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Internal Assessment Test 1 – Sept. 2017

Sub:	Operations Research				Sub Code:	10ME74	Branch:	ME
Date:	21.09.2017	Duration:	90 min's	Max Marks: 50	Sem / Sec:	7th/ A & B		

Answer any FIVE FULL Questions

		MARKS	OBE CO	OBE RBT
1 (a)	A Firm makes two products X & Y And has a total production capacity of 9 ton's per day. X&Y Requiring the same production capacity the firm has a permanent contract to supply at least 2 ton's of X and at least 3 ton's of Y per day to another company each ton of X requires 20 Machine hours production time and each ton of Y requires 50 machine hours Production time the daily maximum possible no. of hours is 360 all the firms output can be Sold and the profit obtained is Rs 80 per ton of X and Rs120 per ton of Y respectively. Formulate The LPP and solve it graphically	[08]	CO3	L3
(b)	List the assumptions made in LPP	[02]	CO1	L1
2 (a)	Solve the LPP Using Simplex method **Maximize** $Z = 6x_1+11x_2$ **ST** $2x_1+x_2 \leq 104$, $x_1+2x_2 \leq 76$, $x_1, x_2 \geq 0$	[06]	CO3	L3
(b)	Explain briefly in LPP Infeasible solution, unbounded solution, Alternate optimal solution, Degenerate solutions with example	[04]	CO1	L2
3 (a)	Solve the LPP Using Simplex method **Minimize** $Z = x_1-3x_2+2x_3$ **ST** $3x_1-x_2+3x_3 \leq 7$, $-2x_1+4x_2 \leq 12$, $-4x_1+3x_2+8x_3 \leq 10$ $x_1, x_2, x_3 \geq 0$	[7.5]	CO3	L3
(b)	Explain slack variable, surplus variable, Artificial variable, Binding & Non- binding constraint	[2.5]	CO1	L2
4 (a)	Solve the LPP Using Penalty method **Minimize** $Z = 4x_1+x_2$ **ST** $3x_1+x_2 = 3$, $4x_1+3x_2 \geq 6$, $x_1+2x_2 \leq 4$ $x_1, x_2 \geq 0$	[10]	CO3	L3
5 (a)	Solve the LPP Using Simplex method **Maximize** $Z = 6x_1+4x_2$ **ST** $2x_1+3x_2 \leq 30$, $3x_1+2x_2 \leq 24$, $x_1+x_2 \geq 3$ $x_1, x_2 \geq 0$ Does the problem have alternative optima; If so find the other solution.	[5+2+3]	CO3	L4
6 (a)	Solve the LPP Using Big-M- method **Minimize** $Z = 3x_1+8x_2$ **ST** $x_1+x_2 = 200$, $x_1 \leq 80$, $x_2 \geq 60$ $x_1, x_2 \geq 0$	[10]	CO3	L3

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1-IAT

Solution

1A] let x_1, x_2 be the no. of tonne of product X & Y
a] the company should manufacture respectively.

$\max Z = 80x_1 + 120x_2$

Sub. to.

$x_1 + x_2 \leq 9$

$x_1 \geq 2$

$x_2 \geq 3$

$20x_1 + 50x_2 \leq 360$

$x_1, x_2 \geq 0$

The LPP is to find x_1, x_2 where:

$\max Z = 80x_1 + 120x_2$

Sub to

$x_1 + x_2 \leq 9$ — (1)

$x_1 \geq 2$ — (2)

$x_2 \geq 3$ — (3)

$20x_1 + 50x_2 \leq 360$ — (4)

$x_1, x_2 \geq 0$

Assuming constraint to be equation

(1) $x_1 + x_2 = 9$

$x_1 = 0, x_2 = 9$

$x_2 = 0, x_1 = 9$

$(x_1, x_2) = (9, 9)$

(2)

$x_1 = 2$

$(x_1, x_2) = (2, 0)$

(3)

$x_2 = 3$

$(x_1, x_2) = (0, 3)$

(4)

$20x_1 + 50x_2 = 360$

$x_1 = 0, x_2 = 7.2$

$x_2 = 0, x_1 = 18$

$\therefore (x_1, x_2) = (18, 7.2)$

From the graph:
the company should produce 3 tonns of X, 6 tonns of Y to get a profit of Rs 960.

2A] a) $\uparrow \max Z = 6x_1 + 11x_2$

sub. to

$2x_1 + x_2 \le 104$

$x_1 + 2x_2 \le 76$

$x_1, x_2 \ge 0$

converting inequalities to equation adding slack variables

$2x_1 + x_2 + S_1 = 104$

$x_1 + 2x_2 + S_2 = 76$ $\qquad S_1, S_2$ — slack variables

$x_1, x_2, S_1, S_2 \ge 0$

∴ new obj. funcⁿ:

$\uparrow \max Z = 6x_1 + 11x_2 + 0S_1 + 0S_2$

C_B	C_j / Basis	6 / x_1	11 / x_2	0 / S_1	0 / S_2	RHS	min. ratio
0	S_1	2	1	1	0	104	104
0	S_2	1	2 PE	0	1	76	38 — LV
	$Z = z_j - c_j$	−6	−11 EV	0	0		
0	S_1	3/2 PE	0	1	−1/2	66	44 — LV
11	x_2	1/2	1	0	1/2	38	76
	$Z = z_j - c_j$	−1/2 EV	0	0	11/2		
6	x_1	1	0	2/3	−1/3	44	
11	x_2	0	1	−1/3	2/3	16	
	$Z = z_j - c_j$	0	0	1/3	16/3		

$Z \ge 0$

Solution is optimal

$x_1 = 44, \; x_2 = 16$

∴ $\max Z = \underline{440}$

3A] a) $\min Z = x_1 - 3x_2 + 2x_3$

sub to

$3x_1 - x_2 + 3x_3 \le 7$

$-2x_1 + 4x_2 + 0x_3 \le 12$

$-4x_1 + 3x_2 + 8x_3 \le 10$

$x_1, x_2, x_3 \ge 0$

converting min to max

$\uparrow \max Z = -x_1 + 3x_2 - 2x_3$

converting inequalities to equation. adding slack variables

$3x_1 - x_2 + 3x_3 + S_1 = 7$

$-2x_1 + 4x_2 + 0x_3 + S_2 = 12$

$-4x_1 + 3x_2 + 8x_3 + S_3 = 10$

$x_1, x_2, x_3, S_1, S_2, S_3 \ge 0$

∴ new obj func^n

$\uparrow$ Max $Z = x_1 - 3x_2 + 2x_3 + 0S_1 + 0S_2 + 0S_3$

C_B	C_j / Basis	-1 / x_1	3 / x_2	-2 / x_3	0 / S_1	0 / S_2	0 / S_3	RHS	min ratio
0	S_1	3	-1	3	1	0	0	7	-7
0	S_2	$\cancel{-2}$	4	0	0	1	0	12	3 — LV
0	S_3	-4	3	8	0	0	1	10	3.2
	$Z = Z_j - C_j$	1	-3	2	0	0	0		
0	S_1	$5/2$	0 EV	3	1	$1/4$	0	10	$20/5$ — LV
3	x_2	$-1/2$	1	0	0	$1/4$	0	3	$3/-1/2$
0	S_3	$-5/2$	0	8	0	$-3/4$	1	1	$1/-5/2$
	$Z = Z_j - C_j$	$-1/2$	0	-2	0	$3/4$	0		
-1	x_1	1 EV	0	$6/5$	$2/5$	$1/10$	0	4	
3	x_2	0	1	$3/5$	$1/5$	$3/10$	0	5	
0	S_3	0	0	11	1	$-1/2$	1	11	
	$Z = Z_j - C_j$	0	0	$13/5$	$1/5$	$8/10$	0		

$Z \geq 0$ solution is optimal

$x_1 = 4, \; x_2 = 5, \; x_3 = 0$

∴ Max $Z = 11$

$\downarrow$ min $Z = -11$

4A) $\downarrow$ min $Z = 4x_1 + x_2$

sub to

$3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$

$x_1, x_2 \geq 0$

converting to max:

$\uparrow$ max $Z = -4x_1 - x_2$

converting inequalities adding slack & artificial variables & subtracting surplus variable

$3x_1 + x_2 + A_1 = 3$

$4x_1 + 3x_2 - S_1 + A_2 = 6$

$x_1 + 2x_2 + S_2 = 4$

$x_1, x_2, A_1, A_2, S_1, S_2 \geq 0$

A_1, A_2 — artifical variable

S_1 — surplus "

S_2 = slack "

∴ new obj func^n:

$\uparrow$ max $Z = -4x_1 - x_2 - MA_1 - 0S_1 - MA_2 + 0S_2$

C_B	Basis	x_1	x_2	A_1	S_1	A_2	S_2	RHS	min ratio
	c_j	-4	-1	$-M$	0	$-M$	0		
$-M$	A_1	3	1	1	0	0	0	3	1 ← LV
$-M$	A_2	4	3	0	-1	1	0	6	1.5
0	S_2	1	2	0	0	0	1	4	4
	$Z = z_j - c_j$	$-7M+4$	$-4M+1$	0	M	0	0		
		↑EV							
-4	x_1	1	$1/3$	$1/3$	0	0	0	1	3
$-M$	A_2	0	$5/3$	$-4/3$	-1	1	0	2	$6/5$ — LV
0	S_2	0	$5/3$	$-1/3$	0	0	1	3	$9/5$
	$Z = z_j - c_j$	0	$-5M - 1/3$	$7/3M - 4/3$	M	0	0		
			↑EV						
-4	x_1	1	0	$3/5$	$1/5$	$-1/5$	0	$3/5$	3
-1	x_2	0	1	$-4/5$	$-3/5$	$3/5$	0	$6/5$	2
0	S_2	0	0	1	1	-1	1	1	1
	$Z = z_j - c_j$	0	0	$-8/5 + M$	$-1/5$	$1/5$	0		
-4	x_1	1	0	$2/5$	0	0	$-1/5$	$2/5$	
-1	x_2	0	1	$-1/5$	0	0	$3/5$	$9/5$	
0	S_1	0	0	1	1	-1	1	1	
	$Z = z_j - c_j$	0	0	M	0	M	$1/5$		

$Z \geq 0$ ∴ soln. is optimal

$x_1 = 2/5$, $x_2 = 9/5$

Max $Z = -17/5$

∴ Min $Z = \underline{17/5}$

5A) Converting inequalities to equations by adding slack variables & artificial variables & subtracting surplus variable

$$2x_1 + 3x_2 + S_1 = 30$$

$$3x_1 + 2x_2 + S_2 = 24$$

$$x_1 + x_2 - S_3 + A_1 = 3$$

$$x_1, x_2, x_3, S_1, S_2, S_3, A_1 \geq 0$$

∴ new obj function

Max $Z = 6x_1 + 4x_2 + 0S_1 + 0S_2 - 0S_3 - MA_1$

C_B	C_j Basis	6 x_1	4 x_2	0 S_1	0 S_2	0 S_3	$-M$ A_1	RHS	min ratio	
0	S_1	2	3	1	0	0	0	30	15	
0	S_2	3	2	0	1	0	0	24	8	
$-M$	A_1	1	1	0	0	-1	1	3	3	LV
	$Z = z_j - c_j$	$-M-6$ EV	$-M-4$	0	0	M	0			
0	S_1	0	1	1	0	2	-2	24	12	
0	S_2	0	-1	0	1	3	-3	15	5	LV
6	x_1	1	1	0	0	-1	1	3	-3	
	$Z = z_j - c_j$	0	2	0	0	-6 EV	$M+6$			
0	S_1	0	$5/3$	1	$-2/3$	0	0	14	8.4	LV
0	S_3	0	$-1/3$	0	$1/3$	1	-1	5	$-$ve	I
6	x_1	1	$2/3$	0	$1/3$	0	0	8	12	
	$Z = z_j - c_j$	0	0 EV	0	2	0	M			
4	x_2	0	1	$3/5$	$-2/5$	0	0	$42/5$		
0	S_3	0	0	$1/5$	$1/5$	1	-1	$39/5$		II
6	x_1	1	0	$-2/5$	$3/5$	0	0	$12/5$		
	$Z = z_j - c_j$	0	0	0	2	0	M			

$Z \geq 0$ $\therefore$ solution is optimal.

Yes the LPP has alternate solution

I optimal solution

$x_1 = 8$, $x_2 = 0$

$\therefore \text{Max } Z = 6(8) + 4(0)$

$= 48$

II optimal solution

$x_1 = 12/5$, $x_2 = 42/5$

$\therefore \max Z = 6(12/5) + 4(42/5)$

$\max Z = 48$

6A) converting min to max

$\uparrow \max Z = -3x_1 - 8x_2$

converting inequalities to equations adding slack, artificial variables & subtracting surplus variable

$x_1 + x_2 + A_1 = 200$

$x_1 + S_1 = 80$

$x_2 - S_2 + A_2 = 60$

$x_1, x_2, S_1, S_2, A_1, A_2 \geq 0$

S_1 - slack, S_2 - surplus, $A_1 A_2$ - artificial

$\therefore$ new obj. func^n $\boxed{\max Z = -3x_1 - 8x_2 - MA_1 + 0S_1 - 0S_2 - MA_2}$

C_B	C_j / Basis	-3 x_1	-8 x_2	$-M$ A_1	0 S_1	0 S_2	$-M$ A_2	RHS	min ratio
$-M$	A_1	1	1	1	0	0	0	200	200
0	S_1	1	0	0	1	0	0	80	∞
$-M$	A_2	0	1	0	0	-1	1	60	60 - LV
	$Z = Z_j - C_j$	$-M+3$	$-2M+8$	0	0	M	0		
$-M$	A_1	1	0	1	0	1	-1	140	140
0	S_1	1	0	0	1	0	0	80	80 - LV
-8	x_2	0	1	0	0	-1	1	60	∞
	$Z = Z_j - C_j$	$-M+3$	0	0	0	$-M+8$	$2M-8$		
$-M$	A_1	0	0	1	-1	1	-1	60	60 - LV
-3	x_1	1	0	0	1	0	0	80	∞
-8	x_2	0	1	0	0	-1	1	60	-60
	$Z = Z_j - C_j$	0	0	0	$M-3$	$-M+8$	$2M-8$		
0	S_2	0	0	1	-1	1	-1	60	
-3	x_1	1	0	0	1	0	0	80	
-8	x_2	0	1	1	-1	0	0	120	
	$Z = Z_j - C_j$	0	0	$+M$	5	0	M		

$Z \geq 0$

$\therefore$ solution is optimal

$x_1 = 80, \ x_2 = 120$

$\uparrow \max Z = -1200$

$\downarrow \min Z = \underline{\underline{1200}}$