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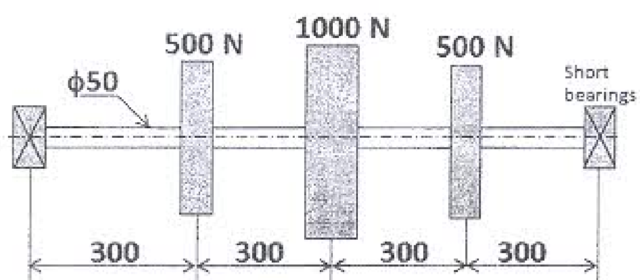
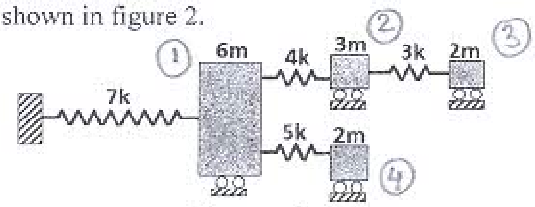
CMR INSTITUTE OF TECHNOLOGY

USN										

Internal Assessment Test I

SUBJECT: MECHANICAL VIBRATIONS								Code:	10ME72
Date:	07/11/2017	Duration:	90 min	Max. Marks:	50	Sem:	07	Branch:	MECH

Note: Answer any FIVE questions.

Q. No.	QUESTION	MARKS	OBE MAP CO	OBE MAP RBT
	PART – A			
1	Explain the principle of operation of an accelerometer with the help of relevant equations and graphs.	[10]	CO2	L2
2	Derive an expression for the critical speed of a rotating shaft carrying a disc, without considering the resistance to motion from the surrounding viscous medium. Discuss the relative locations of geometric center and *CG* of the system at the three speed regimes.	[10]	CO2	L2
3	A torsional accelerometer in the form of a ring is connected to a shaft by means of a spiral spring. The system is damped having a constant 0.12 Nms/radian. If the torsional stiffness of the spring is 1Nm/radian and the mass moment of inertia of the ring is 0.05 kgm^2, determine the maximum acceleration of the shaft when the amplitude of relative angle between the ring and the shaft is 3.5°. Assume the shaft is rotating at a speed of 25 r.p.m.	[10]	CO2	L3
4	A shaft of diameter 12 mm and length 500 mm carries a disc weighing 150 N at its mid span. The shaft has vertical orientation, and is housed in long bearings. If the distance between the geometric center and the center of gravity of the disc is 0.5 mm, what is the critical speed of the shaft? What is the speed range at which the stress due to shaft bending will exceed 120 MPa? Take $E = 2\times10^{11}$ Pa.	[10]	CO2	L3
5	Determine the fundamental frequency of 3 rotor system shown in figure 1 by Rayleigh's method. Take $E = 2.1\times10^{11}$ N/m^2  All dimensions in mm Figure – 1	[10]	CO3	L3
6	Using Stodola's method, determine the lowest natural frequency of the branched system shown in figure 2.  Figure – 2	[10]	CO3	L3
7	List the different machine condition monitoring techniques? Explain the different machine maintenance schemes.	[10]	CO4	L3

Q1

Principle of working of an accelerometer

An accelerometer is an instrument that measures the acceleration of a vibrating body. Accelerometers are widely used for vibration measurements and also to record earthquakes. From the accelerometer record, the velocity and displacements are obtained by integration.

We know that $z(t) = Z\sin(\omega t - \phi)$ and

$$Z = \frac{r^2 Y}{[(1-r^2)^2 + (2\zeta r)^2]^{1/2}} \qquad \text{Substituting } Z = \frac{z(t)}{\sin(\omega t - \phi)}$$

$$\frac{z(t)}{\sin(\omega t - \phi)} = \frac{r^2 Y}{[(1-r^2)^2 + (2\zeta r)^2]^{1/2}}$$

$$= \frac{\omega^2 Y}{\omega_n^2 [(1-r^2)^2 + (2\zeta r)^2]^{1/2}} \qquad r^2 \text{ is substituted}$$

$$z(t)\,\omega_n^2 = \frac{\omega^2 Y \sin(\omega t - \phi)}{[(1-r^2)^2 + (2\zeta r)^2]^{1/2}} \qquad \omega_n^2 \text{ and } \sin(\omega t - \phi) \text{ is relocated}$$

$$-z(t)\,\omega_n^2 = \frac{-\omega^2 Y \sin(\omega t - \phi)}{[(1-r^2)^2 + (2\zeta r)^2]^{1/2}} \qquad \text{Multiplying } -1 \text{ on both sides}$$

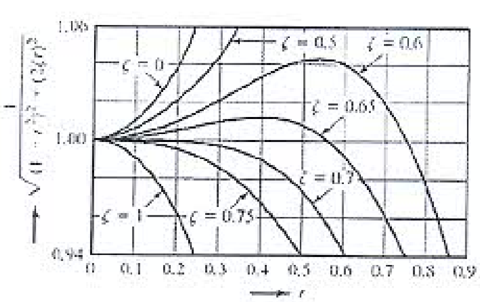
Or,
$$-z(t)\,\omega_n^2 = \frac{1}{[(1-r^2)^2 + (2\zeta r)^2]^{1/2}}\{-Y\omega^2\sin(\omega t - \phi)\}$$

Now if $\dfrac{1}{[(1-r^2)^2 + (2\zeta r)^2]^{1/2}} \simeq 1$ then $-z(t)\,\omega_n^2 \simeq -Y\omega^2\sin(\omega t - \phi)$

Comparing $-z(t)\,\omega_n^2 \simeq -Y\omega^2\sin(\omega t - \phi)$ and $\ddot{y}(t) = -Y\omega^2\sin\omega t$

it is clear that the term $z(t)\,\omega_n^2$ gives the acceleration $\ddot{y}$ of the base, except for the phase lag ϕ. Thus the instrument can be made to give directly the value of $\ddot{y} = -z(t)\,\omega_n^2$. The time by which the record lags the acceleration is given by $t' = \phi/\omega$.

The value of the expression $\dfrac{1}{[(1-r^2)^2 + (2\zeta r)^2]^{1/2}}$ is shown plotted in the following figure.



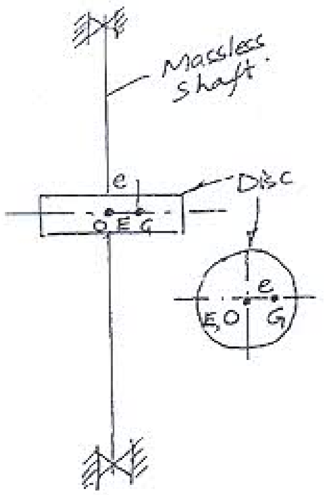
It can be seen that the value of the plotted expression lies between 0.96 and 1.04 for $0 \le r \le 0.6$ if the value of ζ lies between 0.65 and 0.7. Since r is small, the natural frequency of the instrument has to be large compared to the frequency of vibration to be measured.

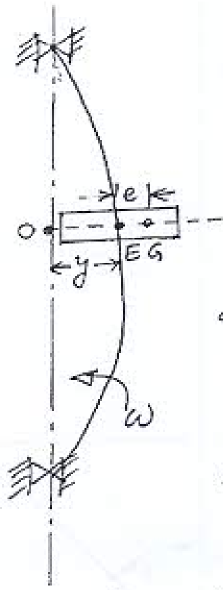
From the relation $\omega_n = \sqrt{k/m}$ we find that the mass needs to be small and the spring needs to have a large value of k (i.e., short spring), so the instrument will be small in size. Due to their small size and high sensitivity, accelerometers are preferred in vibration measurements.

Q2

Whirling of Shafts without damping

Consider a shaft of negligible mass carrying a rotor as shown. Point G is the c.g. of the rotor, point E is the geometric centre of the rotor and lying on the shaft axis, and point O is the axis of rotation of the shaft along with the rotor.





Let.

m = mass of the rotor

e = Eccentricity of cg with Geometric centre of disc

y = Deflection of the shaft wh the shaft rotates at ω.

δ = Static deflection of shaft disc under its own weight

ω = Angular speed of rotor

W = Weight of rotor = mg

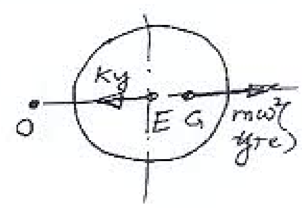
K = Stiffness of the shaft

Stationary State

The shaft–rotor assembly will be in dynamic equilibrium under the action of two forces.

i) Radially outward centrifugal force = $m\omega^2(y+e)$.

ii) Inward elastic pull exerted by the shaft on the pulley resisting shaft deflection = $K\cdot y$.



For equilibrium, the points O, E and G must lie on a straight line. with

$$m\omega^2(y+e) = Ky$$

$$m\omega^2 e = y(K - m\omega^2)$$

$$y = \frac{m\omega^2 e}{(K - m\omega^2)} = \frac{\omega^2 e}{\omega_c^2 - \omega^2}$$ where

$\omega_c = \omega_n$ = Critical speed of shaft.

$$y = \frac{(\omega/\omega_c)^2 e}{1-\left(\frac{\omega}{\omega_c}\right)^2} = \frac{e\cdot r^2}{1-r^2}$$ where $r = \frac{\omega}{\omega_c}$.

Q3

$$\omega_n = \sqrt{\frac{K_t}{I}} = \sqrt{\frac{1}{.05}} = 4.472 \text{ rad/s}$$

$$\omega = 2\pi \times 25/60 = 2.618 \text{ rad/s}$$ (2)

$$r = \frac{\omega}{\omega_n} = 2.618/4.472 = 0.585$$

$$c = 0.12 = 2\sqrt{K_t \cdot I} \cdot \xi \quad \text{or} \quad \xi = \frac{0.12}{2\sqrt{1 \times 0.05}} = 0.268$$ (3)

$$\frac{\theta_z}{\theta_y} = \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} = \frac{0.585^2}{\sqrt{(1-0.585^2)^2 + (2 \times 0.268 \times 0.585)^2}}$$

$$= 0.4696$$

$$\theta_y = \frac{\theta_z}{0.4696} = \frac{3.5}{0.4696} = 7.453^\circ = 0.13 \text{ radians.}$$ (2)

Max. angular acceleration of shaft $\ddot{\theta}_{max} = \omega^2 \theta_y$

$$= (2.618)^2 \times 0.13$$

Affixing correct units : 1 mark.

$$= 0.891 \text{ rad/s}^2$$ (2)

Q4 Static deflection of shaft at mid-span, δ $\Big\}$ $\delta = \frac{Wl^3}{192EI} = \frac{150 \times 0.5^3}{192 \times 2 \times 10^{11} \times \frac{\pi}{64}(12 \times 10^{-3})^4}$

$$= \underline{4.797 \times 10^{-4}} \text{ m.}$$ (2)

$$\text{Critical Speed} = N_c = \frac{0.4985}{\sqrt{\delta}} = 22.76 \text{ rps (Hz)}$$

or $\underline{1365.63}$ rpm. (1)

To find the force at mid span that will produce 120 MPa.

$$\frac{W_1 \times 0.5}{\cancel{8} \times \frac{\pi}{\cancel{64}\,\cancel{8}\,4}(\cancel{12 \times 10^{-3}})^{4\,3}} = \frac{120 \times 10^6}{\left(\frac{\cancel{12 \times 10^{-3}}}{\cancel{2}}\right)}$$

$$W_1\ 0.5 \times 4 = 120 \times 10^6 \times \pi \times (12 \times 10^{-3})^3$$

$$W_1 = \underline{325.72} \text{ N}$$ (2)

150 N at midspan caused a deflection of 4.797×10^{-4} m.

325.72 N acting at mid span would produce a deflection of

$$\frac{325.72}{150} \times 4.797 \times 10^{-4} = 1.042 \times 10^{-3} \text{ m.} = y \quad 1$$

$$y = \pm \frac{e}{\left(\frac{N_c}{N}\right)^2 - 1}$$ where $N_c = 1365.63$ rpm and N = to be computed; $e = 0.5$ mm

$$1.042 \times 10^{-3} = \pm \frac{0.5 \times 10^{-3}}{\left(\frac{1365.63}{N}\right)^2 - 1}$$

$$\left(\frac{1365.63}{N}\right)^2 = \pm \frac{0.5 \times 10^{-3}}{1.042 \times 10^{-3}} + 1 = \pm 0.4798 + 1 \quad 2$$

$$= 1.4798 \text{ or } 0.5202$$

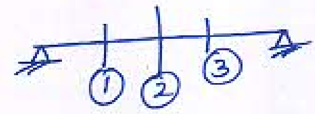
$$N_1 = \sqrt{\frac{1365.63^2}{1.4798}} = 1122.1 \text{ rpm} \quad 1$$

$$N_2 = \sqrt{\frac{1365.63^2}{0.5202}} = 1893.42 \text{ rpm.} \quad 1$$

The stress level will be greater than 120 MPa between the speeds 1122.1 rpm and 1893.42 rpm.

Q5 There are 6 influence coefficients for the given system.

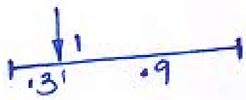
They are $a_{11}, a_{12}, a_{13}, a_{22}, a_{23}, a_{33}$



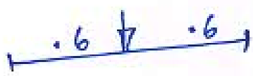
$a_{11} = a_{33}$ due to symmetry.

$a_{12} = a_{32}$ due to symmetry.

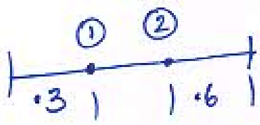
$\therefore$ Distinct influence coeff's are $a_{11}, a_{22}, a_{12}, a_{13}$

a_{11} 

$= \dfrac{.3^2 \times .9^2}{3EI \times 1.2} = \dfrac{0.02025}{EI}$ (1)

a_{22} 

$= \dfrac{.6^2 \times .6^2}{3EI \times 1.2} = \dfrac{0.036}{EI}$ (1)

a_{12} 

$= \dfrac{.3 \times .6\,(1.2^2 - .3^2 - .6^2)}{6EI \times 1.2} = \dfrac{0.02475}{EI}$ (1)

a_{13} 

$= \dfrac{.3 \times .3\,(1.2^2 - 0.3^2 - 0.3^2)}{6EI \times 1.2} = \dfrac{0.01575}{EI}$ (1)

$y_1 = \dfrac{1}{EI}\left\{0.02025 \times 500 + 0.02475 \times 1000 + 0.01575 \times 500\right\} = \dfrac{42.75}{EI}$ (1)

$y_2 = \dfrac{1}{EI}\left\{0.02475 \times 500 + 0.036 \times 1000 + 0.02475 \times 500\right\} = \dfrac{60.75}{EI}$ (1)

$y_3 = \dfrac{1}{EI}\left\{0.01575 \times 500 + 0.02475 \times 1000 + 0.02025 \times 500\right\} = \dfrac{42.75}{EI}$ (1)

$I = \dfrac{\pi}{64}(50 \times 10^{-3})^4 = 3.068 \times 10^{-7}\ m^4$

$f_n = \dfrac{1}{2\pi}\sqrt{\dfrac{g \sum W_i y_i}{\sum W_i y_i^2}}$ Hz.

$\sum W_i y_i = \left(500 \times 42.75 + 1000 \times 60.75 + 500 \times 42.75\right)\dfrac{1}{EI} = \dfrac{103500}{EI}$

$\sum W_i y_i^2 = \left(2 \times 500 \times 42.75^2 + 1000 \times 60.75^2\right)\dfrac{1}{E^2I^2} = \dfrac{5,518,125}{E^2I^2}$ (3)

$f_n = \dfrac{1}{2\pi}\sqrt{\dfrac{9.81 \times 103500}{5,518,125} \times 2.1 \times 10^{11} \times 3.068 \times 10^{-7}} = \dfrac{108.88}{2\pi}$

$= 17.33$ Hz

Refer to question paper (page-1) for mass positions.

		k1=7k	m1=6m	k2=4k	m2=3m	k3=3k	m3=2m	k4=5k	m4=2m
Assume unit disp. For masses	Step-1 (Trial-1)		1.0000		1.0000		1.0000		1.0000
Inertia forces ($m_i x_i \omega^2$)	Step-2 ($\times m\omega^2$)		6.0000		3.0000		2.0000		2.0000
Spring force	Step-3 ($\times m\omega^2$)	13.0000		5.0000		2.0000		2.0000	
Spring deflection	Step-4 ($\times m\omega^2/k$)	13/7=1.857		5/4=1.25		2/3=0.667		2/5=0.4	
Cumulative deflection	Step-5 ($\times m\omega^2/k$)		1.8570		3.1070		3.7740		2.2570
NORMALIZED DEFLECTION	Step-6		1.0000		1.6731		2.0323		1.2154
Assume disp. For masses	Step-1 (Trial-2)		1.0000		1.6731		2.0323		1.2154
Inertia forces ($m_i x_i \omega^2$)	Step-2 ($\times m\omega^2$)		6.0000		5.0194		4.0646		2.4308
Spring force	Step-3 ($\times m\omega^2$)	17.5148		9.0840		4.0646		2.4308	
Spring deflection	Step-4 ($\times m\omega^2/k$)	2.5021		2.2710		1.3549		0.4862	
Cumulative deflection	Step-5 ($\times m\omega^2/k$)		2.5021		4.7731		6.1280		2.9883
NORMALIZED DEFLECTION	Step-6		1.0000		1.9076		2.4491		1.1943
Assume disp. For masses	Step-1 (Trial-3)		1.0000		1.9076		2.4491		1.1943
Inertia forces ($m_i x_i \omega^2$)	Step-2 ($\times m\omega^2$)		6.0000		5.7229		4.8982		2.3886
Spring force	Step-3 ($\times m\omega^2$)	19.0097		10.6211		4.8982		2.3886	
Spring deflection	Step-4 ($\times m\omega^2/k$)	2.7157		2.6553		1.6327		0.4777	
Cumulative deflection	Step-5 ($\times m\omega^2/k$)		2.7157		5.3710		7.0037		3.1934
NORMALIZED DEFLECTION	Step-6		1.0000		1.9778		2.5790		1.1759
Assume disp. For masses	Step-1 (Trial-4)		1.0000		1.9778		2.5790		1.1759
Inertia forces ($m_i x_i \omega^2$)	Step-2 ($\times m\omega^2$)		6.0000		5.9333		5.1580		2.3518
Spring force	Step-3 ($\times m\omega^2$)	19.4431		11.0913		5.1580		2.3518	
Spring deflection	Step-4 ($\times m\omega^2/k$)	2.7776		2.7728		1.7193		0.4704	
Cumulative deflection	Step-5 ($\times m\omega^2/k$)		2.7776		5.5504		7.2697		3.2479
NORMALIZED DEFLECTION	Step-6		1.0000		1.9983		2.6173		1.1693
Assume disp. For masses	Step-1 (Trial-5)		1.0000		1.9983		2.6173		1.1693
Inertia forces ($m_i x_i \omega^2$)	Step-2 ($\times m\omega^2$)		6.0000		5.9949		5.2346		2.3387
Spring force	Step-3 ($\times m\omega^2$)	19.5681		11.2294		5.2346		2.3387	
Spring deflection	Step-4 ($\times m\omega^2/k$)	2.7954		2.8074		1.7449		0.4677	
Cumulative deflection	Step-5 ($\times m\omega^2/k$)		2.7954		5.6028		7.3477		3.2632
NORMALIZED DEFLECTION	Step-6		1.0000		2.0043		2.6284		1.1673

$\left(f_n = 0.1086\sqrt{\frac{k}{m}}\ Hz\right)$

$$\frac{6m\omega^2}{k} = 2.7953$$

$$\omega^2 = \frac{2.7953}{6} \times \frac{k}{m} = 0.46588\frac{k}{m}$$

$$\omega = 0.683\sqrt{\frac{k}{m}}\ \text{rad/s}$$

The basic methods of machine condition monitoring are as follows:

(1) Acoustic and visual monitoring
(2) Vibration Monitoring or Analysis
(3) Thermal Monitoring
(4) Wear Debris Monitoring
(5) Performance Monitoring

The commonly used machine maintenance schemes are as follows:

(1) Breakdown Maintenance (Run-to-Failure): The maintenance performed only when machinery has failed is called breakdown maintenance. This method allows repair or replacement of damaged parts when the machine comes to a complete stop. This is a crude method and the price could be very high in terms of lost output and machine destruction which could be sometime dangerous. This maintenance is applied to non-essential equipment and machinery for which shutdowns do not affect production and where replacements are readily available. Example: Replacement of bulbs after failure.

(2) Preventive (Time-based) maintenance: This involved scheduled maintenance activities at predetermined intervals based on calendar days or running hours of machines. Here, repair or replacement of damaged equipment is carried out before obvious problems occur. This is a good approach that allows the machine to run continuously and where the maintenance personnel have enough skill, knowledge and time to perform the preventive work. Example: Oil and oil-filter changes on a regular basis of cars and bike.

(3) Condition based or Predictive maintenance: This involves periodic monitoring of mechanical and operational conditions on the health of the machine and scheduling maintenance only when a functional failure is detected. This relies on an estimation of time to failure. The machine would then be shut-down at a time when it is most convenient to replace the damaged component. This technique ensures maximum production and avoids catastrophic failures, thereby leading to full utilization of the machine or component. Examples: Tyre changes on cars and bike.