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CMR INSTITUTE OF TECHNOLOGY, BENGALURU.
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Internal Assessment Test 1 – Sept. 2017

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|---|---|---|---|---|---|---|---|
| Sub: | Optimization Techniques | | | Sub Code: | 15ME561 | Branch: | ISE/CSE |
| Date: | 21.09.2017 | Duration: | 90 min's | Max Marks: 50 | Sem / Sec: | 5th Open elective | |

Answer any FIVE FULL Questions

| | | MARKS | CO (OBE) | RBT (OBE) |
|---|---|---|---|---|
| 1 (a) | A Firm makes two products X & Y And has a total production capacity of 9 ton's per day. X&Y Requiring the same production capacity the firm has a permanent contract to supply at least 2 ton's of X and at least 3 ton's of Y per day to another company each ton of X requires 20 Machine hours production time and each ton of Y requires 50 machine hours Production time the daily maximum possible no. of hours is 360 all the firms output can be Sold and the profit obtained is Rs 80 per ton of X and Rs120 per ton of Y respectively. Formulate The LPP and solve it graphically | [08] | CO3 | L3 |
| (b) | List the assumptions made in LPP | [02] | CO1 | L1 |
| 2 (a) | Solve the LPP Using Simplex method<br>**Maximize** $Z = 6x_1+11x_2$ **ST** $2x_1+x_2 \leq 104$, $x_1+2x_2 \leq 76$, $x_1, x_2 \geq 0$ | [06] | CO3 | L3 |
| (b) | Explain briefly in LPP Infeasible solution, unbounded solution, Alternate optimal solution, Degenerate solutions with example | [04] | CO1 | L2 |
| 3 (a) | Solve the LPP Using Simplex method<br>**Minimize** $Z = x_1-3x_2+2x_3$ **ST** $3x_1-x_2+3x_3 \leq 7$, $-2x_1+4x_2 \leq 12$, $-4x_1+3x_2+8x_3 \leq 10$<br>$x_1, x_2, x_3 \geq 0$ | [7.5] | CO3 | L3 |
| (b) | Explain slack variable, surplus variable, Artificial variable, Binding & Non- binding constraint | [2.5] | CO1 | L2 |
| 4 (a) | Solve the LPP Using Penalty method<br>**Minimize** $Z = 4x_1+x_2$ **ST** $3x_1+x_2 = 3$, $4x_1+3x_2 \geq 6$, $x_1+2x_2 \leq 4$ $x_1, x_2 \geq 0$ | [10] | CO3 | L3 |
| 5 (a) | Solve the LPP Using Simplex method<br>**Maximize** $Z = 6x_1+4x_2$ **ST** $2x_1+3x_2 \leq 30$, $3x_1+2x_2 \leq 24$, $x_1+x_2 \geq 3$ $x_1, x_2 \geq 0$<br>Dces the problem have alternative optima; If so find the other solution. | [5+2+3] | CO3 | L4 |
| 6 (a) | Solve the LPP Using Big-M- method<br>**Minimize** $Z = 3x_1+8x_2$ **ST** $x_1+x_2 = 200$, $x_1 \leq 80$, $x_2 \geq 60$ $x_1, x_2 \geq 0$ | [10] | C03 | L3 |

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# Operation Research

## 1 - IAT
### Solution

1A] a] let $x_1, x_2$ be the no. of tonne of product X & Y the company should manufacture respectively.

$\max Z = 80x_1 + 120x_2$

Sub. to.

$x_1 + x_2 \leq 9$

$x_1 \geq 2$

$x_2 \geq 3$

$20x_1 + 50x_2 \leq 360$

$x_1, x_2 \geq 0$

The LPP is to find $x_1, x_2$ where:

$\max Z = 80x_1 + 120x_2$

Sub. to

$x_1 + x_2 \leq 9$ — (1)

$x_1 \geq 2$ — (2)

$x_2 \geq 3$ — (3)

$20x_1 + 50x_2 \leq 360$ — (4)

$x_1, x_2 \geq 0$

Assuming constraints to be equations

(1) $x_1 + x_2 = 9$

$x_1 = 0, x_2 = 9$

$x_2 = 0, x_1 = 9$

$(x_1, x_2) = (9, 9)$

(2)

$x_1 = 2$

$(x_1, x_2) = (2, 0)$

(3) $x_2 = 3$

$(x_1, x_2) = (0, 3)$

(4)

$20x_1 + 50x_2 = 360$

$x_1 = 0, x_2 = 7.2$

$x_2 = 0, x_1 = 18$

$\therefore (x_1, x_2) = (18, 7.2)$

From the graph:

the company should produce 3 tonnes of X, 6 tonnes of Y to get a profit of Rs 960.

2A] a) $\uparrow \max Z = 6x_1 + 11x_2$

sub. to

$2x_1 + x_2 \leq 104$

$x_1 + 2x_2 \leq 76$

$x_1, x_2 \geq 0$

converting inequalities to equation adding slack variable

$2x_1 + x_2 + S_1 = 104$

$x_1 + 2x_2 + S_2 = 76$ $\qquad S_1, S_2$ — slack variable

$x_1, x_2, S_1, S_2 \geq 0$

$\therefore$ new obj. funcn:

$\uparrow \max Z = 6x_1 + 11x_2 + 0S_1 + 0S_2$

| | $C_j$ | 6 | 11 | 0 | 0 | | | |
|---|---|---|---|---|---|---|---|---|
| $C_B$ | Basis | $x_1$ | $x_2$ | $S_1$ | $S_2$ | RHS | min. ratio | |
| 0 | $S_1$ | 2 | 1 | 1 | 0 | 104 | 104 | |
| 0 | $S_2$ | 1 | 2 PE | 0 | 1 | 76 | 38 | — LV |
| | $Z = z_j - c_j$ | -6 | -11 EV | 0 | 0 | | | |
| 0 | $S_1$ | 3/2 PE | 0 | 1 | -1/2 | 66 | 44 | — LV |
| 11 | $x_2$ | 1/2 | 1 | 0 | 1/2 | 38 | 76 | |
| | $Z = z_j - c_j$ | -1/2 EV | 0 | 0 | 11/2 | | | |
| 6 | $x_1$ | 1 | 0 | 2/3 | -1/3 | 44 | | |
| 11 | $x_2$ | 0 | 1 | -1/3 | 2/3 | 16 | | |
| | $Z = z_j - c_j$ | 0 | 0 | 1/3 | 16/3 | | | |

$Z \geq 0$

Solution is optimal

$x_1 = 44, \; x_2 = 16$

$\therefore \max Z = \underline{440}$

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3A] a) $\min Z = x_1 - 3x_2 + 2x_3$

sub to

$3x_1 - x_2 + 3x_3 \leq 7$

$-2x_1 + 4x_2 + 0x_3 \leq 12$

$-4x_1 + 3x_2 + 8x_3 \leq 10$

$x_1, x_2, x_3 \geq 0$

converting min to max

$\uparrow \max Z = -x_1 + 3x_2 - 2x_3$

converting inequalities to equation. adding slack variables

$3x_1 - x_2 + 3x_3 + S_1 = 7$

$-2x_1 + 4x_2 + 0x_3 + S_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + S_3 = 10$

$x_1, x_2, x_3, S_1, S_2, S_3 \geq 0$

∴ new obj func$^n$

↑ $\max Z = x_1 - 3x_2 + 2x_3 + 0S_1 + 0S_2 + 0S_3$

| $C_B$ | $C_j$ Basis | $-1$ $x_1$ | $3$ $x_2$ | $-2$ $x_3$ | $0$ $S_1$ | $0$ $S_2$ | $0$ $S_3$ | RHS | min ratio |
|---|---|---|---|---|---|---|---|---|---|
| 0 | $S_1$ | 3 | $-1$ | 3 | 1 | 0 | 0 | 7 | $-7$ |
| 0 | $S_2$ | ~~$-2$~~ | 4 | 0 | 0 | 1 | 0 | 12 | 3 — LV |
| 0 | $S_3$ | $-4$ | 3 | 8 | 0 | 0 | 1 | 10 | 3.2 |
| | $Z = z_j - c_j$ | 1 | $-3$ ↑EV | 2 | 0 | 0 | 0 | | |
| 0 | $S_1$ | $5/2$ | 0 | 3 | 1 | $1/4$ | 0 | 10 | $20/5$ — LV |
| 3 | $x_2$ | $-1/2$ | 1 | 0 | 0 | $1/4$ | 0 | 3 | $3/-1/2$ |
| 0 | $S_3$ | $-5/2$ | 0 | 8 | 0 | $-3/4$ | 1 | 1 | $1/-5/2$ |
| | $Z = z_j - c_j$ | $-1/2$ ↑EV | 0 | $-2$ | 0 | $3/4$ | 0 | | |
| $-1$ | $x_1$ | 1 | 0 | $6/5$ | $2/5$ | $1/10$ | 0 | 4 | |
| 3 | $x_2$ | 0 | 1 | $3/5$ | $1/5$ | $3/10$ | 0 | 5 | |
| 0 | $S_3$ | 0 | 0 | 11 | 1 | $-1/2$ | 1 | 11 | |
| | $Z = z_j - c_j$ | 0 | 0 | $13/5$ | $1/5$ | $8/10$ | 0 | | |

$Z \geq 0$ solution is optimal

$x_1 = 4,\ x_2 = 5,\ x_3 = 0$

∴ $\max Z = 11$

↓ $\min Z = -11$

4A) ↓ $\min Z = 4x_1 + x_2$

sub to

$3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$

$x_1, x_2 \geq 0$

converting to max:

↑ $\max Z = -4x_1 - x_2$

converting inequalities adding slack & artificial variables & subtracting surplus variable

$3x_1 + x_2 + A_1 = 3$

$4x_1 + 3x_2 - S_1 + A_2 = 6$

$x_1 + 2x_2 + S_2 = 4$

$x_1, x_2, A_1, A_2, S_1, S_2 \geq 0$

$A_1, A_2$ — artifical variable

$S_1$ — surplus "

$S_2$ = slack "

∴ new obj func$^n$:

↑ $\max Z = -4x_1 - x_2 - MA_1 - 0S_1 - MA_2 + 0S_2$

| $C_B$ | $c_j$ Basis | $-4$ $x_1$ | $-1$ $x_2$ | $-M$ $A_1$ | $0$ $S_1$ | $-M$ $A_2$ | $0$ $S_2$ | RHS | min ratio |
|---|---|---|---|---|---|---|---|---|---|
| $-M$ | $A_1$ | 3 | 1 | 1 | 0 | 0 | 0 | 3 | 1 → LV |
| $-M$ | $A_2$ | 4 | 3 | 0 | $-1$ | 1 | 0 | 6 | 1.5 |
| 0 | $S_2$ | 1 | 2 | 0 | 0 | 0 | 1 | 4 | 4 |
| | $Z = z_j - c_j$ | $-7M + 4$ ↑EV | $-4M + 1$ | 0 | $M$ | 0 | 0 | | |
| $-4$ | $x_1$ | 1 | $1/3$ | $1/3$ | 0 | 0 | 0 | 1 | 3 |
| $-M$ | $A_2$ | 0 | $5/3$ | $-4/3$ | $-1$ | 1 | 0 | 2 | $6/5$ — LV |
| 0 | $S_2$ | 0 | $5/3$ | $-1/3$ | 0 | 0 | 1 | 3 | $9/5$ |
| | $Z = z_j - c_j$ | 0 | $-5M - 1/3$ ↑EV | $7/3 M - 4/3$ | $M$ | 0 | 0 | | |
| $-4$ | $x_1$ | 1 | 0 | $3/5$ | $1/5$ | $-1/5$ | 0 | $3/5$ | 3 |
| $-1$ | $x_2$ | 0 | 1 | $-4/5$ | $-3/5$ | $3/5$ | 0 | $6/5$ | 2 |
| 0 | $S_2$ | 0 | 0 | 1 | 1 | $-1$ | 1 | 1 | 1 |
| | $Z = z_j - c_j$ | 0 | 0 | $-8/5 + M$ | $-1/5$ | $1/5$ | 0 | | |
| $-4$ | $x_1$ | 1 | 0 | $2/5$ | 0 | 0 | $-1/5$ | $2/5$ | |
| $-1$ | $x_2$ | 0 | 1 | $-1/5$ | 0 | 0 | $3/5$ | $9/5$ | |
| 0 | $S_1$ | 0 | 0 | 1 | 1 | $-1$ | 1 | 1 | |
| | $Z = z_j - c_j$ | 0 | 0 | $M$ | 0 | $M$ | $1/5$ | | |

$Z \geq 0$ ∴ soln. is optimal

$x_1 = 2/5$, $x_2 = 9/5$

Max $Z = -17/5$

∴ Min $Z = \underline{\underline{17/5}}$

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5A) converting inequalities to equations by adding slack variables & artificial variables & subtracting surplus variable

$$2x_1 + 3x_2 + S_1 = 30$$

$$3x_1 + 2x_2 + S_2 = 24$$

$$x_1 + x_2 - S_3 + A_1 = 3$$

$$x_1, x_2, x_3, S_1, S_2, S_3\ A_1 \geq 0$$

∴ new obj function

∴ Max $Z = 6x_1 + 4x_2 + 0S_1 + 0S_2 - 0S_3 - MA_1$

| $C_B$ | Basis | $x_1$ | $x_2$ | $S_1$ | $S_2$ | $S_3$ | $A_1$ | RHS | min ratio | |
|---|---|---|---|---|---|---|---|---|---|---|
| $C_j$ | | 6 | 4 | 0 | 0 | 0 | $-M$ | | | |
| 0 | $S_1$ | 2 | 3 | 1 | 0 | 0 | 0 | 30 | 15 | |
| 0 | $S_2$ | 3 | 2 | 0 | 1 | 0 | 0 | 24 | 8 | |
| $-M$ | $A_1$ | 1 | 1 | 0 | 0 | $-1$ | 1 | 3 | 3 | LV |
| | $Z = z_j - c_j$ | $-M-6$ EV | $-M-4$ | 0 | 0 | $M$ | 0 | | | |
| 0 | $S_1$ | 0 | 1 | 1 | 0 | 2 | $-2$ | 24 | 12 | |
| 0 | $S_2$ | 0 | $-1$ | 0 | 1 | 3 | $-3$ | 15 | 5 | LV |
| 6 | $x_1$ | 1 | 1 | 0 | 0 | $-1$ | 1 | 3 | $-3$ | |
| | $Z = z_j - c_j$ | 0 | 2 | 0 | 0 | $-6$ EV | $M+6$ | | | |
| 0 | $S_1$ | 0 | $5/3$ | 1 | $-2/3$ | 0 | 0 | 14 | 8.4 | LV, I |
| 0 | $S_3$ | 0 | $-1/3$ | 0 | $1/3$ | 1 | $-1$ | 5 | $-$ve | I |
| 6 | $x_1$ | 1 | $2/3$ | 0 | $1/3$ | 0 | 0 | 8 | 12 | I |
| | $Z = z_j - c_j$ | 0 | 0 EV | 0 | 2 | 0 | $M$ | | | |
| 4 | $x_2$ | 0 | 1 | $3/5$ | $-2/5$ | 0 | 0 | $42/5$ | | II |
| 0 | $S_3$ | 0 | 0 | $1/5$ | $1/5$ | 1 | $-1$ | $39/5$ | | II |
| 6 | $x_1$ | 1 | 0 | $-2/5$ | $3/5$ | 0 | 0 | $12/5$ | | II |
| | $Z = z_j - c_j$ | 0 | 0 | 0 | 2 | 0 | $M$ | | | |

$Z \geq 0$ ∴ solution is optimal.

Yes the LPP has alternate solution

**I optimal solution**

$x_1 = 8, \quad x_2 = 0$

$\therefore \text{Max } Z = 6(8) + 4(0)$

$= \underline{\underline{48}}$

**II optimal solution**

$x_1 = 12/5, \quad x_2 = 42/5$

$\therefore \max Z = 6(12/5) + 4(42/5)$

$\max Z = \underline{48}$

6A) converting min to max

$\uparrow \max Z = -3x_1 - 8x_2$

converting inequalities to equations adding slack, artificial variables & subtracting surplus variable

$x_1 + x_2 + A_1 = 200$

$x_1 + S_1 = 80$

$x_2 - S_2 + A_2 = 60$

$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$

$S_1$ - slack, $S_2$ - surplus, $A_1$ $A_2$ - artificial

$\therefore$ new obj. func$^n$ $\boxed{\max Z = -3x_1 - 8x_2 - MA_1 + 0S_1 - 0S_2 - MA_2}$

| $C_B$ | Basis $c_j$ | $x_1$ (−3) | $x_2$ (−8) | $A_1$ (−M) | $S_1$ (0) | $S_2$ (0) | $A_2$ (−M) | RHS | min ratio |
|---|---|---|---|---|---|---|---|---|---|
| −M | $A_1$ | 1 | 1 | 1 | 0 | 0 | 0 | 200 | 200 |
| 0 | $S_1$ | 1 | 0 | 0 | 1 | 0 | 0 | 80 | ∞ |
| −M | $A_2$ | 0 | 1 | 0 | 0 | −1 | 1 | 60 | 60 LV |
| | $Z = Z_j - C_j$ | −M+3 | −2M+8 | 0 | 0 | M | 0 | | |
| −M | $A_1$ | 1 | 0 | 1 | 0 | 1 | −1 | 140 | 140 |
| 0 | $S_1$ | 1 | 0 | 0 | 1 | 0 | 0 | 80 | 80 LV |
| −8 | $x_2$ | 0 | 1 | 0 | 0 | −1 | 1 | 60 | ∞ |
| | $Z = Z_j - C_j$ | −M+3 | 0 | 0 | 0 | −M+8 | 2M−8 | | |
| −M | $A_1$ | 0 | 0 | 1 | −1 | 1 | −1 | 60 | 60 LV |
| −3 | $x_1$ | 1 | 0 | 0 | 1 | 0 | 0 | 80 | ∞ |
| −8 | $x_2$ | 0 | 1 | 0 | 0 | −1 | 1 | 60 | −60 |
| | $Z = Z_j - C_j$ | 0 | 0 | 0 | M−3 | −M+8 | 2M−8 | | |
| 0 | $S_2$ | 0 | 0 | 1 | −1 | 1 | −1 | 60 | |
| −3 | $x_1$ | 1 | 0 | 0 | 1 | 0 | 0 | 80 | |
| −8 | $x_2$ | 0 | 1 | 1 | −1 | 0 | 0 | 120 | |
| | $Z = Z_j - C_j$ | 0 | 0 | 4M | 5 | 0 | M | | |

$Z \geq 0$

$\therefore$ solution is optimal

$x_1 = 80, x_2 = 120$

$\uparrow \max Z = -1200$

$\downarrow \min Z = \underline{\underline{1200}}$