USN | | | | | | | | | |

Internal Assessment Test II – Nov. 2017

Sub:	ENGINEERING ELECTROMAGNETICS				Sub Code:	15EC36	Branch:	ECE	
Date:	07/11/2017	Duration:	90 min's	Max Marks: 50	Sem / Sec:	3rd ECE(A, B,C)/TCE		OBE	

Answer any FIVE FULL Questions

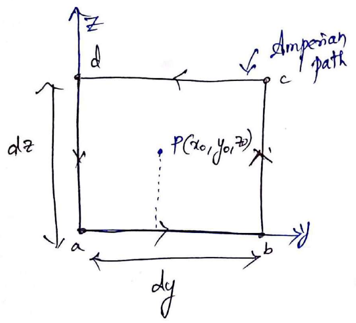
		MARKS	CO	RBT
1 (a)	Derive point form of Ampere's circuital law.	[10]	CO1	L3

CURL OF A VECTOR & STOKES THEOREM:

Curl of $\vec{H}$:

It is an axial (rotational) vector whose magnitude is the maximum circulation of $\vec{H}$ per unit area as the area tends to zero and whose direction is the normal direction of the area when the area is oriented as to make the circulation maximum.

$$\boxed{\text{curl } \vec{H} = \vec{\nabla} \times \vec{H} = \left(\lim_{\Delta S \to 0} \frac{\oint_L \vec{H}\cdot\vec{dl}}{\Delta S} \right)_{max} \cdot \vec{a_n}}$$



$$\oint_L \vec{H}\cdot\vec{dl} = \int_{ab} + \int_{bc} + \int_{cd} + \int_{da} \vec{H}\cdot\vec{dl}$$

By using Taylor Series expansion of H_y about P,

$$\int_{ab} \vec{H}\cdot\vec{dl} = \left[H_y(x_0, y_0, z_0) - \frac{dz}{2} \frac{\partial H_y}{\partial z}\Big|_P \right] dy + \text{higher order term}$$

Three dimensional Taylor series expansion

$$\boxed{\begin{aligned} H_y(x,y,z) = H_y(x_0,y_0,z_0) &+ (x-x_0)\frac{\partial H_y}{\partial x}\Big|_P \\ &+ (y-y_0)\frac{\partial H_y}{\partial y}\Big|_P \\ &+ (z-z_0)\frac{\partial H_y}{\partial z}\Big|_P \\ &+ \text{higher order terms} \end{aligned}}$$

	$$\int_{bc} \vec{H}\cdot\vec{dl} = dz\left[H_z(x_0,y_0,z_0) + \frac{dy}{2}\frac{\partial H_z}{\partial y}\Big|_P\right] + \text{higher order terms}$$ $$\int_{cd} \vec{H}\cdot\vec{dl} = -dy\left[H_y(x_0,y_0,z_0) + \frac{dz}{2}\frac{\partial H_y}{\partial z}\Big|_P\right] + \text{higher order terms}$$ $$\int_{da} \vec{H}\cdot\vec{dl} = -dz\left[H_z(x_0,y_0,z_0) - \frac{dy}{2}\frac{\partial H_z}{\partial y}\Big|_P\right] + \text{higher order terms}$$ $$\lim_{\Delta s\to 0}\frac{\oint \vec{H}\cdot\vec{dl}}{\Delta s} = \lim_{\Delta s\to 0}\frac{\left[\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right]\cdot \cancel{dy\,dz}}{\cancel{dy\,dz}} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}$$ $$\boxed{(\mathrm{curl}\,\vec{H})_x = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}}$$ Similarly y- & z- components of curl can be found. $$\boxed{(\mathrm{curl}\,\vec{H})_y = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}}$$ $$\boxed{(\mathrm{curl}\,\vec{H})_z = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}}$$ $$\mathrm{curl}\,\vec{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\vec{a_x} + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\vec{a_y} + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\vec{a_z}$$ $$\boxed{\mathrm{curl}\,\vec{H} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}}$$ Cartesian systems Cylindrical system: $$\mathrm{Curl}\,\vec{H} = \vec{\nabla}\times\vec{H} = \frac{1}{\rho}\begin{vmatrix} \vec{a_\rho} & \rho\vec{a_\phi} & \vec{a_z} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ H_\rho & \rho H_\phi & H_z \end{vmatrix}$$ Spherical system: $$\mathrm{curl}\,\vec{H} = \vec{\nabla}\times\vec{H} = \frac{1}{r^2\sin\theta}\begin{vmatrix} \vec{a_r} & r\vec{a_\theta} & r\sin\theta\,\vec{a_\phi} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ H_r & rH_\theta & r\sin\theta\,H_\phi \end{vmatrix}.$$			
	OR			
2 (a)	Discuss scalar and vector magnetic potential.	[06]	CO1	L3

Scalar & Vector Magnetic Potentials:

electrostatic potential V → greatly simplified electrostatic field problems.

Can a scalar magnetic potential be defined?

Let us assume the existence of scalar magnetic potential V_m, whose negative gradient gives magnetic field intensity.

Let, $\vec{H} = -\nabla V_m$

W.K.T

$\nabla \times \vec{H} = \vec{J}$

$\nabla \times \vec{H} = \vec{J} = \nabla \times (-\nabla V_m)$

Vector Identity:

$\nabla \times \nabla V_m = 0$

curl of gradient of a scalar is zero.

$\nabla \times (-\nabla V_m) = 0 \Rightarrow \vec{J} = 0$

∴ If a scalar magnetic potential is defined for a region, then current density must be zero throughout the region.

∴ $\boxed{\vec{H} = -\nabla V_m \quad (\vec{J} = 0)}$

Scalar magnetic potential is useful in magnetic problems involving geometries in which current carrying conductors occupy a relatively small fraction of total region of interest & also in case of permanent magnets.

V_m is in amperes.

This scalar potential also satisfies Laplace's equation.

In free space,

$$\vec{B} = \mu_0 \vec{H}$$

$$\nabla \cdot \vec{B} = \nabla \cdot \mu_0 \vec{H} = 0 \cdot \quad (\text{from } \nabla \cdot \vec{B} = 0).$$

$$\mu_0 (\nabla \cdot \vec{H}) = 0$$

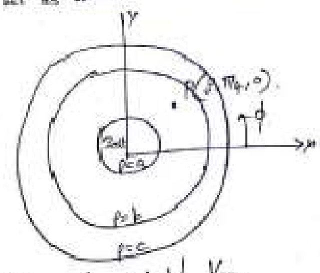
$$\mu_0 \left(\nabla \cdot (-\nabla V_m) \right) = 0$$

$$\boxed{\nabla^2 V_m = 0 \quad \text{for } \vec{J} = 0}$$ (In homogeneous magnetic materials).

One difference between electric scalar potential (V) & magnetic scalar potential (V_m)

V ⇓ It is single valued (only one value of V associated with each point in space if zero reference is assigned). $\boxed{V_{ab} = -\int_b^a \vec{E} \cdot \vec{dL}}$ → independent of the path.	V_m. ⇓ not single valued function of position.

Let us consider coaxial cable



Let $\vec{J} = 0$ in $a < \rho < b$

⇓ we may establish electic scalar potential V_m.

$$\vec{H} = \frac{I}{2\pi\rho} \vec{a_\phi} \quad (a < \rho < b)$$

$$\vec{H} = -\nabla V_m.$$

$$\frac{I}{2\pi\rho} \vec{a_\phi} = -\nabla V_m$$

$$-\frac{1}{\rho}\frac{\partial V_m}{\partial \phi} = \frac{I}{2\pi\rho}$$

$$\frac{\partial V_m}{\partial \phi} = \frac{-I}{2\pi}$$

$$\boxed{V_m = \frac{-I}{2\pi}\phi}$$ where constant of integration is set to zero.

If $\phi = 0 \Rightarrow V_m = 0$
$\phi = 2\pi \Rightarrow V_m = -I$
$\phi = 4\pi \Rightarrow V_m = -2I$

} Represents a same point but with different values of V_m. (multivalued ness).

At P, $(\rho, \pi/4, 0)$ $V_{mp} = \frac{-I}{2\pi}\left(\pi/4 - 2n\pi\right)$

$$= \frac{I}{2\pi}\left(2n - \tfrac{1}{4}\right)\cdot\pi$$

$$\boxed{V_{mp} = \frac{I}{2}\left(n - \tfrac{1}{8}\right)}$$ where $n = 0, \pm1, \pm2, \ldots$

In magnetostatic case,

$\nabla \times \vec{H} = 0$ when $\vec{J} = 0$

$\oint \vec{H}\cdot d\vec{l} = I$ even if $\vec{J} = 0$ along the path of integration

Every time when we take another lap around I, the integration increases by I.

If no current is enclosed by the path, then a single valued potential function may be defined.

In general, $$\boxed{V_{m\,ab} = -\int_b^a \vec{H}\cdot d\vec{l} \quad (\text{specified path})}$$

(6)

Electrostatic potential (V) is a conservative field.

Magnetostatic potential (V_m) is not a conservative field.

Vector Magnetic Potential:

extremely useful in studying radiation from antennas, & radiation leakage from transmission lines, waveguides & microwave ovens.

used in regions where current density is zero or nonzero, & extended to time varying case.

w.k.t.

$$\boxed{\nabla \cdot \vec{B} = 0}$$

Divergence of curl of a vector field is zero.

Let $\boxed{\nabla \cdot (\nabla \times \vec{A}) = 0}$

Therefore we get

Useful definition of $\vec{A}$ $\Rightarrow \boxed{\vec{B} = \nabla \times \vec{A}}$ where $\vec{A}$ signifies Vector magnetic potential

$$\therefore \vec{H} = \frac{1}{\mu_0}(\nabla \times \vec{A})$$

$$\boxed{\nabla \times \vec{H} = \vec{J} = \frac{1}{\mu_0}(\nabla \times \nabla \times \vec{A})}$$

taking curl twice

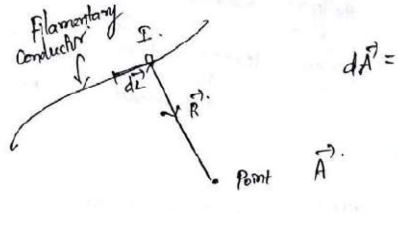
Vector Identity

$$\boxed{\nabla \times \nabla \times \vec{A} = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}}$$

$\vec{A}$ is in Wb/m.

Magnetic Vector potential $\vec{A}$, may be determined from differential current elements as

$$\boxed{\vec{A} = \oint_L \frac{\mu_0 I \, d\vec{L}}{4\pi |\vec{R}|}}$$



$$d\vec{A} = \frac{\mu_0 I \, d\vec{L}}{4\pi |\vec{R}|}$$

(no significance of this expression without considering any closed path in which current flows).

(b)	The magnetic field intensity is given in a region of space as $\boldsymbol{H} = \frac{(x+2y)}{z^2}\boldsymbol{a_y} + \frac{2}{Z}\boldsymbol{a_z}\, A/m$. Calculate current density.	[04]	CO1	L3

Soln

$$\vec\nabla \times \vec H = \begin{vmatrix} \hat a_x & \hat a_y & \hat a_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{z} \\ 0 & \frac{(x+2y)}{z^2} & \frac{2}{z}\end{vmatrix} = \frac{-2(x+2y)}{z^3}\,\hat a_x + \frac{1}{z^2}\,\hat a_z$$

$$\therefore \vec J = \vec\nabla\times\vec H = \frac{2(x+2y)}{z^3}\,\hat a_x + \frac{1}{z^2}\,\hat a_z$$

(c) $$I = \int \vec J\cdot d\vec s = \int\left(\frac{2(x+2y)}{z^3}\hat a_x + \frac{1}{z^2}\hat a_z\right)\cdot dx\,dy\,\hat a_x$$

$$= \frac{1}{z^2}\int_{x=1}^{2}\int_{y=3}^{5} dx\,dy = \frac{1}{16}\cdot[2-1]\,[5-3] = \frac{1}{8}\,A$$

3 (a)	State and explain Stoke's theorem.	[06]	CO2	L3

From the definition of curl,

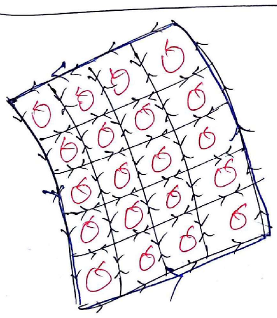
$$\frac{\left(\oint_L \vec H\cdot d\vec l\right)_{\Delta s}}{\Delta s} = (\vec\nabla\times\vec H)_N \Rightarrow \left(\oint_L \vec H\cdot d\vec l\right)_{\Delta s} = (\vec\nabla\times\vec H)_N\cdot\Delta s\ \vec a_N$$

$\vec a_N$ — unit vector normal to the surface Δs

Stokes theorem.

$$\boxed{\oint_L \vec H\cdot d\vec l = \iint_S (\vec\nabla\times\vec H)\cdot d\vec s}$$

The Circulation of a vector field $\vec H$ around a closed path L is equal to surface integral of curl of $\vec H$ over the open surface S bounded by L, provided $\vec H$ & $\vec\nabla\times\vec H$ are continuous on S.



$$\oint_L \vec H\cdot d\vec l = \sum_k \oint_{L_k} \vec H\cdot d\vec l$$

$$= \sum_k \frac{\oint_{L_k} \vec H\cdot d\vec l}{\Delta s_k}\cdot\Delta s_k$$

If $\Delta s_k \to 0$

$$= \sum_k \left(\lim_{\Delta s_k\to 0}\frac{\oint_{L_k}\vec H\cdot d\vec l}{\Delta s_k}\right)\cdot\Delta s_k$$

$$\boxed{\oint_L \vec H\cdot d\vec l = \iint_S (\vec\nabla\times\vec H)\cdot d\vec s}$$

(b)	At a point P(x,y,z) the components of vector magnetic potential are given as $A_x = 4x+3y+2z$, $A_y = 5x+6y+3z$, $A_z = 2x+3y+5z$. Determine **B** at point P.	[04]	CO2	L3

$$\vec{B} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (4x+3y+2z) & (5x+6y+3z) & (2x+3y+5z) \end{vmatrix}$$

$$= \vec{a_x}\left[3-3\right] - \vec{a_y}\left[2-2\right] + \vec{a_z}\left[5-3\right]$$

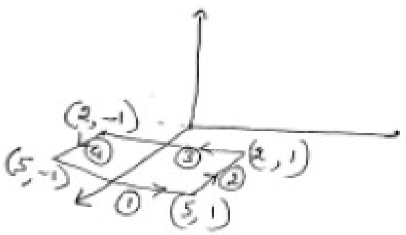
$$\boxed{\vec{B} = 2\,\vec{a_z}}\ Wb/m^2$$

OR

4 Evaluate both sides of Stoke's theorem for the field $\mathbf{H} = 6xy\,\boldsymbol{a_x} - 3y^2\,\boldsymbol{a_y}$ A/m and the rectangular path region, $-2 \le x \le 5$, $-1 \le y \le 1$, $z = 0$. Let the positive direction of $\mathbf{ds}$ be $\boldsymbol{a_z}$. [10] CO2 L3

Soln.

$$\vec{H}\cdot\vec{dl} = (6xy\,\hat{a}_x - 3y^2\,\hat{a}_y)\cdot(dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z)$$
$$= (6xy\,dx - 3y^2 dy)$$



Along path ①,

$$\int \vec{H}\cdot\vec{dl} = \int_{y=-1}^{1} (6xy\,\hat{a}_x - 3y^2\,\hat{a}_y)\cdot dy\,\hat{a}_y$$
$$= -3\int_{-1}^{1} y^2 dy = -3\left[\frac{y^3}{3}\right]_{-1}^{1}$$
$$= -2$$

Along path ②,

$$\int \vec{H}\cdot\vec{dl} = \int_{x=5}^{2} (6xy\,\hat{a}_x - 3y^2\,\hat{a}_y)\cdot dx\,\hat{a}_x$$
$$= \int_{x=5}^{2} 6xy\,dx = 6\cdot 1\cdot\left[\frac{x^2}{2}\right]_5^2 = 3\cdot(4-25) =$$

Along path ③

$$\int \vec{H}\cdot\vec{dl} = \int_{y=1}^{-1} (6xy\,\hat{a}_x - 3y^2\,\hat{a}_y)\cdot dy\,\hat{a}_y$$
$$= -3\int_{y=1}^{-1} y^2 dy = -3\left[\frac{y^3}{3}\right]_1^{-1} = -3\cdot\left[\frac{-2}{3}\right] = 2$$

Along path ④

$$\int \vec{H}\cdot d\vec{l} = \int_{x=2}^{5} 6xy\,dx = 6\cdot(-1)\left[\frac{x^2}{2}\right]_2^5$$
$$= -6\cdot[25-4] = -63.$$

$$\therefore \oint \vec{H}\cdot d\vec{l} = -2-63+2-63 = -126.$$

$$\nabla\times\vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6xy & -3y^2 & 0 \end{vmatrix} = -6x\,\hat{a}_z.$$

$$\therefore \iint(\nabla\times\vec{H})\cdot d\vec{s} = -\iint 6x\,\hat{a}_z\cdot dx\,dy\,\hat{a}_z$$
$$= -6\cdot\left[\frac{x^2}{2}\right]_2^5\left[y\right]_{-1}^{1}$$
$$= -\frac{6}{\cancel{2}}[25-4]\cdot\cancel{2} = -6\cdot 21 = -126.$$

$$\therefore \oint \vec{H}\cdot d\vec{l} = \iint(\nabla\times\vec{H})\cdot d\vec{s} \quad \text{(Proved)}$$

5 (a) Derive the expression for force on a current element placed in a steady magnetic field. [06] CO1 L3

Consider the case of current carrying conductors

convection current density,

$$\boxed{\vec{J} = \rho_v\vec{v}}$$

$$dQ = \rho_v\,dv.$$
$$\therefore d\vec{F} = \rho_v\,dv\,\vec{v}\times\vec{B}$$
$$\Rightarrow \boxed{d\vec{F} = \vec{J}\times\vec{B}\,dv} \quad [\because \vec{J} = \rho_v\vec{v}]$$
$$\vec{J}\,dv = \vec{K}\,ds = I\,d\vec{L}$$
$$\therefore d\vec{F} = \vec{K}\times\vec{B}\,ds$$
$$d\vec{F} = I\,d\vec{L}\times\vec{B}.$$

Integrating,

$$\vec{F} = \int_{vol}\vec{J}\times\vec{B}\,dv$$
$$\vec{F} = \int_{s}\vec{K}\times\vec{B}\,ds$$
$$\vec{F} = \oint I\,d\vec{L}\times\vec{B} = -I\oint\vec{B}\times d\vec{L}$$

For a straight conductor in a uniform mag. field,

$$\boxed{\vec{F} = I\vec{L}\times\vec{B}}$$

$\hat{a}_x \quad \hat{a}_y \quad \hat{a}$
$-1 \quad 0$
$0 \quad 0$

(b) Calculate **M** for the material in which: (a) μ_r=1.4 and $\boldsymbol{H} = 350\,\boldsymbol{a_x}$ A/m. (b) μ_r= 6 and there are $2.5\,x\,10^{29}$atoms/m^3, each having magnetic dipole moment of $2.8\times10^{-30}\,\boldsymbol{a_x}$ A.m^2. [04] CO1 L3

Soln. (a) Given $\mu_R = 1.4$, $\vec{H} = 350\,\hat{a}_x$ A/m.

$\therefore\ \chi_m = (\mu_R - 1) = 0.4$

$\therefore$ Magnetization, $\vec{M} = \chi_m \vec{H} = 0.4 \times 350\,\hat{a}_x = 140$ A/m.

(b) Here $\mu_R = 6$, $n = 2.5 \times 10^{29}$ atoms/m^3

$\therefore\ M = n\,m = 2.5 \times 10^{29} \times 2.8 \times 10^{-30}\,\hat{a}_x = 0.7\,\hat{a}_x$ A/m

OR

6 (a) Write the equations to compare electric circuit and magnetic circuit with respect to i) e.m.f and m.m.f, ii) Electric and magnetic scalar potential, iii) Electric current and magnetic flux, iv) Resistance and reluctance, v) $\oint E.dl$ and $\oint H.dl$ — [05] CO1 L3

$\vec{E} = -\vec{\nabla} V$ $\qquad$ $\vec{H} = -\vec{\nabla} V_m$

$V_{a,b} = -\int_a^b \vec{E}\cdot\vec{dL}$ $\qquad$ $V_{m\,a,b} = \int_a^b \vec{H}\cdot\vec{dL}$

$\vec{D} = \epsilon\vec{E}$, $\vec{J} = \sigma\vec{E}$ $\qquad$ $\vec{B} = \mu\vec{H}$

$I = \int_S \vec{J}\cdot\vec{ds}$ $\qquad$ $\phi = \int_S \vec{B}\cdot\vec{ds}$

Resistance $\qquad$ Reluctance

$V = IR$ $\qquad$ $V_m = \phi\,\mathcal{R}$

Conductance $\qquad$ Permeance

$G = \frac{1}{R}$ $\qquad$ $\mathcal{P} = \frac{1}{\mathcal{R}}$

$R = \frac{l}{\sigma S}$ $\qquad$ $\boxed{\mathcal{R} = \frac{l}{\mu S}}$

KVL:

$\oint_L \vec{E}\cdot\vec{dL} = 0$ $\qquad$ $\oint_L \vec{H}\cdot\vec{dL} = I_{total}$

For N turns coil,

$\oint_L \vec{H}\cdot\vec{dL} = NI$.

Magneto motive force

$\boxed{m.m.f = \phi_m\,\mathcal{R}}$

$m.m.f. = \mathcal{F} = \phi_m\,\mathcal{R}$

(b) Two differential current elements are present as follows: $I_1\boldsymbol{dl_1} = -3\boldsymbol{a_y}$ A.m at P$_1$(5,2,1) and $I_2\boldsymbol{dl_2} = -4\boldsymbol{a_z}$ A.m at P$_2$ (1,8,5). Determine differential force on element 2 due to element 1. — [05] CO1 L3

1) Two differential current elements are present as follows:
$I_1 \vec{dL_1} = -3\hat{a}_y$ A.m at $P_1(5,2,1)$ and
$I_2 \vec{dL_2} = -4\hat{a}_z$ A.m at $P_2(1,8,5)$. Determine different force on element 2 due to element 1.

$(5,2,1) \xrightarrow{\vec{R_{12}}} (1,8,5)$
$P_1 \qquad P_2$

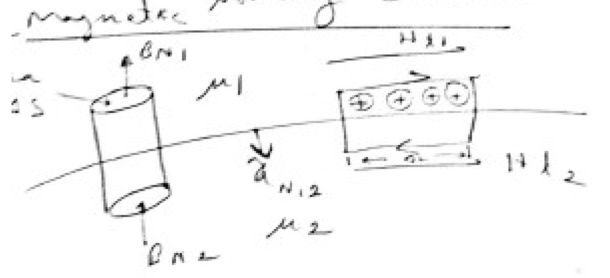
Soln.
$$\vec{R_{12}} = (1-5)\hat{a}_x + (8-2)\hat{a}_y + (5-1)\hat{a}_z$$
$$= -4\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z$$

∴ Differential force on element 2 due to element 1

$$d(dF_2) = I_2 \vec{dL_2} \times d\vec{B_2}$$
$$= I_2 \vec{dL_2} \times \frac{\mu_0 I_1 (dL_1 \times \hat{a}_{R_{12}})}{4\pi R_{12}^2}$$
$$= \frac{\mu_0 I_1 I_2}{4\pi R_{12}^2} \vec{dL_2} \times (\vec{dL_1} \times \hat{a}_{R_{12}})$$
$$= \frac{4\pi \times 10^{-7} (-4\hat{a}_z) \times [(-3\hat{a}_y) \times (-4\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z)]}{(4^2+6^2+4^2)^{3/2}}$$
$$= 8.56\ \hat{a}_y \text{ nN}.$$

7 (a)	Obtain the boundary conditions at the interface between two magnetic materials.	[06]	CO1	L3

Magnetic boundary conditions



Gauss's law of mag.

$$\oint \vec{B} \cdot \vec{ds} = 0$$

$$B_{N1}\Delta S - B_{N2}\Delta S = 0$$

$$\Rightarrow \boxed{B_{N1} = B_{N2}}$$

The fig shows a boundary b/w two isotropic homogeneous linear materials

	$\therefore H_{N2} = \frac{\mu_1}{\mu_2} H_{N1}$ Normal comp. of B is continuous. " " of H is discontinuous by ratio $\left(\frac{\mu_1}{\mu_2}\right)$. $M_{N2} = \chi_{m2}\left(\frac{\mu_1}{\mu_2} H_{N1}\right)$ → H_{N2} $= \frac{\chi_{m2}\mu_1}{\chi_{m1}\mu_2} M_{N1}$ $[M = \chi_m H]$ $H_{N1} = \frac{M_{N1}}{\chi_{m1}}$ $\therefore \oint$ Now, $\oint \vec{H}\cdot d\vec{l} = I$. Small closed path in a plane normal to the boundary surface. $H_{t1}\Delta L - H_{t2}\Delta L = K\Delta L$. Surface current $\vec{K}$ → comp. normal to the plane of closed path is K. $\therefore \boxed{H_{t1} - H_{t2} = K}$ $\Rightarrow (\vec{H}_1 - \vec{H}_2) \times \hat{a}_{N12} = \vec{K}$ $\hat{a}_{N12}$ → unit normal at the boundary directed from region 1 to region 2 $(\vec{H}_{t1} - \vec{H}_{t2}) = \hat{a}_{N12} \times \vec{K}$ $\frac{B_{t1}}{\mu_1} - \frac{B_{t2}}{\mu_2} = K$ $M_{t2} = \frac{\chi_{m2}}{\chi_{m1}} M_{t1} - \chi_{m2} K$ $H_{t1} - H_{t2} = k$ $\Rightarrow \frac{M_{t1}}{\chi_{m1}} - \frac{M_{t2}}{\chi_{m2}} = k$ $\Rightarrow M_{t1}\chi_{m2} - M_{t2}\chi_{m1} = k\,\chi_{m1}\chi_{m2}$ $\Rightarrow M_{t2}\chi_{m1} = -k\,\chi_{m1}\chi_{m2} + M_{t1}\chi_{m2}$ $\Rightarrow \boxed{M_{t2} = \frac{\chi_{m2}}{\chi_{m1}} M_{t1} - k\,\chi_{m2}}$			
(b)	An air core toroid has 500 turns, mean radius of 15 cm, cross-sectional area of 6 cm^2. The magneto motive force is 2000 A.t. Calculate total reluctance, flux, flux-density, field intensity inside the core.	[04]	CO1	L3

4) An air core toroid has 500 turns, mean radius of 15 cm, cross-sectional area of 6 cm². The magnetomotive force is 2000 A.t. Calculate total reluctance, flux, flux-density, field intensity inside the core.

Sol: Reluctance, $\mathcal{R} = \frac{d}{\mu S} = \frac{2\pi \times (0.15)}{4\pi\times10^{-7}\times 6\times10^{-4}}$

$= 1.25\times10^{9}$ A.t/Wb.

The flux, $\phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{2000}{1.25\times10^{9}} = 1.6\times10^{-6}$ W

Flux-density, $B = \frac{\phi}{S} = \frac{1.6\times10^{-6}}{6\times10^{-4}} = 2.67\times10$

The field intensity, $H = \frac{B}{\mu_0} = \frac{2.67\times10^{-3}}{4\pi\times10^{-7}} = 2120$ A

OR

8 (a)	Using Faraday's law derive an expression for e.m.f induced in stationary conductor placed in a time varying magnetic field. Also explain motional e.m.f.	[06]	CO4	L3

$$emf = -N\frac{d\phi}{dt}$$

where, $\phi \rightarrow$ flux passing through any one of the N coincident paths.

e.m.f is defined as,

$emf = \oint \vec{E}\cdot d\vec{l} \rightarrow$ voltage about a specific closed path.

for electrostatic $\oint \vec{E}\cdot d\vec{l} = 0$

$$e.m.f = \oint \vec{E}\cdot d\vec{l} = -\frac{d\phi}{dt} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{S}$$

fingers of our right hand indicate the direction of the closed path, thumb indicates the direction of $d\vec{S}$.

- A flux-density $\vec{B}$ in the direction of $d\vec{S}$ and increasing with time thus produces an average value of $\vec{E}$ which is opposite to the +ve direction about the path.

Stationary path.

magnetic flux → time varying quantity.

$\therefore emf = \oint \vec{E}\cdot d\vec{L} = -\int \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$

Applying Stoke's theorem

$$\int_s (\vec{\nabla}\times\vec{E})\cdot d\vec{s} = -\int_s \frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$$

Surface integrals taken over identical general surfaces.

$$\therefore (\vec{\nabla}\times\vec{E})\cdot d\vec{s} = -\frac{\partial \vec{B}}{\partial t}\cdot d\vec{s}$$

$$\Rightarrow \boxed{(\vec{\nabla}\times\vec{E}) = -\frac{\partial \vec{B}}{\partial t}} \rightarrow \text{differential or point form}$$

$$\boxed{\oint \vec{E}\cdot d\vec{L} = -\int \frac{\partial B}{\partial t}\cdot d\vec{s}} \rightarrow \text{integral form.}$$

If $\vec{B}$ = constant, back to electrostatics

$$\oint \vec{E}\cdot d\vec{L} = 0$$

$$\Rightarrow (\vec{\nabla}\times\vec{E}) = 0$$

Enclosed flux ...

Motional emf: (Conductor moving in a uniform constant magnetic field)

Force on charge Q moving at a velocity $\vec{v}$ in the magnetic field $\vec{B}$.

$$\vec{F} = Q(\vec{v}\times\vec{B})$$

$$\boxed{\frac{\vec{F}}{Q} = \vec{v}\times\vec{B}}$$

Sliding bar → positive & negative charges experiencing this force.

Force per unit charge → motional electric field intensity (E_m)

$$\boxed{\vec{E}_m = \vec{v}\times\vec{B}}$$

$$\boxed{emf = \oint \vec{E}_m\cdot d\vec{L} = \oint (\vec{v}\times\vec{B})\cdot d\vec{L}}$$

non zero only if $\vec{v}$ is non zero

$$emf = \oint_L (\vec{v}\times\vec{B})\cdot d\vec{L} = \int_d^0 vB\,dx = -Bvd.$$

B is not a function of time

$$\boxed{emf = -Bvd}$$

(b)	Let $\mu = 10^{-5}$ H/m, $\epsilon = 4 \times 10^{-9}$ F/m, $\sigma = 0$ and $\rho_v = 0$. Find K so that the following pair of fields satisfy Maxwell's equations: $\boldsymbol{E} = (20y - Kt)\boldsymbol{a_x}$ V/m & $\boldsymbol{H} = (y + 2 \times 10^6 t)\, \boldsymbol{a_z}$ A/m. $\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$ $\quad \vec{J} = \sigma \cdot (\because \sigma = 0)$ $\nabla \times \vec{H} = \frac{\partial \vec{D}}{\partial t}$ $\begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 0 & y + 2\times10^6 t \end{vmatrix} = \varepsilon \frac{\partial}{\partial t}\left((20y - Kt)\,\vec{a_x}\right)$ $\vec{a_x}\left(\frac{\partial}{\partial y}(y + 2\times10^6 t) - 0\right) - \vec{a_y}(0-0) + \vec{a_z}(0-0) = 4\times10^{-9} \cdot (-K)\, \vec{a_x}$ $\vec{a_x} = 4\times10^{-9}(-K)\,\vec{a_x}$ $\boxed{K = -2.5\times10^{8}}$ V/(ms)	[04]	CO4	L3
9	Let $\mu = 3 \times 10^{-5}$ H/m, $\epsilon = 1.2 \times 10^{-10}$ F/m and $\sigma = 0$ everywhere. If $\boldsymbol{H} = 2\cos(10^{10}t - \beta x)\boldsymbol{a_z}$ A/m, use Maxwell's equations to obtain the expressions for **B**, **D**, **E** and β.	[10]	CO4	L3

$$\left.\begin{array}{l}\mu = 3\times10^{-5}\ H/m \\ \epsilon = 1.2\times10^{-10}\ F/m\end{array}\right\} \quad \sigma = 0 \qquad \vec{H} = 2\cos(10^{10}t - \beta x)\hat{a}_z$$

Calculate $\vec{B}$, $\vec{D}$, $\vec{E}$ and β.

$$\vec{B} = \mu\vec{H} = 6\times10^{-5}\cos(10^{10}t - \beta x)\,\hat{a}_z$$

$$\vec{\nabla}\times\vec{H} = \frac{\partial\vec{D}}{\partial t}$$

$$\vec{\nabla}\times\vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & 2\cos(10^{10}t - \beta x) \end{vmatrix}$$

$$= -\hat{a}_y\frac{\partial}{\partial x}\left[2\cos(10^{10}t - \beta x)\right]$$

$$= -\hat{a}_y\, 2\left(-\sin(10^{10}t - \beta x)\right)(-\beta)$$

$$= -2\sin(10^{10}t - \beta x)\,\beta\,\hat{a}_y$$

$$\therefore\ \vec{D} = -2\beta\int\sin(10^{10}t - \beta x)\,\hat{a}_y\,dt$$

$$= \frac{+2\beta\cos(10^{10}t - \beta x)}{10^{10}}\,\hat{a}_y$$

$$= \frac{2\beta\cos(10^{10}t - \beta x)}{10^{10}}\,\hat{a}_y \quad C/m^2$$

	Now, $\vec{D} = \epsilon \vec{E}$ $\therefore \vec{E} = \frac{\vec{D}}{\epsilon} = \frac{2\beta \cos(10^{10}t - \beta x)}{1.2\times10^{-10}\times10^{10}} \hat{a}_y$ $= 1.66\,\beta \cos(10^{10}t - \beta x)\,\hat{a}_y$ V/m $\vec{\nabla}\times\vec{E} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 1.66\beta\cos(10^{10}t - \beta x) & 0 \end{vmatrix}$ $= \hat{a}_z(-1.66\beta \sin(10^{10}t - \beta x))(-\beta)$ $= +1.66\beta^2 \sin(10^{10}t - \beta x)\,\hat{a}_z$ Now, $\vec{\nabla}\times\vec{E} = -\frac{\partial \vec{B}}{\partial t}$ $\therefore \vec{B} = -\int \vec{\nabla}\times\vec{E}\, dt = -\int 1.66\beta^2 (\sin(10^{10}t - \beta x))\,\hat{a}_z\, dt$ $= 1.66\beta^2 \hat{a}_z \frac{\cos(10^{10}t - \beta x)}{10^{10}}$ $= 1.66\times10^{-10}\,\beta^2 \cos(10^{10}t - \beta x)\,\hat{a}_z$ $\therefore 1.66\times10^{-10}\,\beta^2 = 6\times10^{-5}$ $\therefore \beta^2 = \frac{6\times10^{-5}}{1.66\times10^{-10}} = 3.61\times10^5 = 361000$ $\therefore \beta = \pm 600.$			
	OR			
10(a)	What is the inconsistency of Ampere's law with the equation of continuity? Derive modified form of Ampere's law. From Ampere's circuital law, $(\vec{\nabla}\times\vec{H}) = \vec{J}$ --- (1) $\therefore \vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = 0 = \vec{\nabla}\cdot\vec{J}$ Identically. $\Rightarrow \vec{\nabla}\cdot\vec{J} = 0.$ But from eqn. of continuity. $\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho_v}{\partial t}$ $\therefore$ For time varying field we add an unknown term $\vec{G}$ to eqn. (1) $\vec{\nabla}\times\vec{H} = \vec{J} + \vec{G}$ $\Rightarrow \vec{\nabla}\cdot(\vec{\nabla}\times\vec{H}) = \vec{\nabla}\cdot\vec{J} + \vec{\nabla}\cdot\vec{G}$ $\Rightarrow \vec{\nabla}\cdot\vec{J} = -\vec{\nabla}\cdot\vec{G}$ $\therefore \vec{\nabla}\cdot\vec{G} = \frac{\partial \rho_v}{\partial t}$ $\therefore \vec{\nabla}\cdot\vec{G} = \frac{\partial}{\partial t}(\vec{\nabla}\cdot\vec{D}) = \vec{\nabla}\cdot\frac{\partial \vec{D}}{\partial t}$ $\therefore \vec{G} = \frac{\partial \vec{D}}{\partial t}.$	[07]	CO4	L3

	∴ Ampere's circuital law in point form, $(\vec{\nabla} \times \vec{H}) = \left(\vec{J} + \frac{\partial D}{\partial t}\right)$			
(b)	List Maxwell's equations in point and integral forms for time varying field.	[03]	CO4	L1

- Maxwell's eqn. in point form,

$$\vec{\nabla} \cdot \vec{D} = \rho_v$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

Maxwell's eqn. in Integral form,

$$\oint \vec{D} \cdot \vec{ds} = Q$$

$$\oint \vec{B} \cdot \vec{ds} = 0$$

$$\oint \vec{E} \cdot \vec{dl} = -\frac{\partial}{\partial t} \int_s \vec{B} \cdot \vec{ds}$$

$$\oint \vec{H} \cdot \vec{dl} = I + \int_s \frac{\partial \vec{D}}{\partial t} \cdot \vec{ds}$$