CMR
INSTITUTE OF
TECHNOLOGY

Internal Assessment Test 1 – Sep. 2017

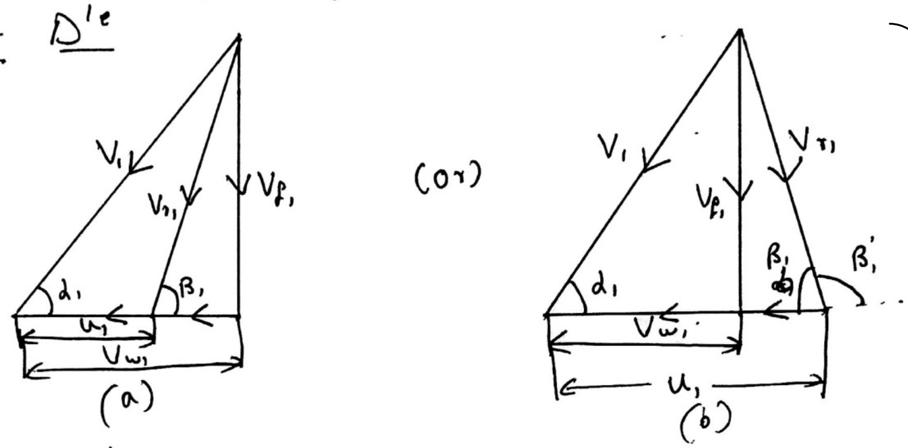
Sub:	Turbo-Machines							Code:	15ME53
Date:	21/09/2017	Duration:	90 mins	Max Marks:	50	Sem:	5(A&B)	Branch:	ME

PART-A (Answer any THREE FULL Questions)

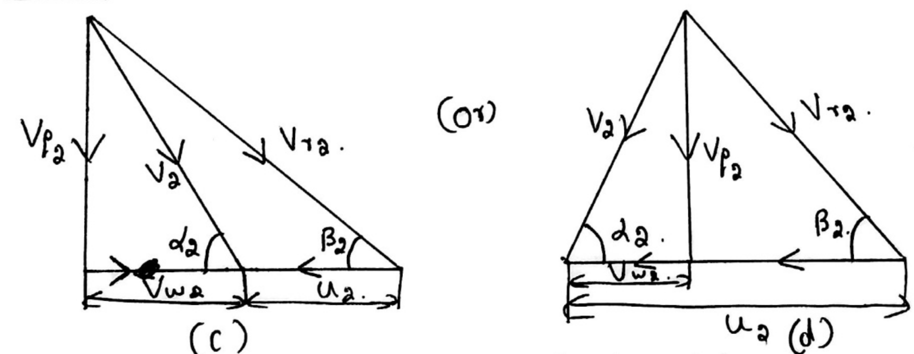
1. Draw the velocity triangle at inlet and exit of a turbomachine in general and derive modified Euler's equation. Also explain the significance of each term in the equation.(10 Marks)

Possible Velocity triangles

Inlet Δ^{le}



Outlet Δ^{le}



2 MARKS

$V_1 \Rightarrow$ Absolute Velocity of fluid at inlet.

$\alpha_1 \Rightarrow$ Angle made by V_1

$U_1 \Rightarrow$ Tangential Velocity of the rotor. at inlet.

$V_{r_1} \Rightarrow$ Relative Velocity of fluid at inlet.

$\beta_1 \Rightarrow$ Blade angle or Rotor angle.

$V_{w_1} \Rightarrow$ Tangential Component of V_1 at inlet or whirl velocity.

$V_{f_1} \Rightarrow$ Flow Velocity at inlet $= V_R$ or V_A.

The Subscript '2' are corresponding components at outlet.

Consider the outlet velocity Δ'le [ⓒ and ⓓ].

In both the cases, there are two possible right angle triangles. [consider either ⓒ or ⓓ].

Δ'le 1 ⇒ Comprising of V_{f_2}, V_2 and V_{w_2}.

Δ'le 2 ⇒ Comprising of V_{f_2}, V_{r_2} and $(U_2 - V_{w_2})$.

From Δ'le 1,

$$V_2^2 = V_{f_2}^2 + V_{w_2}^2.$$

$$\Rightarrow V_{f_2}^2 = V_2^2 - V_{w_2}^2 \quad \text{—— (1)}$$

From Δ'le 2,

$$V_{r_2}^2 = V_{f_2}^2 + (U_2 - V_{w_2})^2. \quad \text{—— (2)}$$

But in fig(c), U_2 and V_{w_2} are opp in direction. Hence $(U_2 - V_{w_2})$. In fig(d), U_2 and V_{w_2} are in same direction. Hence for Δ'le 2 in fig(d), the base is $(U_2 - V_{w_2})$.

$$(2) \Rightarrow V_{f_2}^2 = V_{r_2}^2 - [U_2 - V_{w_2}]^2 \quad \text{—— (3)}$$

(3) = (1)

$$\Rightarrow V_2^2 - \cancel{V_{w_2}^2} = V_{r_2}^2 - U_2^2 - \cancel{V_{w_2}^2} + 2U_2V_{w_2}.$$

$$2U_2V_{w_2} = V_2^2 + U_2^2 - V_{r_2}^2.$$

$$U_2V_{w_2} = \frac{1}{2}\left[V_2^2 + U_2^2 - V_{r_2}^2\right]. \quad \text{—— (4)}$$

III'ly from inlet velocity Δ'les, we get.

$$U_1V_{w_1} = \frac{1}{2}\left[V_1^2 + U_1^2 - V_{r_1}^2\right]. \quad \text{—— (5)}$$

3 MARKS

But Euler Turbine equation is.

$$E = U_1 V_{u_1} - U_2 V_{u_2}.$$

Substituting (4) and (5) in above eqn,

$$E = \frac{1}{2}\left[V_1^2 + U_1^2 - V_{r_1}^2 - V_2^2 - U_2^2 + V_{r_2}^2 \right].$$

$$E = \frac{1}{2}\left[(V_1^2 - V_2^2] + [U_1^2 - U_2^2] + [V_{r_2}^2 - V_{r_1}^2] \right] \quad \text{—— (6)}$$

Eqn (6) is applicable for Power producing type.

For Power Absorbing Machines,

$$E = \frac{1}{2}\left[(V_2^2 - V_1^2) + (U_2^2 - U_1^2) + (V_{r_1}^2 - V_{r_2}^2) \right] \quad \text{—— (7).}$$

Eqn (6) and (7) are different forms of Euler Turbine Equation.

The three terms inside the bracket of (6) and (7) indicates nature of energy transfer.

2 MARKS

Significance of Each term

(1) $\frac{1}{2}[V_1^2 - V_2^2]$ represent the change in absolute kinetic energy of the fluid, during its passage. Hence, this term represents the change in dynamic head.

1 MARK

(2) $\frac{1}{2}[U_1^2 - U_2^2]$ represent the change in fluid energy due to movement of rotation of fluid from one radius to another. i.e., Centrifugal energy. Hence this term represents the change in static head.

1 MARK

(3) $\frac{1}{2}[V_{r_2}^2 - V_{r_1}^2]$ represents kinetic energy change due to relative velocity change. This will result in change in static head within the rotor.

1 MARK

2.Define utilization factor for a turbine and derive an expression for the same involving degree of reaction (10 Marks)

"The ratio of ideal work to the energy supplied is called diagram efficiency or utilization factor ($\mathcal{E}$)".

The energy available to the rotor are:

i) Kinetic energy of the fluid at inlet $\left(\frac{1}{2} V_1^2\right)$.

ii) The static head available $\left[\frac{1}{2}\left[(U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)\right]\right]$.

Hence Total energy available is,

$$E_{avail} = \frac{1}{2}\left[V_1^2 + (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)\right] \quad \text{—— (1)}$$

The energy utilized by the turbine is given by Euler turbine eqn (or) the components of Euler turbine eqn.

$$\Rightarrow E_{utilized} = \frac{1}{2}\left[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)\right] \quad \text{—— (2)}$$

Hence, by definition of $\mathcal{E}$,

$$\mathcal{E} = \frac{E_{utilized}}{E_{avail}} = \frac{\frac{1}{2}\left[(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)\right]}{\frac{1}{2}\left[V_1^2 + (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)\right]} \quad \text{—— (3)}$$

4 MARKS

0

Relationship between Utilization factor and Degree of Reaction

$$\text{Degree of Reaction} = R = \frac{(U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)}{(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2)}.$$

1 MARK

Let, $(U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2) = S.$

$(V_1^2 - V_2^2) = D.$

$\Rightarrow R = \frac{S}{D+S} \Rightarrow (D+S)R = S.$

$\Rightarrow DR + SR = S. \Rightarrow DR = S - SR \Rightarrow DR = S(1-R).$

$\Rightarrow S = \frac{R}{1-R} D$

$$\Rightarrow (U_1^2 - U_2^2) + (V_{r_2}^2 - V_{r_1}^2) = \frac{R}{1-R}(V_1^2 - V_2^2). \quad \text{—— (1)}$$

$$\text{Utilization Factor} = \varepsilon = \frac{(V_1^2 - V_2^2) + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)}{V_1^2 + (U_1^2 - U_2^2) + (V_{r2}^2 - V_{r1}^2)} \quad \text{—(2)}$$

Substitute (1) in (2)

$$\Rightarrow \varepsilon = \frac{(V_1^2 - V_2^2) + \frac{R}{1-R}(V_1^2 - V_2^2)}{V_1^2 + \frac{R}{1-R}(V_1^2 - V_2^2)}$$

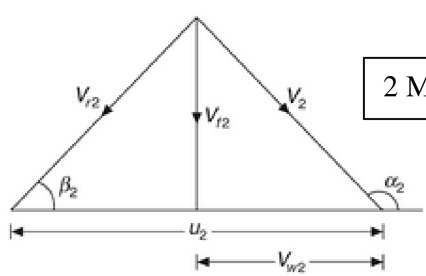
$$\Rightarrow \varepsilon = \frac{(1-R)(V_1^2 - V_2^2) + R(V_1^2 - V_2^2)}{(1-R)\,V_1^2 + R(V_1^2 - V_2^2)}$$

$$= \frac{V_1^2 - V_2^2 - \cancel{RV_1^2} + \cancel{RV_2^2} + \cancel{RV_1^2} - \cancel{RV_2^2}}{V_1^2 - \cancel{RV_1^2} + \cancel{RV_1^2} - RV_2^2}$$

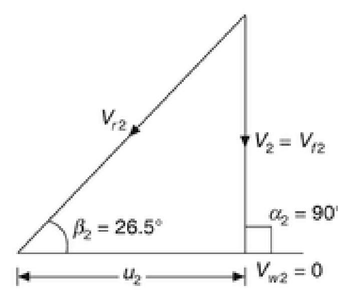
$$\boxed{\varepsilon = \frac{V_1^2 - V_2^2}{V_1^2 - RV_2^2}}$$

5 MARKS

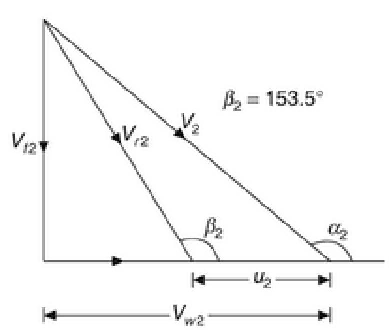
3. Draw inlet and exit velocity triangles for a radial flow machine with i) Backward blade ii) Radial blade iii) Forward blade (5 marks)



2 MARKS



1 MARK



2 MARKS

5. Performance of a turbomachine depends on the following variables, Discharge (Q), Speed (N), Rotor diameter (D), Energy per unit mass flow (gH), Power (P), Density (ρ), Dynamic viscosity (μ). Using dimensional analysis, obtain the π-terms. (Do not expalin the significance) (12 Marks)

SCHEME: EACH π-term carries 4 Marks each

Solution

General Relationship is.

$f(Q, N, D, gH, P, \rho, \mu) = \text{Constant}$

No of Variables, $n = 7$.

No of fundamental variables, $m = 3$.

No of π-terms $= (n-m) = 7-3 = 4$.

Dimensions

$Q = m^3/s = L^3T^{-1}$

$N = rpm = 1/s = T^{-1}$

$D = m = L$

$gH = m^2/s^2 = L^2T^{-2}$

$P = J/s = ML^2T^{-3}$

$\rho = kg/m^3 = ML^{-3}$

$\mu = N\text{-}s/m^2 = ML^{-1}T^{-1}$

Repeating Variables

Geometric Property → D

Flow property → N

Fluid Property → ρ.

π-terms

$\pi_1 = D^{a_1} N^{b_1} \rho^{c_1} Q.$

$\pi_2 = D^{a_2} N^{b_2} \rho^{c_2} gH$

$\pi_3 = D^{a_3} N^{b_3} \rho^{c_3} P$

$\pi_4 = D^{a_4} N^{b_4} \rho^{c_4} \mu$

$\underline{\pi_1 \text{ - term}}$

$$\pi_1 = D^{a_1} N^{b_1} \rho^{c_1} Q.$$

$$M^0 L^0 T^0 = L^{a_1} (T^{-1})^{b_1} (ML^{-3})^{c_1} (L^3 T^{-1})$$

Equating Powers of M,

$\boxed{0 = c_1}$

Equating Powers of L,

$0 = a_1 - 3c_1 + 3$

$0 = a_1 + 3$

$\boxed{a_1 = -3}$

Equating Powers of T,

$0 = -b_1 - 1$

$\boxed{b_1 = -1}$

$\therefore \pi_1 = D^{-3} N^{-1} \rho^0 Q.$

$$\boxed{\pi_1 = \frac{Q}{ND^3}}$$

$\underline{\pi_2 \text{ - term}}$

$$\pi_2 = D^{a_2} N^{b_2} \rho^{c_2} gH.$$

$$M^0 L^0 T^0 = L^{a_2} (T^{-1})^{b_2} (ML^{-3})^{c_2} (L^2 T^{-2}).$$

Equating Powers of M,

$\boxed{0 = c_2}$

Equating Powers of L,

$0 = a_2 - 3c_2 + 2.$

$\Rightarrow \boxed{a_2 = -2}$

Equating Powers of T,

$0 = -b_2 - 2.$

$\Rightarrow \boxed{b_2 = -2}$

$$\pi_2 = D^{-2} N^{-2} \rho^0 gH$$

$$\boxed{\pi_2 = \frac{gH}{D^2 N^2}}$$

$\underline{\pi_3 \text{ - term}}$

$$\pi_3 = D^{a_3} N^{b_3} \rho^{c_3} P.$$

$$M^0 L^0 T^0 = L^{a_3} (T^{-1})^{b_3} (ML^{-3})^{c_3} ML^2 T^{-3}$$

Equating Powers of M,

$0 = c_3 + 1$

$\Rightarrow \boxed{c_3 = -1}$

Equating Powers of L,

$0 = a_3 - 3c_3 + 2$

$\Rightarrow \boxed{a_3 = -5}$

Equating Powers of T,

$0 = -b_3 - 3$

$\Rightarrow \boxed{b_3 = -3}$

$$\pi_3 = D^{-5} N^{-3} \rho^{-1} P$$

$$\boxed{\pi_3 = \frac{P}{\rho N^3 D^5}}$$

$\underline{\pi_4 \text{ - term}}$

$$\pi_4 = D^{a_4} N^{b_4} \rho^{c_4} \mu$$

$$M^0 L^0 T^0 = L^{a_4} (T^{-1})^{b_4} (ML^{-3})^{c_4} (ML^{-1} T^{-1}).$$

Equating Powers of M,

$0 = c_4 + 1$

$\Rightarrow \boxed{c_4 = -1}$

Equating Powers of T,

$0 = -b_4 - 1$

$\Rightarrow \boxed{b_4 = -1}$

Equating Powers of L,

$0 = a_4 - 3c_4 - 1$

$\Rightarrow \boxed{a_4 = -2}$

$$\pi_4 = D^{-2} N^{-1} \rho^{-1} \mu$$

$$\boxed{\pi_4 = \frac{\mu}{\rho N D^2}}$$

PART- C (Answer any one)

6. In a certain turbo machine, the inlet whirl velocity is 15m/s, inlet flow velocity is l0m/s, blade speeds are 30m/s and 8m/s respectively. Discharge is radial with an absolute velocity of 15 m/s. If water is the working fluid, flowing at the rate of 1500 litre/s, calculate: i) Power in kW ii) the change in total pressure in bar iii) the degree of reaction and iv) Utilization factor. (10 Marks)

Data

$R = 0.5$

$U = 98.5\ m/s.$

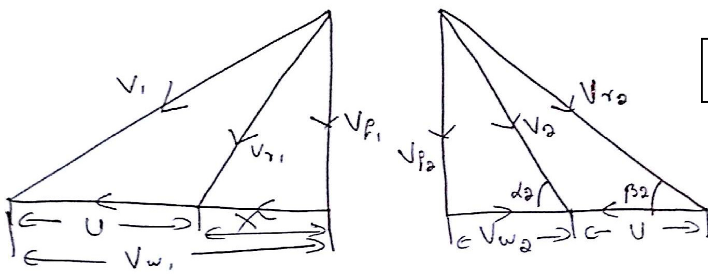
$V_1 = V_{r2} = 155\ m/s$

$\beta_2 = \alpha_1 = 18^\circ$

$\dot{m} = 10\ kg/s.$

To find,

$\beta_1, P\ \&\ \varepsilon.$



1 MARKS

$V_{w1} = V_1 \cos\alpha_1 = 155 \cos 18 = 147.4\ m/s.$

$V_{f1} = V_1 \sin\alpha_1 = 155 \sin 18 = 47.9\ m/s$

$X = V_{w1} - U = 147.4 - 98.5 = 48.9\ m/s = V_{w2}$

$V_{r1} = \sqrt{X^2 + V_{f1}^2} = 68.45\ m/s = V_2.$

$$\boxed{\beta_1 = \tan^{-1}\left[\frac{V_{f1}}{X}\right] = 44.4^\circ = \alpha_2.}$$ → 4 MARKS

Power Output, $P = \dot{m}\, U (V_{w1} + V_{w2})$

$= 10 \times 98.5 (147.4 + 48.9)$

$$\boxed{P = 19.34\ kW}$$ → 3 MARKS

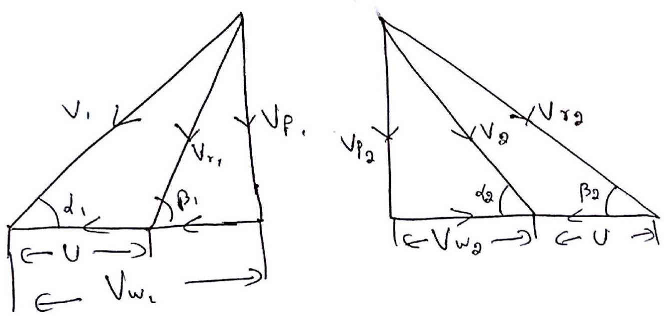
$$\varepsilon = \frac{E}{E + V_2^2/2} = \frac{19.34}{19.34 + \left(\frac{68.45}{2}\right)} = 0.892.$$ → 2 MARKS

7. The velocity of fluid from the nozzle in an axial flow impulse turbine is 1200 m/s. The nozzle angle is 22°. If the rotor blades are equiangular and the rotor tangential blade speed is 400m/s, find i) The rotor blade angles ii) The tangential force on the blade rings iii) Power Output iv) Utilization Factor. Assume $V_{r1}=V_{r2}$ (10 Marks)

Data : $V_1 = 1200$ m/s $\quad \alpha_1 = 22°$ $\quad \beta_1 = \beta_2$

$u = 400$ m/s $\quad V_{r_1} = V_{r_2}$

To find : $\beta_1 = \beta_2 = ?$, $F_T = ?$, $P = ?$, $\varepsilon = ?$



$$\cos\alpha_1 = \frac{V_{w_1}}{V_1} \Rightarrow \cos 22 = \frac{V_{w_1}}{1200}$$

$V_{w_1} = 1112.6$ m/s.

$$V_1^2 = V_{f_1}^2 + V_{w_1}^2 \Rightarrow 1200^2 = V_{f_1}^2 + 1112.6^2$$

$\Rightarrow V_{f_1} = 449.53$ m/s

$$\tan\beta_1 = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{449.53}{1112.6 - 400}$$

$$\Rightarrow \boxed{\beta_1 = 32.24° = \beta_2}$$

2 MARKS

$$\cos\beta_2 = \frac{u_2 + V_{w_2}}{V_{r_2}} \Rightarrow \cos 32.24 = \frac{400 + V_{w_2}}{842.6}$$

$\Rightarrow V_{w_2} = 312.66$ m/s.

$$F_T = \dot{m}(V_{w_1} + V_{w_2})$$

$$= 1 \times (1112.6 + 312.66)$$

$$\boxed{F_T = 1425.27N}$$

2 MARKS

Power Developed, $P = F_T \times u$

$= 1425.27 \times 400$

$\boxed{P = 570.09 \text{ kW}}$

2 MARKS

<u>Utilization Factor</u>

$$\varepsilon = \frac{V_1^2 - V_2^2}{V_1^2}$$

$$V_2^2 = V_{r_2}^2 + u^2 - 2 V_{r_2} u \cos\beta_2$$

$$= 842.6^2 + 400^2 - 2 \times 842.6 \times 400 \times \cos 32.24$$

$$V_2 = 547.49 \text{ m/s.}$$

$$\boxed{\varepsilon = 0.79}$$

2 MARKS