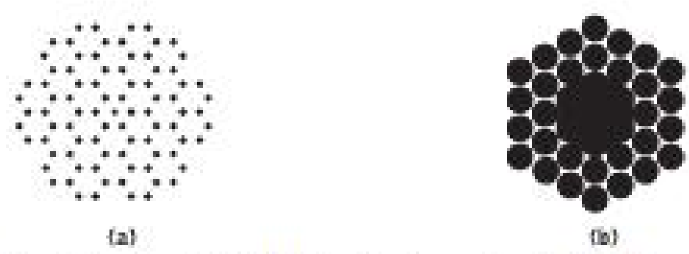


Figure 2.38 Photonic bandgap (PBG) fiber structures in which the dark areas are air (lower refractive index) and the lighter area is the higher refractive index: (a) honeycomb PBG fiber; (b) air-guiding PBG fiber

while single-mode operation is possible, it is not as straightforward to achieve in comparison with index-guiding PCFs.

More recently, the fabrication and characterization of a new type of solid silica-based photonic crystal fiber which guides light using the PBG mechanism has been reported [Refs 87, 88]. This fiber employed a two-dimensional periodic array of germanium-doped rods in the core region. It was therefore referred to as a nanostructure core fiber and exhibited a minimum attenuation of 2.6 dB km^{-1} at a wavelength of 1.59 μm [Ref. 87]. Furthermore, the fiber displayed greater bending sensitivity than conventional single-mode fiber as a result of the much smaller index difference between the core and the leaky modes which could provide for potential applications in the optical sensing of curvature and stress. In addition, it is indicated that the all-solid silica structure would facilitate fiber fabrication using existing technology (see Sections 4.2 to 4.4), and birefringence (see Section 3.13.1) of the order of 10^{-4} is easily achievable with a large mode field diameter up to 10 μm, thus enabling its use within fiber lasers (see Section 6.10.3) and gyroscope applications [Ref. 88].

(b) Graded index fiber (with mathematical equations):

Graded index fibers do not have a constant refractive index in the core* but a decreasing core index $n(r)$ with radial distance from a maximum value of n_1 at the axis to a constant value n_2 beyond the core radius a in the cladding. This index variation may be represented as:

$$n(r) = \begin{cases} n_1(1 - 2\Delta(r/a)^{\alpha})^{\frac{1}{2}} & r < a \quad \text{(core)} \\ n_1(1 - 2\Delta)^{\frac{1}{2}} = n_2 & r \geq a \quad \text{(cladding)} \end{cases} \tag{2.75}$$

where Δ is the relative refractive index difference and α is the profile parameter which gives the characteristic refractive index profile of the fiber core. Equation (2.75) which is a convenient method of expressing the refractive index profile of the fiber core as a variation of α, allows representation of the step index profile when $\alpha = \infty$, a parabolic profile when $\alpha = 2$ and a triangular profile when $\alpha = 1$. This range of refractive index profiles is illustrated in Figure 2.22.

The graded index profiles which at present produce the best results for multimode optical propagation have a near parabolic refractive index profile core with $\alpha \simeq 2$. Fibers

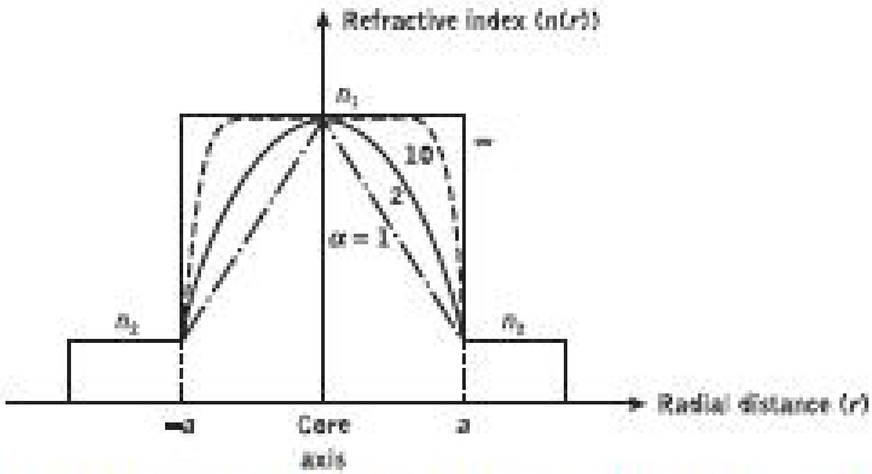


Figure 2.22 Possible fiber refractive index profiles for different values of α (given in Eq. (2.75))

* Graded index fibers are therefore sometimes referred to as inhomogeneous core fibers.

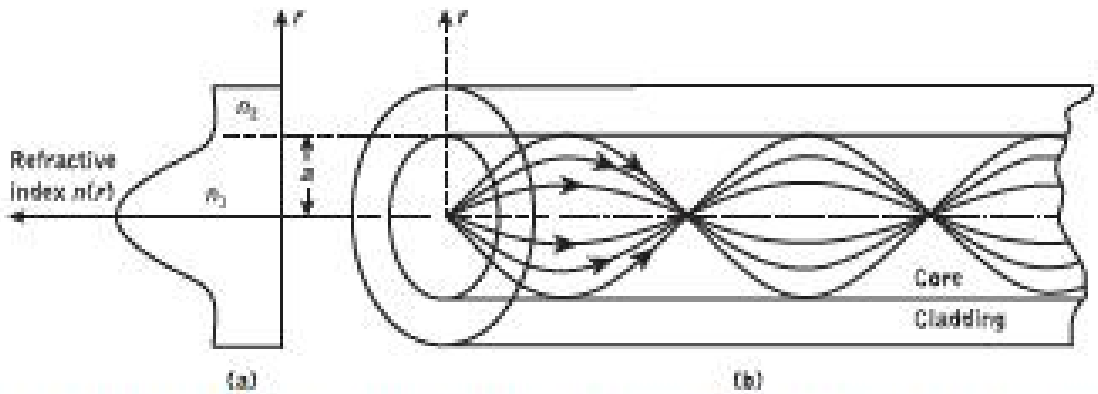


Figure 2.23 The refractive index profile and ray transmission in a multimode graded index fiber

with such core index profiles are well established and consequently when the term 'graded index' is used without qualification it usually refers to a fiber with this profile. For this reason in this section we consider the waveguiding properties of graded index fiber with a parabolic refractive index profile core.

A multimode graded index fiber with a parabolic index profile core is illustrated in Figure 2.23. It may be observed that the meridional rays shown appear to follow curved paths through the fiber core. Using the concepts of geometric optics, the gradual decrease in refractive index from the center of the core creates many refractions of the rays as they are effectively incident on a large number or high to low index interfaces. This mechanism is illustrated in Figure 2.24 where a ray is shown to be gradually curved, with an ever-increasing angle of incidence, until the conditions for total internal reflection are met, and the ray travels back towards the core axis, again being continuously refracted.

Multimode graded index fibers exhibit far less intermodal dispersion (see Section 3.10.2) than multimode step index fibers due to their refractive index profile. Although many different modes are excited in the graded index fiber, the different group velocities of the modes tend to be normalized by the index grading. Again considering ray theory, the rays traveling close to the fiber axis have shorter paths when compared with rays which travel

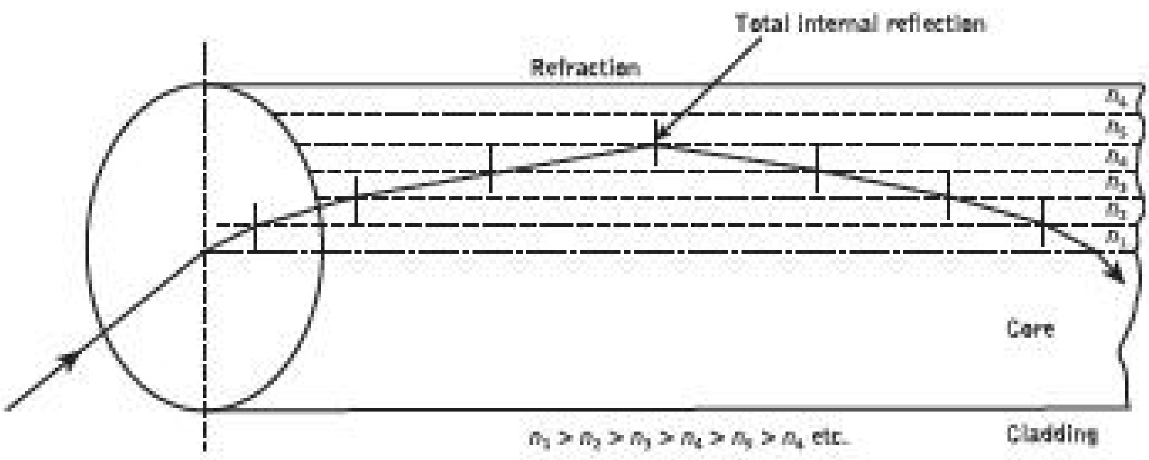


Figure 2.24 An expanded ray diagram showing refraction at the various high to low index interfaces within a graded index fiber, giving an overall curved ray path

8. Silica has an estimated fictive temperature of 1400K with an isothermal compressibility of $7\times10^{-11} m^2N^{-1}$.The refractive index and photoelastic coefficient for silica are 1.46 and 0.286 respectively. Determine the theoretical attenuation in decibels per kilometer due to the fundamental Rayleigh scattering in silica at optical wavelengths of 0.63, 1.00 and 1.30µm. Boltzmann's constant is 1.381×10^{-21} JK^{-1}

$$\gamma_R = \frac{8\pi^3 n^8 p^2 \beta_c K T_F}{3\lambda^4}$$

$$= \frac{248.15 \times 20.65 \times 0.082 \times 7 \times 10^{-11} \times 1.381 \times 10^{-23} \times 1400}{3 \times \lambda^4}$$

$$= \frac{1.895 \times 10^{-28}}{\lambda^4} \text{ m}^{-1}$$

At a wavelength of 0.63 µm:

$$\gamma_R = \frac{1.895 \times 10^{-28}}{0.158 \times 10^{-24}} = 1.199 \times 10^{-3} \text{ m}^{-1}$$

The transmission loss factor for 1 kilometer of fiber may be obtained using Eq. (3.5):

$$\mathcal{L}_{km} = \exp(-\gamma_R L) = \exp(-1.199 \times 10^{-3} \times 10^3) = 0.301$$

The attenuation due to Rayleigh scattering in decibels per kilometer may be obtained from Eq. (3.1) where:

$$\text{Attenuation} = 10 \log_{10}(1/\mathcal{L}_{km}) = 10 \log_{10} 3.322 = 5.2 \text{ dB km}^{-1}$$

At a wavelength of 1.0 µm:

$$\gamma_R = \frac{1.895 \times 10^{-28}}{10^{-24}} = 1.895 \times 10^{-4} \text{ m}^{-1}$$

Using Eq. (3.5):

$$\mathcal{L}_{km} = \exp(-1.895 \times 10^{-4} \times 10^3) = \exp(-0.1895) = 0.827$$

and Eq. (3.1):

$$\text{Attenuation} = 10 \log_{10} 1.209 = 0.8 \text{ dB km}^{-1}$$

At a wavelength of 1.30 µm:

$$\gamma_R = \frac{1.895 \times 10^{-28}}{2.856 \times 10^{-24}} = 0.664 \times 10^{-4}$$

Using Eq. (3.5):

$$\mathcal{L}_{km} = \exp(-0.664 \times 10^{-4} \times 10^3) = 0.936$$

and Eq. (3.1):

$$\text{Attenuation} = 10 \log_{10} 1.069 = 0.3 \text{ dB km}^{-1}$$