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INTERNAL ASSESSMENT TEST – II

Sub:	DIGITAL SIGNAL PROCESSING							Code:	15EC52
Date:	07 / 11 / 2017	Duration:	90 mins	Max Marks:	50	Sem:	V	Branch:	ECE(D),TCE(B)

Answer any 5 full questions

	Question	Marks	CO	RBT
1.	Derive the Radix-2 DIT-FFT algorithm to compute the DFT of an 8-point sequence. Draw the complete signal flow graph.	[10]	CO502.2	L2
2.	Compute the DFT of $x[n] = [0,1,2,3,4,5,6,7]$ using DIF-FFT.	[10]	CO502.2	L2
3.	Compute the IDFT of $X[k] = [\,36,\ 1-2.4142j,\ -8+8j,\ 1-0.4142j,\ -8, 1+0.4142j,\ -8-8j,\ 1+2.4142j\,]$ using DIF-IFFT.	[10]	CO502.2	L2
4	Compute the circular convolution of $x[n] = [2,4,6,8]$ and $h[n] = [1,3,5,7]$ using DIT-FFT algorithm.	[10]	CO502.2	L2

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5	Explain Goertzel algorithm using difference equation. Obtain the direct form-II realization of the system which computes DFT using Goertzel algorithm.	[10]	CO502.2	L2
6	Derive an expression for the frequency response of the filters having a) symmetric and odd length impulse response b) antisymmetric and even length impulse response	[10]	CO502.3	L3
7	The desired response of a filter is $H_d(e^{j\omega}) = \begin{cases} e^{-j3\omega}, & -\frac{3\pi}{4} \le \omega \le \frac{3\pi}{4} \\ 0, & \frac{3\pi}{4} < \lvert\omega\rvert \le \pi \end{cases}$ Determine the impulse response using Hamming window.	[10]	CO502.3	L3
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Solutions

1 $$X(k) = \sum_{n=0}^{N-1} x(n) W_N^{kn}$$

$0 \le k \le N-1$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_N^{2kn} + \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_N^{k(2n+1)}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} x(2n) W_{\frac{N}{2}}^{kn} + W_N^{k} \sum_{n=0}^{\frac{N}{2}-1} x(2n+1) W_{\frac{N}{2}}^{kn}$$

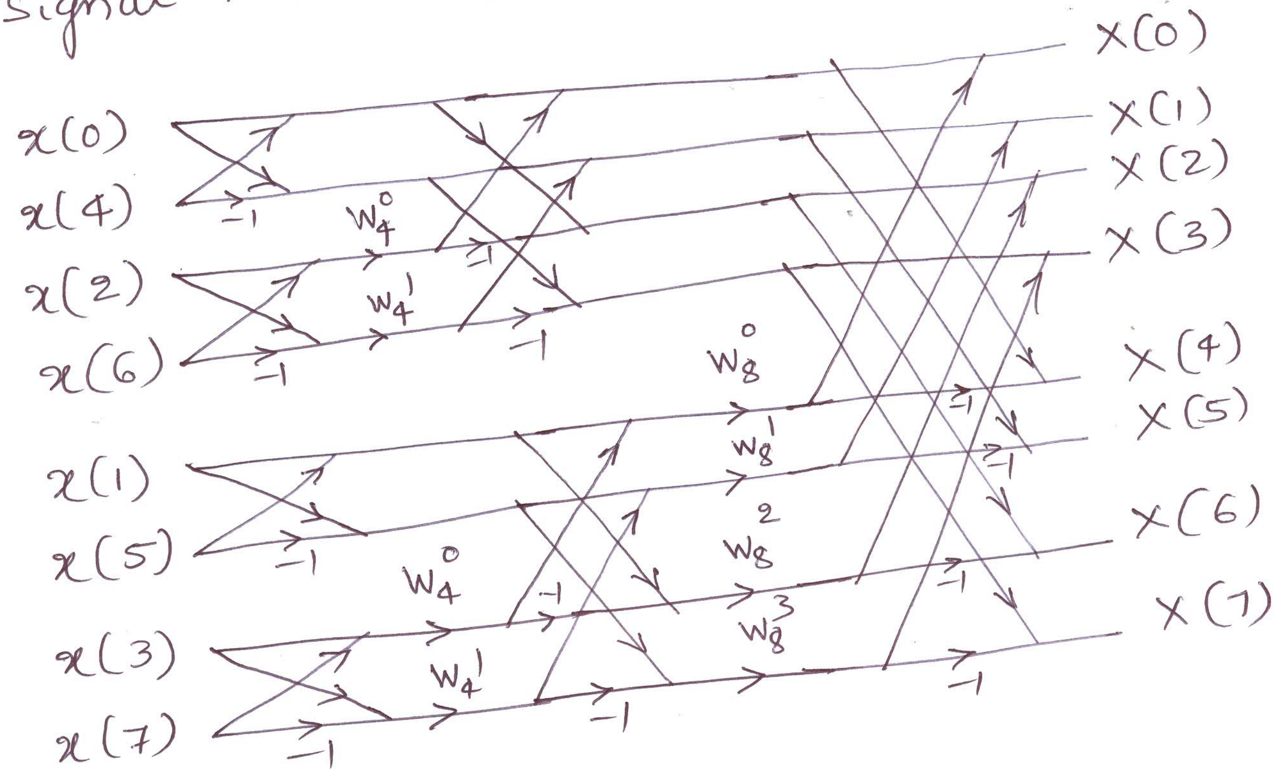
$0 \le k \le N-1$

$$X(k) = X_1(k) + W_N^k X_2(k)$$

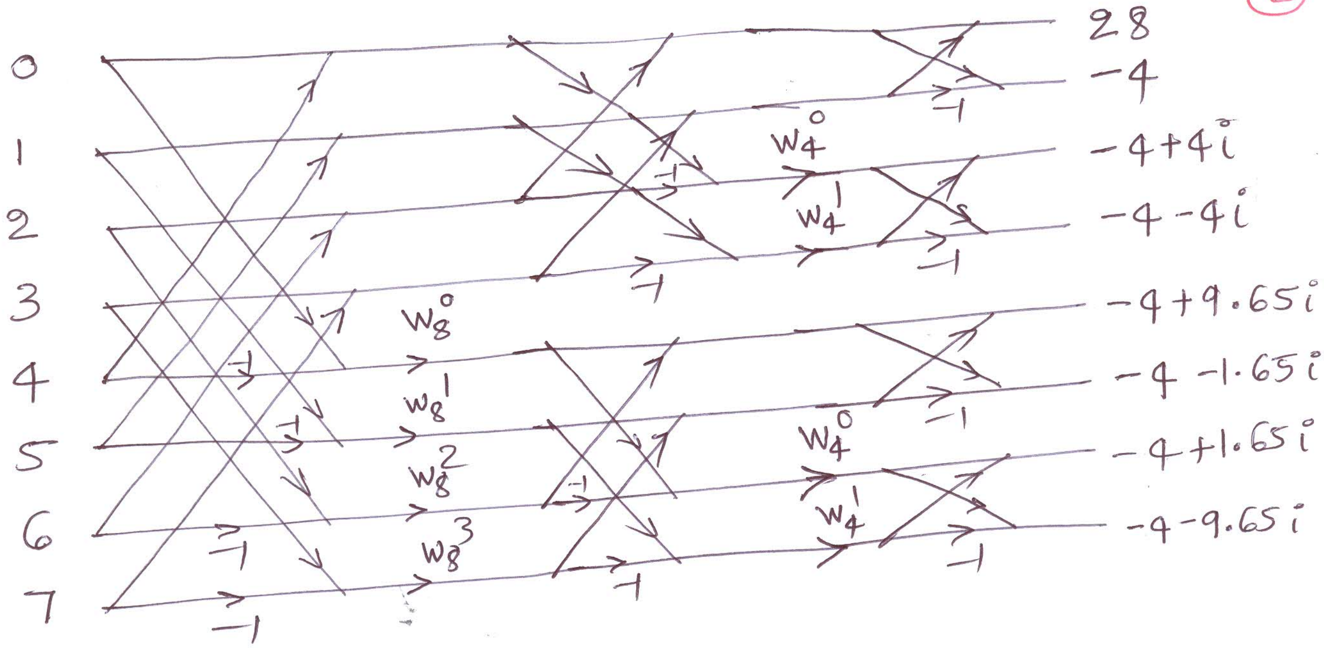
$0 \le k \le \frac{N}{2}-1$

$$X\left(k+\frac{N}{2}\right) = X_1(k) - W_N^k X_2(k)$$

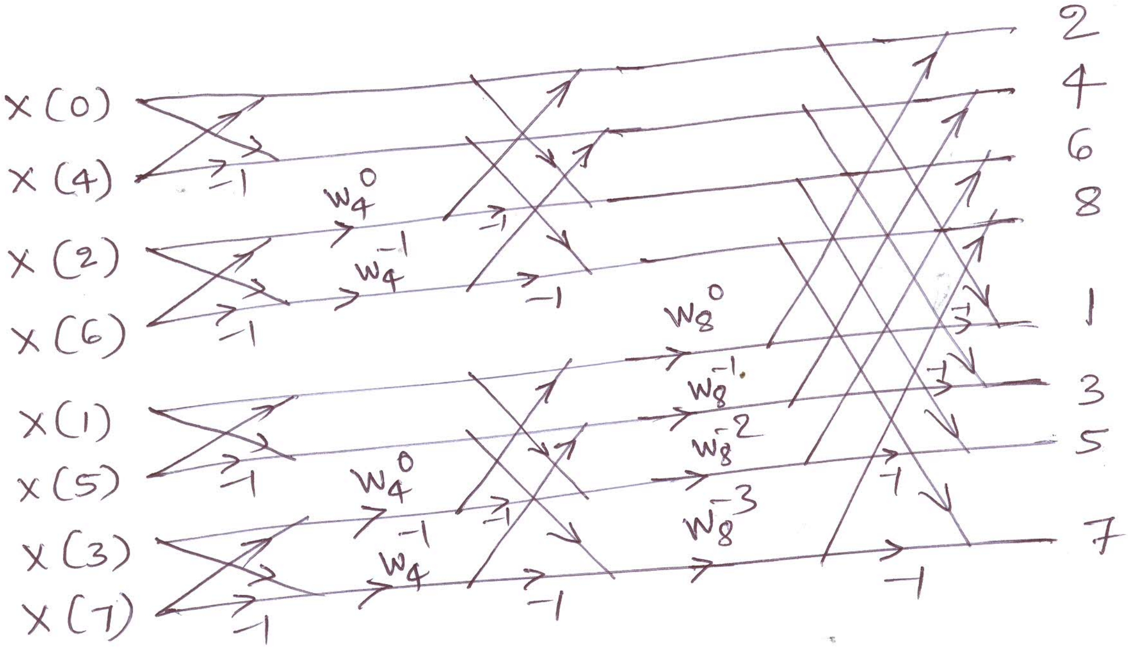
Signal Flow Graph



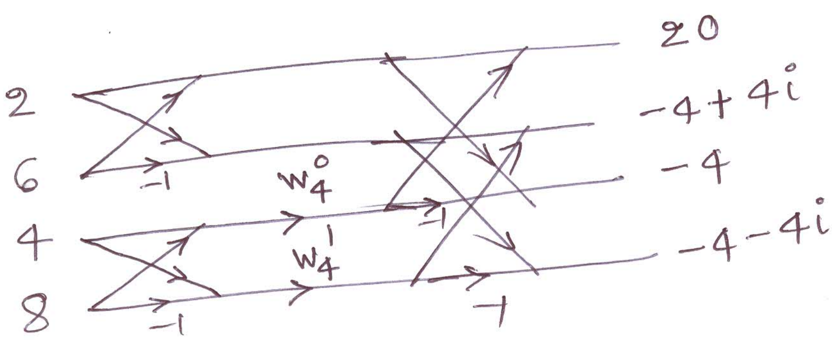
2

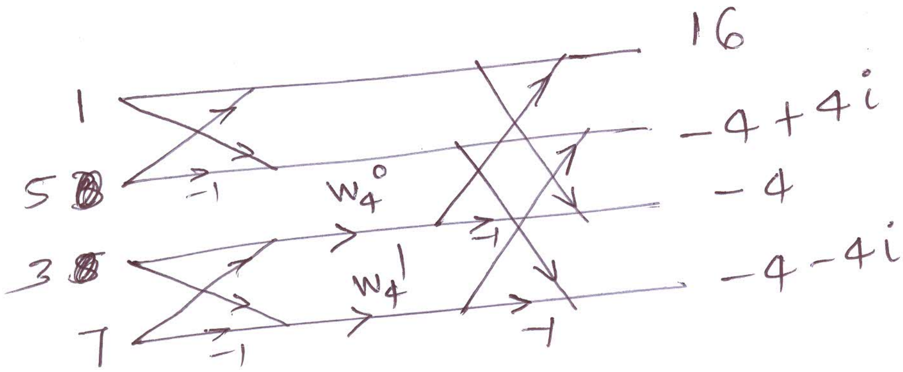

0
1
2
3
4
5
6
7
W_8^0
W_8^1
W_8^2
W_8^3
W_4^0
W_4^1
W_4^0
W_4^1
-1
28
-4
-4+4i
-4-4i
-4+9.65i
-4-1.65i
-4+1.65i
-4-9.65i

3

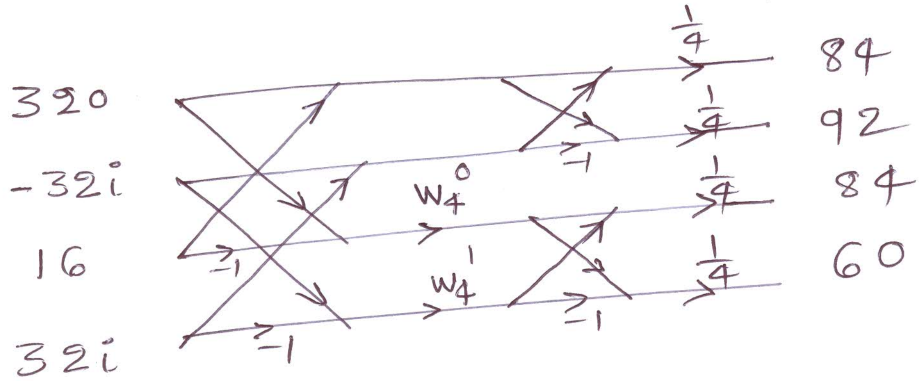

x(0)
x(4)
x(2)
x(6)
x(1)
x(5)
x(3)
x(7)
W_4^0
W_4^{-1}
W_4^0
W_4^{-1}
W_8^0
W_8^{-1}
W_8^{-2}
W_8^{-3}
-1
2
4
6
8
1
3
5
7

4


2
6
4
8
W_4^0
W_4^1
-1
20
-4+4i
-4
-4-4i


1
5
3
7
W_4^0
W_4^1
-1
16
-4+4i
-4
-4-4i

$X(k)H(k) = [320, -32i, 16, 32i]$



5 $h(n) = W_N^{-kn} u(n)$

$$y(n) = x(n) * h(n)$$

$$= \sum_{m=0}^{N-1} x(m)\, h(n-m)$$

$$= \sum_{m=0}^{N-1} x(m)\, W_N^{-k(n-m)}$$

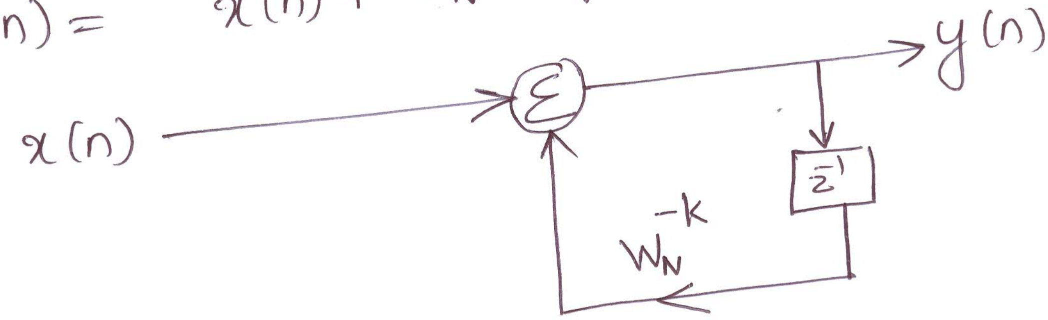
$$y(n)\Big|_{n=N} = \sum_{m=0}^{N-1} x(m)\, W_N^{km}$$

$$= X(k)$$

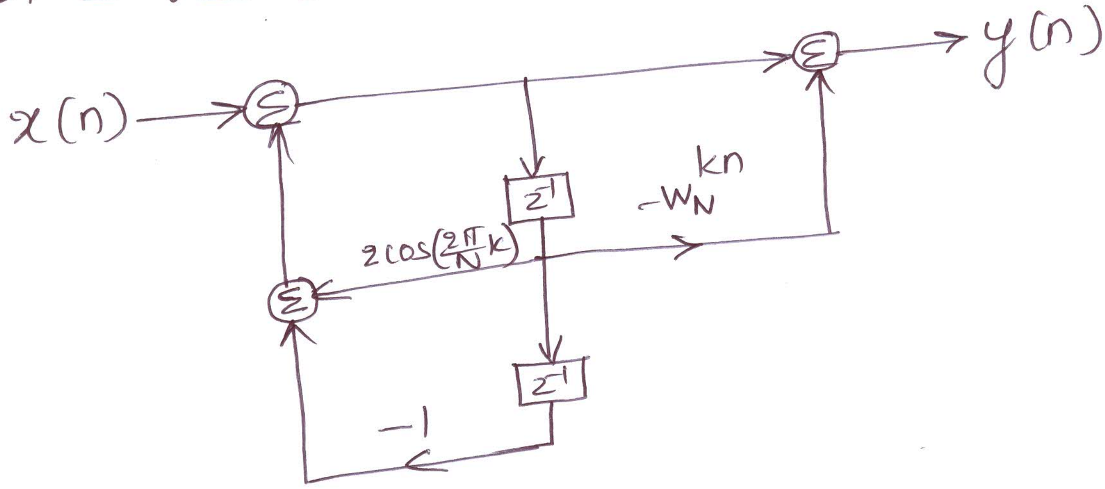
$$H(z) = \sum_{n=0}^{\infty} W_N^{-kn} z^{-n}$$

$$= \frac{1}{1 - W_N^{-k} z^{-1}}, \quad |W_N^{-k} z^{-1}| < 1$$

$$\therefore y(n) = x(n) + W_N^{-k}\, y(n-1)$$



DF-II realization.



6 h(n) - symmetric

N - odd

$$H(z) = z^{-\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + \sum_{n=0}^{\frac{N-3}{2}} h(n)\left\{z^{\frac{N-1}{2}-n} + z^{-\left(\frac{N-1}{2}-n\right)}\right\}\right]$$

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)}\left[h\left(\frac{N-1}{2}\right) + 2\sum_{n=0}^{\frac{N-3}{2}} h(n)\cos\left(\omega\left(\frac{N-1-2n}{2}\right)\right)\right]$$

h(n) - antisymmetric

N - even

$$H(z) = z^{-\left(\frac{N-1}{2}\right)}\sum_{n=0}^{\frac{N}{2}-1} h(n)\left\{z^{\frac{N-1}{2}-n} - z^{-\left(\frac{N-1}{2}-n\right)}\right\}$$

$$H(\omega) = e^{-j\omega\left(\frac{N-1}{2}\right)}\sum_{n=0}^{\frac{N}{2}-1} h(n)\, 2j\sin\left(\omega\left(\frac{N-1-2n}{2}\right)\right)$$

7

$$h(n) = \begin{cases} \dfrac{\sin\left(\frac{3\pi}{4}(n-3)\right)}{\pi(n-3)}, & n \neq 3 \\ \dfrac{3}{4}, & n = 3 \end{cases}$$

$$\omega(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{N-1}\right), \quad 0 \le n \le N-1$$

$$h'(n) = h(n)\,\omega(n), \quad 0 \le n \le N-1$$

8 $h(n) = \frac{1}{N} \sum_{k=0}^{N-1} H(k) e^{j\frac{2\pi}{N}kn}$ (5)

$0 \le n \le N-1$

$$= \frac{1}{N}\left[H(0) + H(1)e^{j\frac{2\pi}{N}n} + H(2)e^{j\frac{2\pi}{N}2n} + \dots H(N-2)e^{j\frac{2\pi}{N}(N-2)} + H(N-1)e^{j\frac{2\pi}{N}(N-1)}\right]$$

$$= \frac{1}{N}\left[H(0) + \sum_{k=1}^{\frac{N-1}{2}} 2Re\left(H(k)e^{j\frac{2\pi}{N}kn}\right)\right]$$

$$H(z) = \sum_{n=0}^{N-1} \frac{1}{N} \sum_{k=0}^{N-1} H(k) e^{j\frac{2\pi}{N}kn} z^{-n}$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} H(k) \sum_{n=0}^{N-1} \left(e^{j\frac{2\pi}{N}k} z^{-1}\right)^n$$

$$= \frac{1}{N} \sum_{k=0}^{N-1} H(k) \frac{1 - z^{-N}}{1 - e^{j\frac{2\pi}{N}k} z^{-1}}$$