CMR
INSTITUTE OF
TECHNOLOGY

USN

Internal Assesment Test - II									
Sub:	ENGINEERING ELECTROMAGNETICS							Code:	15EC36
Date:	07 / 11 / 2017	Duration:	90 mins	Max Marks:	50	Sem:	3rd	Branch:	ECE-Diploma
Answer FIVE FULL Questions									

		Marks	OBE	
			CO	RBT
1 (a)	Derive Lorentz force equation and mention the applications of its solutions.	[6]	CO1	L3
1 (b)	A charged particle moves with a uniform velocity of $4\ \boldsymbol{a_x}\ m/s$ in a region where $\boldsymbol{E} = 20\ \boldsymbol{a_y}\ \ V/m$ and $\boldsymbol{B} = B_0\ \boldsymbol{a_z}\ \ wb/m^2$. Determine B_o such that the velocity of the particle remains constant.	[4]	CO1	L3
	[OR]			
2	Two infinitely long straight conductors, both carrying current of 10mA each in the same direction are located at x=0; y=0 and x=0; y=10m respectively. Derive the expression for force per unit length between them. Also calculate the force per unit length between them.	[6+4]	CO1	L3
3	Obtain the boundary conditions at the interface between two magnetic materials.	[10]	CO1	L3
	[OR]			
4(a)	Given B = 0.05 T and $\mu_r = 50$. Find magnetic susceptibility, Magnetization and Magnetic field Intensity.	[5]	CO1	L3
4(b)	Two differential current elements are present as follows: $I_1\boldsymbol{dl}_1 = -3\boldsymbol{a_y}$ A.m at P$_1$(5,2,1) and $I_2\boldsymbol{dl}_2 = -4\boldsymbol{a_z}$ A.m at P$_2$(1,8,5). Determine differential force on element 2 due to element 1.	[5]	CO1	L3
5(a)	Derive the expression for force on a current element placed in a steady magnetic field.	[6]	CO1	L3
5(b)	Calculate **M** for the material in which: (a) μ_r=1.4 and $\boldsymbol{H} = 350\ \boldsymbol{a_x}$ A/m. (b) μ_r= 6 and there are 2.5×10^{29} atoms/m^3, each having magnetic dipole moment of $2.8 \times 10^{-30}\ \boldsymbol{a_x}$ A.m^2	[4]	CO1	L3
	[OR]			
6	Write the equations to compare electric circuit and magnetic circuit with respect to i) e.m.f and m.m.f, ii) Electric and magnetic scalar potential, iii) Electric current and magnetic flux, iv) Resistance and reluctance, v) $\oint E.dl$ and $\oint H.dl$	[10]	CO1	L3
7(a)	Derive the expression for magnetic torque due to a rectangular current loop.	[5]	CO1	L3
7(b)	An air core toroid has 500 turns, mean radius of 15 cm, cross-sectional area of 6 cm^2. The magneto motive force is 2000 A.t. Calculate total reluctance, flux, flux-density, field intensity inside the core.	[5]	CO1	L3
	[OR]			
8(a)	Derive the expression for force between differential current elements	[5]	CO1	L3
8(b)	The magnetic field intensity is given in a region of space as $\boldsymbol{H} = \frac{(x+2y)}{z^2}\boldsymbol{a_y} + \frac{2}{Z}\boldsymbol{a_z}\ A/m$. Calculate current density in that region.	[5]	CO1	L3
9(a)	At a point P(x,y,z) the components of vector magnetic potential are given as $Ax = 4x + 3y + 2z, Ay = 5x + 6y + 3z, Az = 2x + 3y + 5z$. Determine **B** at P.	[4]	CO1	L3
9(b)	Find the torque on the square wire loop of current 0.6 A, bounded by $x = -1\ to\ x = 1$ and $y = -1\ to\ y = 1$ about (2, 3, 4) by the field $\boldsymbol{B} = (0.4\boldsymbol{ax} + 0.6\boldsymbol{ay} - 0.7\boldsymbol{az})$ T acting only on the side of the loop for which y = 1.	[6]	CO1	L3
	[OR]			
10	Let $\mu_1 = 4\ \mu_0$ H/m in region 1 where z > 0, while $\mu = 7\ \mu_0$ H/m where z < 0. Let $\boldsymbol{K} = 80\ \boldsymbol{ax}\ A/m$ on the surface z = 0. In region1, the magnetic flux density is $\boldsymbol{B1} = 2\boldsymbol{ax} - 3\boldsymbol{ay} + \boldsymbol{az}\ mT$. Find $\boldsymbol{B_2}$.	[10]	CO1	L3

SCHEME OF EVALUATION

		Mark Split-Up
1(a)	Derivation Final Expression	4 2
(b)	Formula Approach & Answer	2 2
2	Diagram Derivation Final Expression Solution	1 4 1 4
3	Normal Conditions Tangential Conditions	3 3
4(a)	Formula Approach Answer	1 3 1
(b)	Formula Approach Answer	1 3 1
5(a)	Derivation Final Expression	4 2
5(b)	(i) Approach & Answer (ii) Approach & Answer	2 2
6	Comparison of each (i),(ii),(iii),(iv) and (v)	5*2=10
7(a)	Diagram Derivation Final Expression	1 3 1
(b)	Formula & Approach Answer	4 1
8(a)	Derivation Final Expression	4 1
(b)	Formula Approach Answer	1 3 1
9(a)	Formula Approach Answer	1 2 1
(b)	Formula Approach Answer	1 4 1
10	Formula & Approach Answer	8 2

SOLUTION

1)a)

Electric field causes a force to be exerted on a stationary or moving charge

Steady magnetic field → exerts force only on a moving charge
(produced by moving charges)

Force on a moving charge:

Electric force on a charged particle, $\boxed{\vec{F} = Q\vec{E}}$ → ①
↓
Same dirn as $\vec{E}$ for a positive charge.

If the charge is in motion, the above equation gives the force at any point in its trajectory.

Force on a charged particle is in motion in a magnetic field of flux density, $\vec{B}$
↓
$\boxed{\vec{F} = Q\vec{v} \times \vec{B}}$ → ②
direction of force is $\perp^r$ to both $\vec{v}$ and $\vec{B}$

$\vec{F}$ applied $\perp^r$ to the dirn in which charge is moving
↓
can never change its velocity

Acceleration vector is always normal to velocity vector

Kinetic energy of particle remains unchanged.

Steady magnetic field is incapable of transferring energy to a moving charge

Electric field → exerts force on particle which is independent of the dirn of progressing charge
& effects an energy transfer between field and particle in general

Force on a moving particle arising from combined electric & magnetic field.
(by superposition)

$\boxed{\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})}$ → ③ → Lorentz force equation
↓
Solution is required in determining
1) electron orbits in magnetron
2) Proton paths in cyclotron
3) plasma characteristics in a magneto hydro dynamic (MHD) generator
↓
In general charged particle motion in combined electric and magnetic fields

1)b)

<u>Solution:</u> If a particle moves with constant velocity, its acceleration is zero.

$$F = ma = 0 = Q(\vec{E} + \vec{v} \times \vec{B})$$

$$0 = Q(20\vec{a_y} + 40\vec{a_x} \times B_0\vec{a_z})$$

$$0 = Q(20\vec{a_y} - B_04\vec{a_y})$$

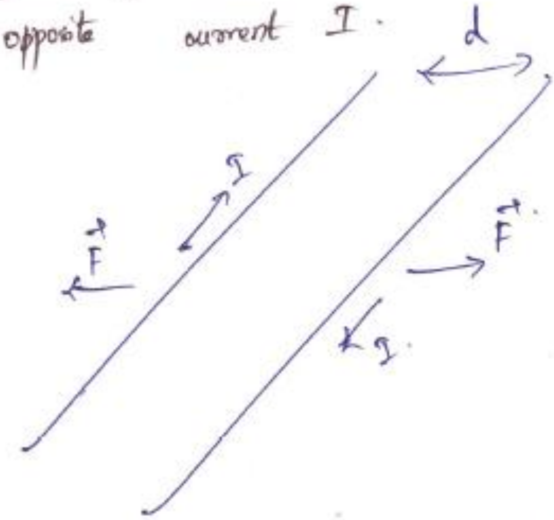
$Q \neq 0$

$$\therefore 20\vec{a_y} = 4B_0\vec{a_y}$$

$$\boxed{B_0 = 5}$$

2)

Find Force of repulsion between two infinitely long, straight, parallel filamentary conductors with separation 'd' and carrying equal but opposite current I.



Magnetic field intensity at either wire due to the current at either wire

$$|\vec{H}| = \frac{I}{2\pi d}$$

$$|\vec{B}| = \frac{\mu_0 I}{2\pi d}$$

$$F = BIL$$

Force per unit length of infinite conductors

$$F/L = BI$$

Repulsive force $$\boxed{F/L = \frac{\mu_0 I^2}{2\pi d}}$$ N/m

Force per unit length of two infinitely long parallel current carrying conductors $= F/L = \frac{\mu_0 I^2}{2\pi d}$

$$= \frac{\overset{2}{\cancel{4}}\cancel{\pi}\times 10^{-7}\times(10\times10^{-3})^2}{\cancel{2\pi}\times \underset{5}{\cancel{10}}}$$

$$= 20\times10^{-7}\times10^{-6}$$

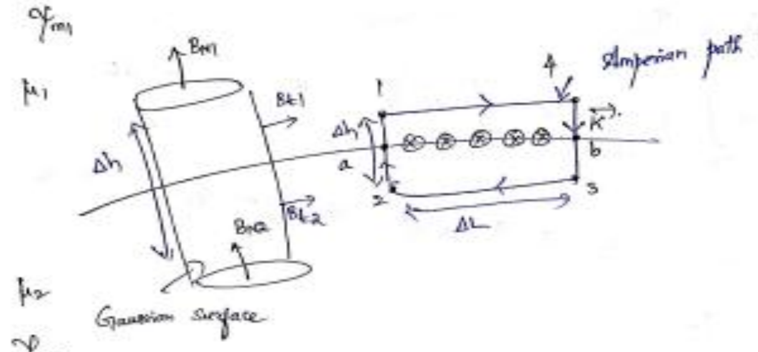
$$= 20\times10^{-13}$$

$$F/L = 2\times10^{-12}\ N/m$$

$$\boxed{F/L = 2}\ p\ N/m$$

3)

Magnetic boundary conditions:



$\vec{B_1} = \vec{B_{t1}} + \vec{B_{N1}}$

$\vec{B_2} = \vec{B_{t2}} + \vec{B_{N2}}$

$$\boxed{\begin{matrix}\vec{B} = \mu\vec{H} \\ \vec{M} = \chi_m \vec{H}\end{matrix}}$$

$\vec{H_1} = \vec{H_{t1}} + \vec{H_{N1}}$

$\vec{H_2} = \vec{H_{t2}} + \vec{H_{N2}}$

① Gauss's law for Magnetic fields:

$$\phi = \oint_S \vec{B}\cdot\vec{ds} = 0$$

$$\Rightarrow \int_{top} + \int_{bottom} + \int_{lateral} \vec{B}\cdot\vec{ds} = 0$$

$$\Rightarrow B_{N1}\cdot\Delta S - B_{N2}\cdot\Delta S + B_{t_1}\underset{0}{\cancel{\frac{\Delta h}{2}}}2\pi r + B_{t2}\underset{0}{\cancel{\frac{\Delta h}{2}}}2\pi r = 0$$

We are obtaining conditions at the boundary.

$\therefore \Delta h \longrightarrow 0.$

$$B_{N1}\cdot\Delta S - B_{N2}\,\Delta S = 0$$

$\boxed{B_{N1} = B_{N2}}$ $\Rightarrow \mu_1 H_{N1} = \mu_2 H_{N2}$

$\Rightarrow \boxed{H_{N1} = \frac{\mu_2}{\mu_1} H_{N2}}$

$$\frac{M_{N1}}{\chi_{m_1}} = \frac{\mu_2}{\mu_1}\frac{M_{N2}}{\chi_{m2}} \Rightarrow \boxed{M_{N1} = \frac{\chi_{m1}}{\chi_{m2}}\cdot\frac{\mu_2}{\mu_1} M_{N2}}$$

② Ampere's circuital Law:

$$\oint_L \vec{H} \cdot \vec{dL} = I_{enc}$$

$$\int_{14} + \int_{4b} + \int_{b3} + \int_{32} + \int_{2a} + \int_{a1} \vec{H} \cdot \vec{dL} = K \cdot \Delta L$$

$$H_{t1} \cdot \Delta L + \left(-H_{N1} \underset{0}{\cancel{\frac{\Delta h}{2}}}\right) + \left(-H_{N2} \cdot \underset{0}{\cancel{\frac{\Delta h}{2}}}\right) - H_{t2}\, \Delta L + H_{N2} \underset{0}{\cancel{\frac{\Delta h}{2}}} + H_{t1} \underset{0}{\cancel{\frac{\Delta h}{2}}} = K \cdot \Delta L$$

At the boundary $\Delta h \longrightarrow 0$

$$(H_{t1} - H_{t2}) \cdot \Delta L = K \cdot \Delta L$$

$$\boxed{H_{t1} - H_{t2} = K} \Rightarrow \boxed{\frac{B_{t1}}{\mu_1} - \frac{B_{t2}}{\mu_2} = k}$$

$$\Downarrow$$

$$\frac{M_{t1}}{\chi_{m1}} - \frac{M_{t2}}{\chi_{m2}} = K \Rightarrow \boxed{M_{t2} = \frac{M_{t1}}{\chi_{m1}} \cdot \chi_{m2} - K\, \chi_{m2}}$$

4) a)

$|\vec{B}| = 0.05T$

$\mu_r = 50$

Find:

Magnetic susceptibility, $\chi_m = \mu_r - 1.$

$$\boxed{\chi_m = 49}$$

$\vec{B} = \mu \vec{H}$

Magnetization, $M = \chi_m H$

$$= \chi_m \cdot \frac{B}{\mu}$$

$$= \frac{\chi_m \cdot B}{\mu_0 \mu_r}$$

$$= \frac{49 \times 0.05}{4\pi \times 10^{-7} \times 50}$$

$$= \frac{3.9012 \times 10^{-3}}{10^{-7}}$$

$$\boxed{M = 3.9012 \times 10^{4}} \text{ A/m}$$

Magnetic field Intensity, $H = \frac{B}{\mu} = \frac{0.05}{4\pi \times 10^{-7} \times 50}$

$$= \frac{7.961 \times 10^{-5}}{10^{-7}}$$

$$\boxed{H = 796.1} \text{ A/m}$$

4) b)

Problem:

Given $P_1(5,2,1)$, $P_2(1,8,5)$,

$$I_1 d\vec{L_1} = -3\vec{a_y} \text{ A.m}$$

$$I_2 d\vec{L_2} = -4\vec{a_z} \text{ A.m}$$

Find: $d(d\vec{F_2}) = ?$ and $d(d\vec{F_1}) = ?$

Solution:

$$\vec{R_{12}} = -4\vec{a_x} + 6\vec{a_y} + 4\vec{a_z}.$$

$$\vec{R_{21}} = 4\vec{a_x} - 6\vec{a_y} - 4\vec{a_z}.$$

$$d(d\vec{F_2}) = \frac{4\pi \times 10^{-7}}{4\pi} \times \left[\frac{-4\vec{a_z} \times \left((-3\vec{a_y}) \times (-4\vec{a_x} + 6\vec{a_y} + 4\vec{a_z})\right)}{(16+36+16)^{3/2}} \right]$$

$$= \frac{10^{-7}}{560.742} \left[-4\vec{a_z} \times \left(-12\vec{a_z} - 12\vec{a_x}\right) \right]$$

$$= \frac{+10^{-7}}{560.742} \left[48\vec{a_y} \right]$$

$$\boxed{d(d\vec{F_2}) = 8.56\,\vec{a_y}\ \text{nN}}$$

$$d(d\vec{F_1}) = \frac{4\pi \times 10^{-7}}{4\pi} \times \left(\frac{-3\vec{a_y} \times \left[(-4\vec{a_z}) \times (4\vec{a_x} - 6\vec{a_y} - 4\vec{a_z})\right]}{560.742} \right)$$

$$= \frac{10^{-7}}{560.742} \times \left(-3\vec{a_y} \times \left(-16\vec{a_y} - 24\vec{a_x}\right)\right) = \frac{10^{-7}}{560.742}\left(-72\vec{a_z}\right)$$

$$\boxed{d(d\vec{F_1}) = -12.84\,\vec{a_z}}\ \text{nN}$$

5) a)

Convection current density

$$\boxed{\vec{J} = \rho_v \vec{v}}$$

Differential element of charge,

$$\boxed{dQ = \rho_v\, dv.}$$

$$d\vec{F} = \rho_v\, dv\ \vec{v} \times \vec{B}$$

$$\boxed{d\vec{F} = \vec{J} \times \vec{B}\ dv} \rightarrow ⑤$$

W.K.T: differential current elements

$$\vec{J}\, dv = \vec{K}\, ds = I\, d\vec{L}$$

Lorentz force equation to different current elements

$$\boxed{d\vec{F} = \vec{K} \times \vec{B}\ ds} \rightarrow ⑥$$

$$\boxed{d\vec{F} = I\, d\vec{L} \times \vec{B}} \rightarrow ⑦$$

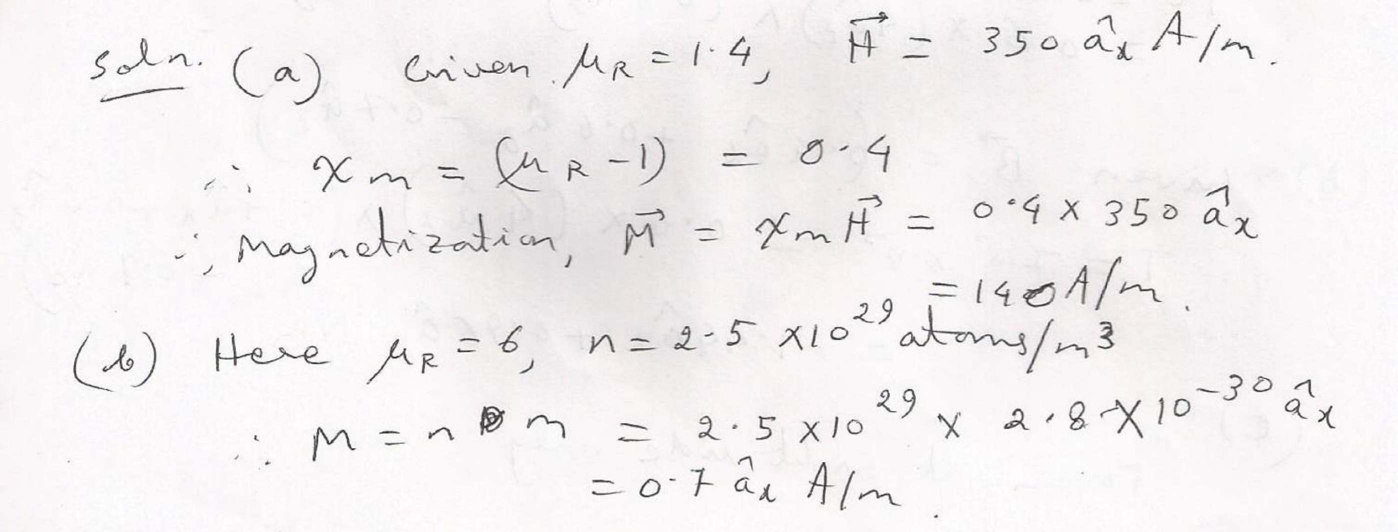
Force

$$\boxed{\vec{F} = \int_{vol} (\vec{J} \times \vec{B})\, dv} \rightarrow ⑧$$

$$\boxed{\vec{F} = \int_{s} (\vec{K} \times \vec{B})\, ds} \rightarrow ⑨$$

$$\vec{F} = \oint_{L} I\, d\vec{L} \times \vec{B}$$

5) b)



Soln. (a) Given $\mu_R = 1{\cdot}4$, $\vec{H} = 350\,\hat{a}_x$ A/m.

$\therefore\ \chi_m = (\mu_R - 1) = 0{\cdot}4$

$\therefore$ Magnetization, $\vec{M} = \chi_m \vec{H} = 0{\cdot}4 \times 350\,\hat{a}_x$

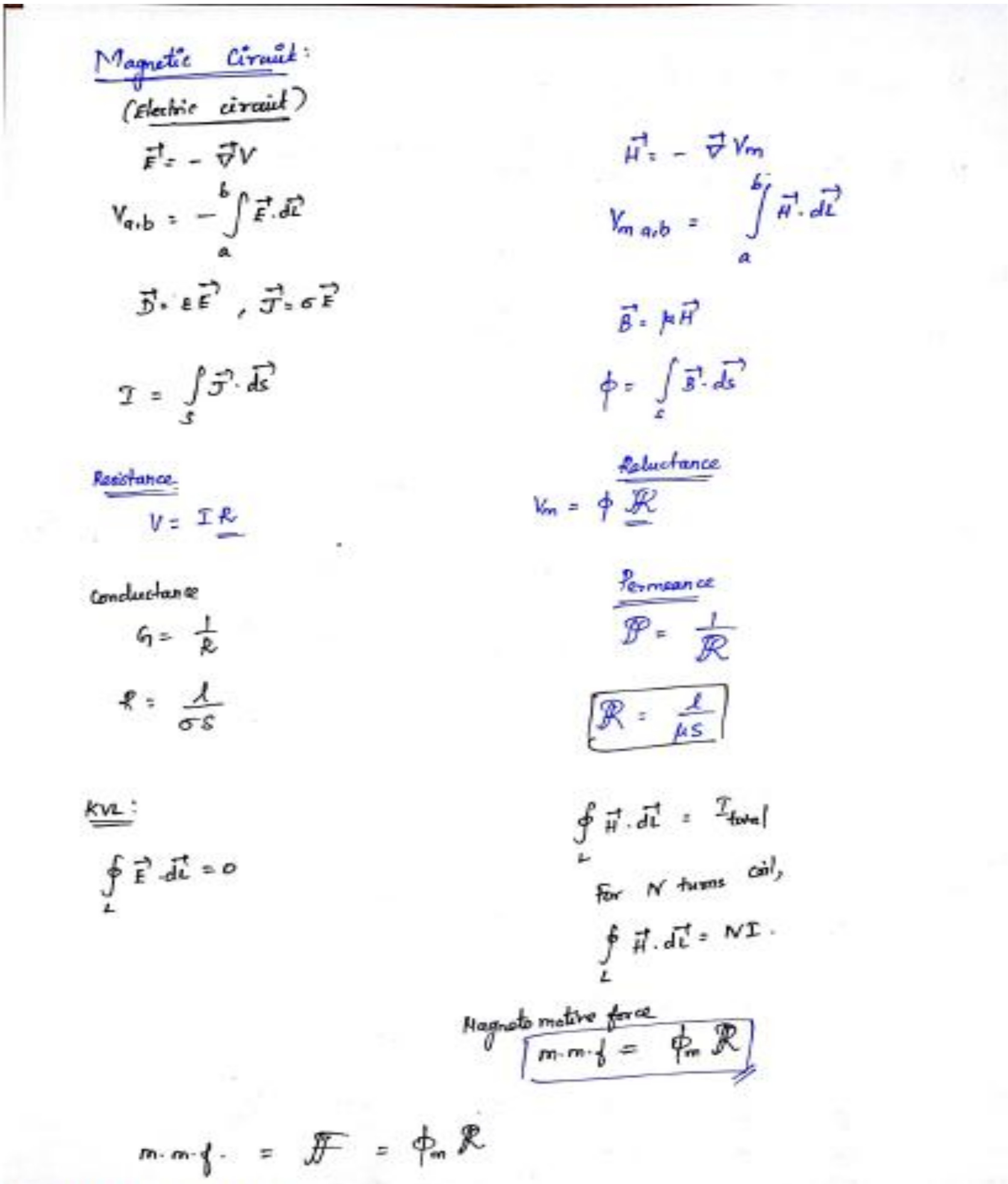
$= 140$ A/m.

(b) Here $\mu_R = 6$, $n = 2{\cdot}5 \times 10^{29}$ atoms/m^3

$\therefore\ M = n\,m = 2{\cdot}5 \times 10^{29} \times 2{\cdot}8 \times 10^{-30}\,\hat{a}_x$

$= 0{\cdot}7\,\hat{a}_x$ A/m.

6)



Magnetic Circuit:

(Electric circuit)

$\vec{E} = -\vec{\nabla} V$ | $\vec{H} = -\vec{\nabla} V_m$

$V_{a,b} = -\int_a^b \vec{E}\cdot\vec{dL}$ | $V_{m\,a,b} = \int_a^b \vec{H}\cdot\vec{dL}$

$\vec{D} = \epsilon\vec{E}$, $\vec{J} = \sigma\vec{E}$ | $\vec{B} = \mu\vec{H}$

$I = \int_S \vec{J}\cdot\vec{ds}$ | $\phi = \int_S \vec{B}\cdot\vec{ds}$

Resistance

$V = IR$

Reluctance

$V_m = \phi\,\mathcal{R}$

Conductance

$G = \frac{1}{R}$

$R = \frac{l}{\sigma S}$

Permeance

$\mathcal{P} = \frac{1}{\mathcal{R}}$

$\boxed{\mathcal{R} = \frac{l}{\mu S}}$

KVL:

$\oint_L \vec{E}\cdot\vec{dL} = 0$

$\oint_L \vec{H}\cdot\vec{dL} = I_{total}$

For N turns coil,

$\oint_L \vec{H}\cdot\vec{dL} = NI$.

Magneto motive force

$\boxed{m.m.f = \phi_m \mathcal{R}}$

$m.m.f. = \mathcal{F} = \phi_m \mathcal{R}$

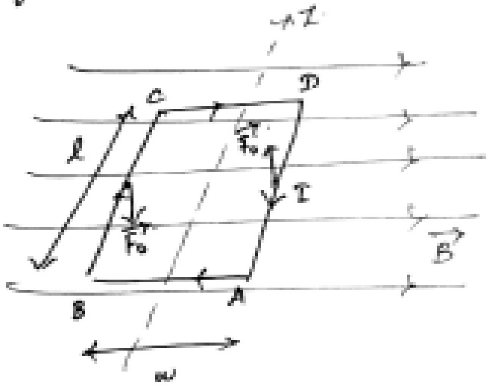
7)a)

Magnetic Torque and Moment :

Torque (or mechanical moment of force) on the loop is the vector product of force $\vec{F}$ and the moment arm $\vec{r}$

$$\boxed{\vec{T} = \vec{r} \times \vec{F}} \quad \text{N-m.}$$

Rectangular loop of length 'l' and width 'w' placed in a uniform magnetic field "$\vec{B}$".



$$\text{Force} = \oint_L I \, \vec{dL} \times \vec{B}$$

$$= \int_{AB} I \underbrace{\cancel{\vec{dL} \times \vec{B}}}_{0} + \int_{BC} I \, \vec{dL} \times \vec{B}.$$

$$+ \int_{CD} I \underbrace{\cancel{\vec{dL} \times \vec{B}}}_{0} + \int_{DA} I \, \vec{dL} \times \vec{B}$$

since $\vec{B}$ & $\vec{dL}$ are parallel

$$\text{Force } \vec{F} = I \int_{BC} \vec{dL} \times \vec{B} + I \int_{DA} \vec{dL} \times \vec{B}$$

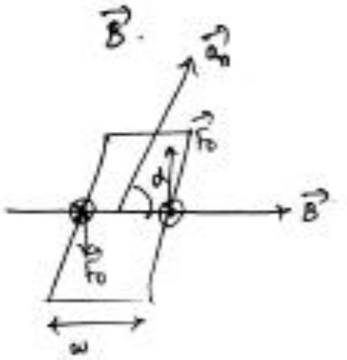
$$= I \int_0^l dz \, \vec{a_z} \times \vec{B} + I \int_l^0 dz \, \vec{a_z} \times \vec{B}$$

$$= \vec{F_0} - \vec{F_0}$$ where $|\vec{F_0}| = |\vec{B}| I l$ because $\vec{B}$ is uniform

$$\vec{F} = 0$$ ∴ No force on the loop as a whole when $\vec{B}$ is uniform

However, $\vec{F_0}$ and $-\vec{F_0}$ act at different points on the loop, thereby creating a couple.

Let Normal to the plane of the loop makes an angle α with $\vec{B}$.



Torque (moment of couple) on the loop

$$\vec{T} = \vec{r} \times \vec{F}$$

$$|\vec{T}| = |\vec{F_0}| \, w \sin \alpha$$

$$|\vec{T}| = BIlw \sin\alpha$$

Area of the loop $\boxed{S = lw}$

$$\boxed{|\vec{T}| = BIS \sin\alpha}$$

Magnetic dipole moment,

$$\boxed{\vec{m} = IS\vec{a_n}} \quad Am^2$$

$\downarrow$

→ product of current and area of the loop.
→ its direction normal to the loop.

$$\boxed{\vec{T} = \vec{m} \times \vec{B}}$$

Torque is in the direction of axis of rotation.
It is directed with the aim of reducing α so that $\vec{m}$ and $\vec{B}$ are in same direction (ie at equilibrium).

If loop is perpendicular to magnetic field,
torque & sum of forces on the loop will be zero.

7) b)

Solution:

① $\mathcal{R} = \frac{l}{\mu S} = \frac{2\pi\rho}{\mu S}$

$= \frac{\overset{1}{\cancel{2\pi}} \times 10^{5} \times 10^{-2}}{\underset{2}{\cancel{4\pi}} \times 10^{-7} \times \underset{2}{\cancel{6}} \times 10^{-2} \times 10^{-2}}$

$$\boxed{\mathcal{R} = 1.25 \times 10^{9} \ At/wb}$$

② $\phi \cdot \mathcal{R} = \mathcal{F}$

$\phi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{2000}{1.25 \times 10^{9}}$

$= 1600 \times 10^{-9}$

$$\boxed{\phi = 1.6 \ \mu \ Wb}$$

③ Flux density, $B = \frac{\phi}{S} = \frac{1.6 \times 10^{-6}}{6 \times 10^{-4}} = 2.667 \times 10^{-3} \ Wb/m^2$

$$\boxed{B = 2.667 \ m \ Wb/m^2 \text{ (or) } T}$$

④ Field intensity, $H = \frac{B}{\mu} = \frac{2.667 \times 10^{-3}}{4\pi \times 10^{-7}} = 2123.4 \ At/m.$

$$\boxed{H = 2123.4 \ At/m}$$

8)a)

Force between differential current elements:

Consider two current elements, to find force b/w two current elements

W.K.T: Magnetic field at point 2 due to current element at point 1.

$$d\vec{H_2} = \frac{I_1 \, d\vec{L_1} \times \vec{a_{R_{12}}}}{4\pi \, |\vec{R_{12}}|^2}$$

Differential force on a differential current element

$$d\vec{F} = \vec{I} \, d\vec{L} \times \vec{B}$$

Differential Flux density $(d\vec{B}_2)$ at point 2 caused by current element 1 $(I\,d\vec{L}_1)$

Differential amount of differential force on element 2,

$$d(d\vec{F}_2) = I_2\, d\vec{L}_2 \times d\vec{B}_2$$

where

$$d\vec{B}_2 = \mu_0\, d\vec{H}_2 = \frac{\mu_0 I_1\, d\vec{L}_1 \times \vec{a}_{R12}}{4\pi |\vec{R}_{12}|^2}.$$

$$\therefore \boxed{d(d\vec{F}_2) = \mu_0 \frac{I_1 I_2}{4\pi |\vec{R}_{12}|^2} \cdot d\vec{L}_2 \times (d\vec{L}_1 \times \vec{a}_{R12})} \rightarrow ⑬$$

$d(d\vec{F}_1) \neq d(d\vec{F}_2)$ because of the non physical nature of the current element.

From the differential force, we can get

Total force between two filamentary circuits

$$\boxed{\vec{F}_2 = \frac{\mu_0 I_1 I_2}{4\pi} \oint \left[d\vec{L}_2 \times \oint \frac{d\vec{L}_1 \times \vec{a}_{R12}}{|\vec{R}_{12}|^2} \right]}$$

(or)

$$\boxed{\vec{F}_2 = \frac{\mu_0 I_1 I_2}{4\pi} \oint \left[\oint \frac{\vec{a}_{R12} \times d\vec{L}_1}{|\vec{R}_{12}|^2} \right] \times d\vec{L}_2}$$

8)b)

Soln

$$\vec{\nabla} \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{z} \\ 0 & \frac{(x+2y)}{z^2} & \frac{2}{z} \end{vmatrix} = \frac{2(x+2y)}{z^3}\,\hat{a}_x + \frac{1}{z^2}\,\hat{a}_z$$

$$\therefore \vec{J} = \vec{\nabla} \times \vec{H} = \frac{2(x+2y)}{z^3}\,\hat{a}_x + \frac{1}{z^2}\,\hat{a}_z.$$

ⓒ $$I = \int \vec{J} \cdot d\vec{s} = \int \left(\frac{2(x+2y)}{z^3}\,\hat{a}_x + \frac{1}{z^2}\,\hat{a}_z \right) \cdot dx\,dy\,\hat{a}_z$$

$$= \frac{1}{z^2} \int_{x=1}^{2} \int_{y=3}^{5} dx\,dy = 16 \cdot [2-1][5-3] = \frac{1}{8} A$$

9) a)

$$\vec{B} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (4x+3y+2z) & (5x+6y+3z) & (2x+3y+5z) \end{vmatrix}$$

$$= \vec{a_x}\left[3-3\right] - \vec{a_y}\left[2-2\right] + \vec{a_z}\left[5-3\right]$$

$$\boxed{\vec{B} = 2\vec{a_z}}\ Wb/m^2$$

9) b)

$\vec{T} = \vec{R} \times \vec{F}$.

$\vec{F}$ is acting on right side only.

$$\vec{F} = I\,\vec{L} \times \vec{B}$$

$$I\vec{L} = 0.6 \times (-2\vec{a_x})$$

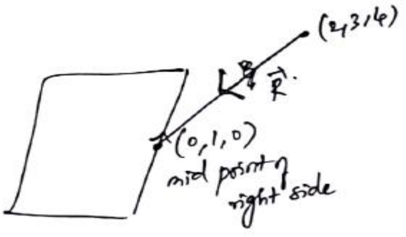
$$\vec{B} = 0.4\vec{a_x} + 0.6\vec{a_y} - 0.7\vec{a_z}.$$

$$\vec{F} = I\vec{L} \times \vec{B} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ -1.2 & 0 & 0 \\ 0.4 & 0.6 & -0.7 \end{vmatrix}$$

$$= \vec{a_x}(0-0) - \vec{a_y}(1.2\times0.7-0) + \vec{a_z}(-1.2\times0.6-0)$$

$$\boxed{\vec{F} = -0.84\vec{a_y} - 0.72\vec{a_z}}\ N$$

$\vec{R} = ?$



$$\vec{R} = -2\vec{a_x} - 2\vec{a_y} - 4\vec{a_z}.$$

$$\vec{T} = \vec{R} \times \vec{F} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_z} \\ -2 & -2 & -4 \\ \cancel{-0.84}\ 0 & -0.84 & -0.72 \end{vmatrix} = \vec{a_x}(-1.92) - \vec{a_y}(1.44) + \vec{a_z}(1.68)$$

$$\boxed{\vec{T} = -1.92\vec{a_x} - 1.44\vec{a_y} + 1.68\vec{a_z}}\ Nm$$

10)

Soln. The normal component of B_1 is,

$$\vec{B}_{N_1} = (\vec{B_1} \cdot \hat{a}_{N12}) \cdot \hat{a}_{N12}$$

$$= \left[(2\hat{a}_x - 3\hat{a}_y + \hat{a}_z) \cdot (-\hat{a}_z)\right] \cdot (-\hat{a}_z) \text{ mT}$$

$$= \hat{a}_z \text{ mT}.$$

$$\therefore \boxed{B_{N_2} = B_{N_1} = \hat{a}_z \text{ mT}}$$

The tangential component of flux-density in region 1 is,

$$\vec{B}_{t_1} = \vec{B}_1 - \vec{B}_{N_1} = 2\hat{a}_x - 3\hat{a}_y + \hat{a}_z - \hat{a}_z$$

$$= (2\hat{a}_x - 3\hat{a}_y) \text{ mT}$$

∴ The tangential field intensity in region 1 is,

$$\vec{H}_{t_1} = \frac{\vec{B}_{t_1}}{\mu_1} = \frac{(2\hat{a}_x - 3\hat{a}_y)}{4 \times 10^{-6}} = (500\hat{a}_x - 750\hat{a}_y) \text{ A/m}$$

The tangential field intensity in region 2,

$$\vec{H}_{t_2} = \vec{H}_{t_1} - \hat{a}_{N12} \times \vec{K}$$

$$= (500\hat{a}_x - 750\hat{a}_y) - \left[(-\hat{a}_z) \times 80\hat{a}_x\right]$$

$$= 500\hat{a}_x - 670\hat{a}_y \text{ A/m}.$$

The tangential flux-density in region 2 is,

$$\vec{B}_{t_2} = \mu_2 \vec{H}_{t_2} = 7 \times 10^{-6}(500\hat{a}_x - 670\hat{a}_y)$$

$$= 3.5\hat{a}_x - 4.69\hat{a}_y \text{ mT}.$$

∴ Flux-density in region 2 is,

$$\vec{B}_2 = \vec{B}_{N_2} + \vec{B}_{t_2} = 3.5\hat{a}_x - 4.69\hat{a}_y + \hat{a}_z \text{ mT}$$