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Improvement Test

Sub: Mechanics of Materials

Code: 15ME34

Date: 18/11/2017

Duration: 90 mins

Max Marks: 50

Sem: 3

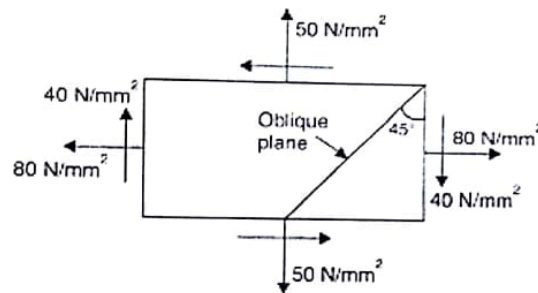
Branch (sections): ME (A,B)

Answer any five questions. Good luck!

**PART A**

	Marks	OBE	
		CO	RBT
1 A simply supported beam of span L carries a point load W at mid-span. Find the strain energy stored by the beam.	[10]	CO1	L3
2 A beam of length L is simply supported at its ends. The beam carries a uniformly distributed load of w per unit length run over the whole span. Find the strain energy stored by the beam.	[10]	CO1	L3
3 Derive an expression for circumferential and longitudinal stress for thin cylinder.	[10]	CO1	L3
4 A thin cylindrical shell 1.2 m in diameter and 3m long has a metal wall thickness of 12mm. It is subjected to an internal fluid pressure of 3.2 MPa. Find the circumferential and longitudinal stress in the wall. Determine change in length, diameter and volume of the cylinder. Assume $E=210\text{GPa}$ and $\mu=0.3$ .	[10]	CO1	L4
5 The internal and external diameters of a thick cylinder are 300mm and 500mm respectively. It is subjected to an external pressure of 4MPa. Find the internal pressure that can be applied if the permissible stress in cylinder is limited to 13MPa. Sketch radial and hoop stresses distribution across the section.	[10]	CO1	L4

- 6 A thick cylinder of 500 mm inner diameter is subjected to an internal pressure of 9 MPa. Taking the allowable stress for the material of the cylinder as 40 MPa, determine the wall thickness of the cylinder. Also plot the stress distribution across the wall thickness of the cylinder. [10]
- 7 A point in a strained material is subjected to stresses shown in Fig. below. Using Mohr's circle method, determine the normal, tangential and resultant stresses across the oblique plane. [10]



- 8 A rectangular block of material is subjected to a tensile stress of  $110 \text{ N/mm}^2$  on one plane and a tensile stress of  $47 \text{ N/mm}^2$  on the plane at right angles to the former. Each of the above stresses is accompanied by a shear stress of  $63 \text{ N/mm}^2$  and that associated with the former tensile stress tends to rotate the block anticlockwise. Find: i) The direction and magnitude of each of the principal stress and (ii) Magnitude of the greatest shear stress. [10]

CO1	L4
CO2	L4
CO2	L4

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CI

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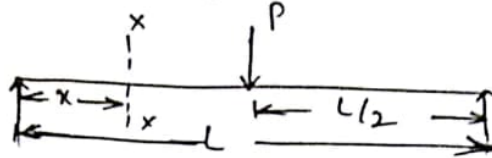
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18-11-2017

## Mechanics of Materials [15ME34]

Improvement Test  
SolutionsSEM: 3<sup>rd</sup> A & B  
Max Marks: 50

1. Fig. below shows a simply supported beam AB of span length  $l$  carries a point load  $P$  at its midspan 'c'.

Reactions at supports,  $R_A = R_B = P/2$ 

Consider section  $x-x$  at distance  $x$  from A i.e., between 'A' and 'c'.

Bending moment at  $x-x$ ,  $M = \frac{P}{2} \cdot x$ 

Strain energy stored by the part Ac =  $\int_0^{l/2} \frac{M^2 dx}{2EI}$

$$= \int_0^{l/2} \frac{\left(\frac{P}{2}x\right)^2 dx}{2EI}$$

$$= \frac{P^2}{8EI} \left( \frac{x^3}{3} \right)_0^{l/2}$$

$$= \frac{P^2 l^3}{192EI}$$

Since length,  $Ac = CB$  and  $R_A = R_B = P/2$

Strain energy stored by the part BC = strain energy stored by the part Ac

$$= \frac{P^2 l^3}{192EI}$$

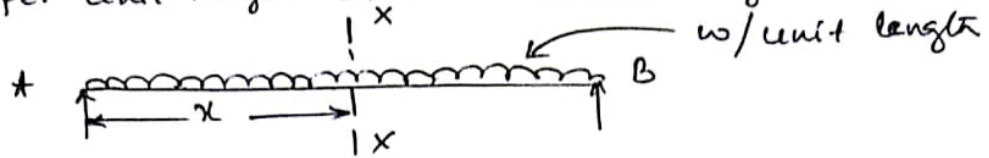
$\therefore$  Total strain energy stored by the beam

$$U = \frac{P^2 l^3}{192EI} + \frac{P^2 l^3}{192EI}$$

$$\therefore \boxed{U = \frac{P^2 l^3}{96EI}} \quad [\text{Ans}]$$

2.

Fig. below shows a cantilever simply supported beam AB of span length  $l$  carrying uniformly distributed load  $w$  per unit length over the entire length.



$$\text{Reaction at A} = \text{Reaction at B} = \frac{wl}{2}$$

Consider section x-x at distance  $x$  from 'A'.

$$\begin{aligned} \therefore \text{Bending moment at } x-x, M &= \frac{wl}{2}x - wx \cdot \frac{x}{2} \\ &= \frac{wl}{2}x - \frac{wx^2}{2} \\ &= \frac{wx}{2}(l-x) \end{aligned}$$

$$\text{Strain energy stored by beam } U = \int_0^l \frac{M^2 dx}{2EI}$$

$$= \int_0^l \frac{\left\{ \frac{wx}{2}(l-x) \right\}^2 dx}{2EI} = \frac{w^2}{8EI} \int_0^l x^2 (l^2 - 2lx + x^2) dx$$

$$= \frac{w^2}{8EI} \int_0^l (l^2 x^2 - 2lx^3 + x^4) dx$$

$$= \frac{w^2}{8EI} \left[ l^2 \cdot \frac{x^3}{3} - 2l \frac{x^4}{4} + \frac{x^5}{5} \right]_0^l$$

$$= \frac{w^2}{8EI} \left[ l^2 \cdot \frac{l^3}{3} - l \cdot \frac{l^4}{4} + \frac{l^5}{5} \right]$$

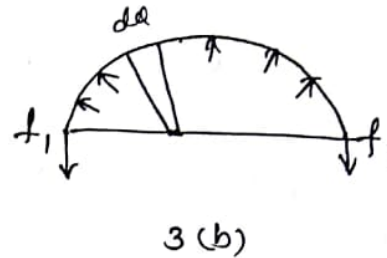
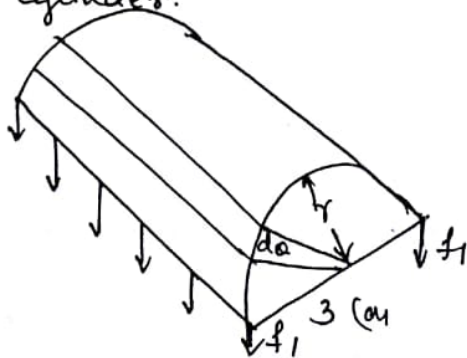
$$= \frac{w^2 l^5}{8EI} \left[ \frac{10 - 15 + 6}{30} \right]$$

$$\therefore \text{Strain energy stored by the beam, } U = \frac{w^2 l^5}{240EI}$$



3.

Circumferential stress: This is also called as hoop stress. Consider the longitudinal and diametrical section x-x of the cylinder.



Let 'r' be the radius. Then normal pressure on elemental length shown in fig. b =  $\underline{p r d \theta}$

Bursting force on the element normal to the section x-x  
 $= \underline{p r L d \theta \cos \theta}$

$\therefore$  Total bursting force normal to the section

$$2 \int_0^{\pi/2} p r L \cos \theta d\theta = 2 p r L [\sin \theta]_0^{\pi/2}$$

$$= 2 p r L [1] = 2 p r L$$

$$= \underline{p d L} \quad \text{--- (1)}$$

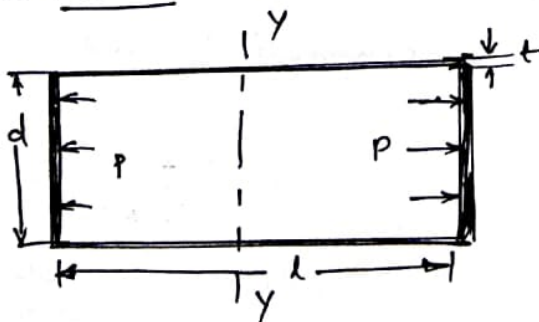
Resisting force for this bursting force is =  $f_1 \times t \times L \times 2$   
 $= 2 f_1 t L \quad \text{--- (2)}$

Equating (1) and (2) we get,

$$2 f_1 t L = p d L$$

$$\textcircled{5} \quad \underline{f_1 = \frac{p d}{2 t}} \quad \text{(Ans)}$$

Longitudinal stress:



3(c)

Consider a thin cylindrical shell subjected to an internal pressure as shown in fig. As a result of the internal pressure, the cylinder also has tendency to split up into two cylinders along section Y-Y as shown in fig. 3(a)

\* Bursting will take place along the section Y-Y, if the force due to internal pressure acting at the ends of the cylinder is more than the resisting force due to longitudinal stress developed in the material.

Force due to internal pressure at the ends of the cylinder or

Bursting force

= Internal pressure  $\times$  Area on which the pressure is acting

$$= P \times \frac{\pi d^2}{4}$$

Force due to longitudinal stress or Resisting force

= Longitudinal stress  $\times$  Area on which the stress is acting

$$= \sigma_2 \times \pi d t$$

For equilibrium, Resisting force = Bursting force

$$\text{i.e. } \sigma_2 \times \pi d t = P \times \frac{\pi d^2}{4}$$

$$\therefore \text{Longitudinal stress, } \sigma_2 = \frac{P d}{4 t} \text{ (Ans)}$$

\* Longitudinal stress is one half of circumferential stress

$$\text{i.e., } \boxed{\sigma_2 = \frac{\sigma_1}{2}}$$

4.

Data:

$$d = 1.2 \text{ m} = 1200 \text{ mm}$$

$$l = 3 \text{ m} = 3000 \text{ mm}$$

$$t = 12 \text{ mm}$$

$$P_i = 3.2 \text{ MPa}$$

$$\sigma_1 \text{ or } \sigma_c = ?$$

$$\sigma_2 \text{ or } \sigma_L = ?$$

$$E = 210 \text{ GPa}$$

$$\mu = 0.3$$

(i) We know that,

$$\text{Circumferential stress, } \sigma_1 = \frac{P d}{2 t}$$

$$= \frac{3.2 \times 1200}{2 \times 12}$$

$$\therefore \sigma_1 \text{ or } \sigma_c = \underline{160 \text{ N/mm}^2}$$

4

$$\text{Longitudinal stress, } \sigma_2 \text{ or } \sigma_L = \frac{Pd}{4t} = \frac{3.2 \times 1200}{4 \times 12}$$

$$= \underline{80 \text{ N/mm}^2 \text{ (Ans)}}$$

(ii) Circumferential, longitudinal and volumetric strain

$$\text{Circumferential strain, } \epsilon_1 = \frac{\sigma_1}{E} - \frac{1}{m} \frac{\sigma_2}{E}$$

$$= \frac{160}{210 \times 10^3} - 0.3 \times \frac{80}{210 \times 10^3}$$

$$= \underline{6.476 \times 10^{-4}}$$

$$\text{Longitudinal strain, } \epsilon_2 = \frac{\sigma_2}{E} - \frac{1}{m} \frac{\sigma_1}{E}$$

$$= \frac{80}{210 \times 10^3} - 0.3 \times \frac{160}{210 \times 10^3}$$

$$= \underline{1.523 \times 10^{-4}}$$

$$\text{Volumetric strain, } \epsilon_v = 2\epsilon_1 + \epsilon_2 = (2 \times 6.476 + 1.523) \times 10^{-4}$$

$$= \underline{14.475 \times 10^{-4}}$$

Change in length, diameter and volume.

$$\text{Longitudinal strain, } \epsilon_2 = \frac{\delta l}{l}$$

$$1.523 \times 10^{-4} = \frac{\delta l}{3000}$$

$$\therefore \text{Change in length, } \delta l = \underline{0.4569 \text{ mm (Ans)}}$$

$$\text{Circumferential strain, } \epsilon_1 = \frac{\delta d}{d}$$

$$6.476 \times 10^{-4} = \frac{\delta d}{1200}$$

$$\therefore \text{Change in diameter, } \delta d = \underline{0.77712 \text{ mm}}$$

$$\text{Volumetric strain, } \epsilon_v = \frac{\delta V}{V}$$

$$\text{where, } V = \frac{\pi d^2}{4} \times l$$

$$= \frac{\pi}{4} \times 1200^2 \times 3000$$

$$14. 475 \times 10^{-4} = \frac{8V}{\frac{\pi}{4} \times 1200^2 \times 3000}$$

$$\therefore 8V = \underline{49.1125 \times 10^5 \text{ mm}^3} \text{ (Ans)}$$

$$\text{(iii) Max. Shear stress, } \tau_{\max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{160 - 80}{2} \\ = \underline{\underline{40 \text{ N/mm}^2}}$$

5.  $d_2 = 300 \text{ mm}; r_2 = 150 \text{ mm}$  (Inner radius)

$d_1 = 500 \text{ mm}; r_1 = 250 \text{ mm}$  (Outer radius)

$t = r_1 - r_2 = 250 - 150 = 100 \text{ mm}$

$r_m = \frac{r_1 + r_2}{2} = \frac{250 + 150}{2} = 200 \text{ mm}$  (Mean radius)

$P/4 = 13 \text{ MPa} = P_{250} = P_0$

$f_{r_2} = f_{150} = 13 \text{ MPa} = 13 \text{ N/mm}^2$

From Lamé's theorem

Radial pressure at any radius  $x$ ,  $P_x = \frac{b}{x^2} - a$

when  $x = r_1$ ,  $P_x = P_r = P_{250} = 13 \text{ MPa} = 13 \text{ N/mm}^2$

$$\therefore 13 = \frac{b}{250^2} - a \quad \text{--- (i)}$$

Circumferential stress at any radius  $x$ ,  $f_x = \frac{b}{x^2} + a$

when,  $x = r_2$ ,  $f_x = f_{r_2} = f_{150} = 13 \text{ N/mm}^2$

$$\therefore f_{150} = \frac{b}{150^2} + a$$

$$13 = \frac{b}{150^2} + a \quad \text{--- (ii)}$$

From (i) and (ii)

$$17 = \frac{b}{150^2} + \frac{b}{250^2}$$

$$\therefore b = \underline{281250} \text{ (cm)}$$

From eqn (i)  $a = \underline{0.5}$



when  $x = r_2 = 150 \text{ mm}$ ,  $P_x = \text{Internal pressure}$

$$\therefore \text{Internal pressure } P_{r_2} = P_{150} = \frac{281250}{150^2} - 0.5$$

$$= \underline{12 \text{ N/mm}^2} \text{ (Ans)}$$

when  $x = r_m = 200 \text{ mm}$ ,  $P_x = P_{200} = P_{r_m}$

$$P_{r_m} = P_{200} = \frac{281250}{200^2} - 0.5$$

$$\therefore P_{200} = \underline{6.53125 \text{ N/mm}^2}$$

Radial  
press

### Circumferential stress

when  $x = r_1 = 250 \text{ mm}$ ,  $f_x = f_{250} \text{ N/mm}^2$

$$\therefore f_{250} = \frac{281250}{250^2} + 0.5$$

$$= \underline{5 \text{ N/mm}^2}$$

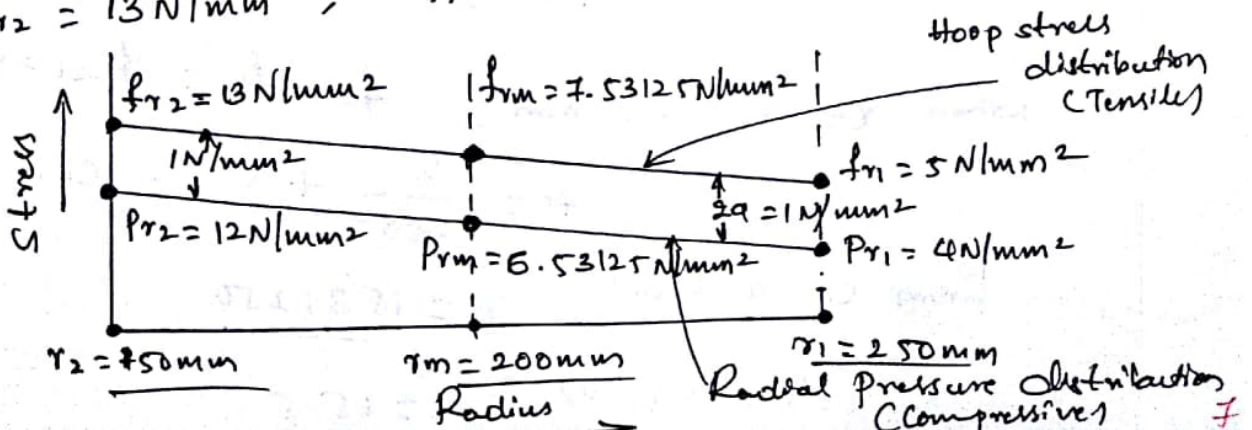
when  $x = r_m = 200 \text{ mm}$ ,  $f_x = f_n = f_{200}$

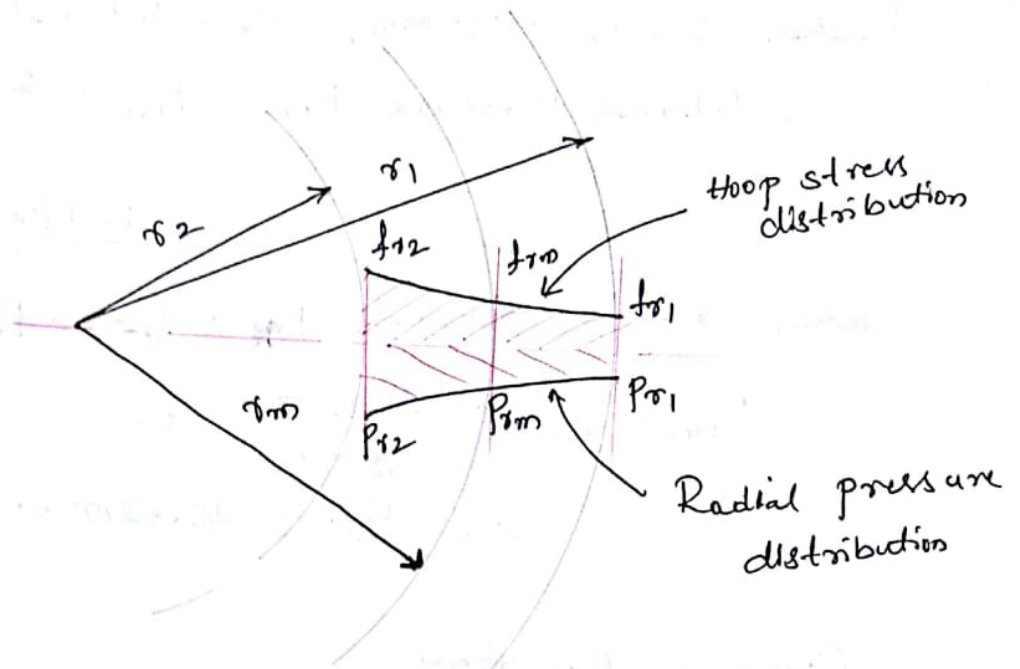
$$\therefore f_{200} = \frac{281250}{200^2} + 0.5$$

$$= \underline{7.53125 \text{ N/mm}^2}$$

$$P_{r_2} = 12 \text{ N/mm}^2 ; \quad P_{r_m} = 6.53125 \text{ N/mm}^2 \quad P_{r_1} = 4 \text{ N/mm}^2$$

$$f_{r_2} = 13 \text{ N/mm}^2 ; \quad f_{r_m} = 7.53125 \text{ N/mm}^2 \quad f_{r_1} = 5 \text{ N/mm}^2$$





6. Data:  $d_2 = 500\text{mm}$  ;  $r_2 = 250\text{mm}$

$$P_i = 9\text{MPa} = P_{r2} = P_{250}$$

$$f_{r2} = f_{250} = 40\text{MPa} = 40\text{N/mm}^2$$

thickness,  $t = ?$

From Lamé's theorem

Radial pressure at any radius,  $P_x = \frac{b}{x^2} - a$

⊗ Circumferential stress at any radius,  $f_x = \frac{b}{x^2} + a$

When  $x = r_2 = 250\text{mm}$  ;  $P_x = P_{r2} = P_{250} = 9\text{MPa}$

$$9 = \frac{b}{250^2} - a \quad \text{--- (i)}$$

When  $x = r_2 = 250\text{mm}$  ;  $f_x = f_{r2} = f_{250} = 40\text{MPa}$

$$40 = \frac{b}{250^2} + a \quad \text{--- (ii)}$$

From (i) and (ii)

$$b = \underline{1531250}$$

From eqn (i)

$$a = \underline{\underline{15.5}}$$

$$\text{At } x = r_1, \quad P_{r_1} = 0$$

$$\therefore P_{r_1} = \frac{b}{r_1^2} - a$$

$$0 = \frac{1531250}{r_1^2} - 15.5$$

$$\frac{1531250}{r_1^2} = 15.5$$

$$r_1^2 = 98790.3226$$

$$\therefore r_1 = 314.309 \approx \underline{\underline{315 \text{ mm}}}$$

$$\text{At } x = r_1, \quad f_{r_1} = f_{315}$$

$$f_{315} = \frac{b}{315^2} + a$$

$$= \frac{1531250}{315^2} + 15.5$$

$$\therefore f_{315} = \underline{\underline{31 \text{ N/mm}^2}}$$

$$\text{Mean radius, } r_m = \frac{r_1 + r_2}{2} = \frac{315 + 250}{2} = \underline{\underline{282.5 \text{ mm}}}$$

$$\text{At } x = r_m; \quad P_x = P_{r_m} = P_{282.5}$$

$$P_{282.5} = \frac{1531250}{282.5^2} - 15.5$$

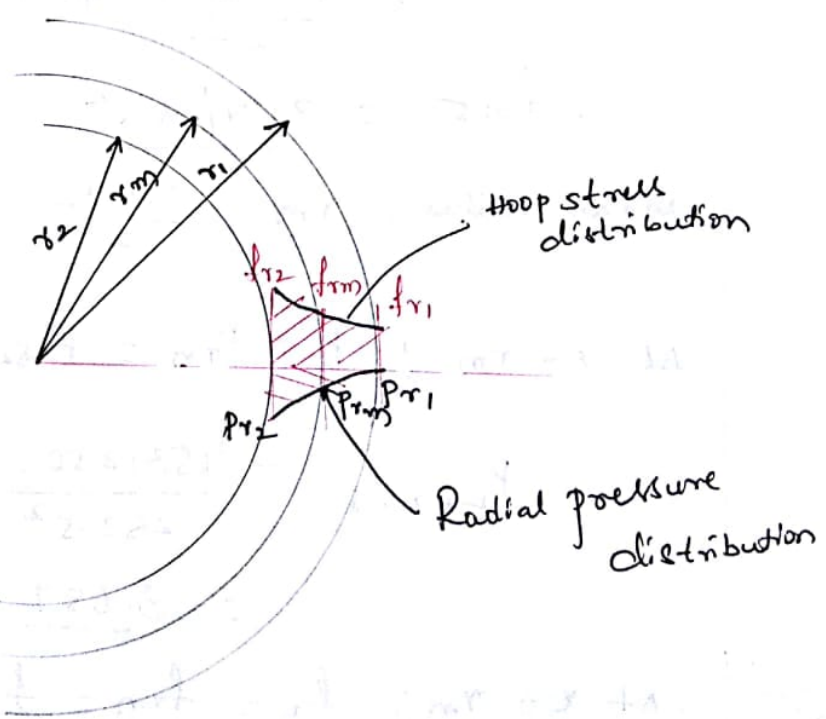
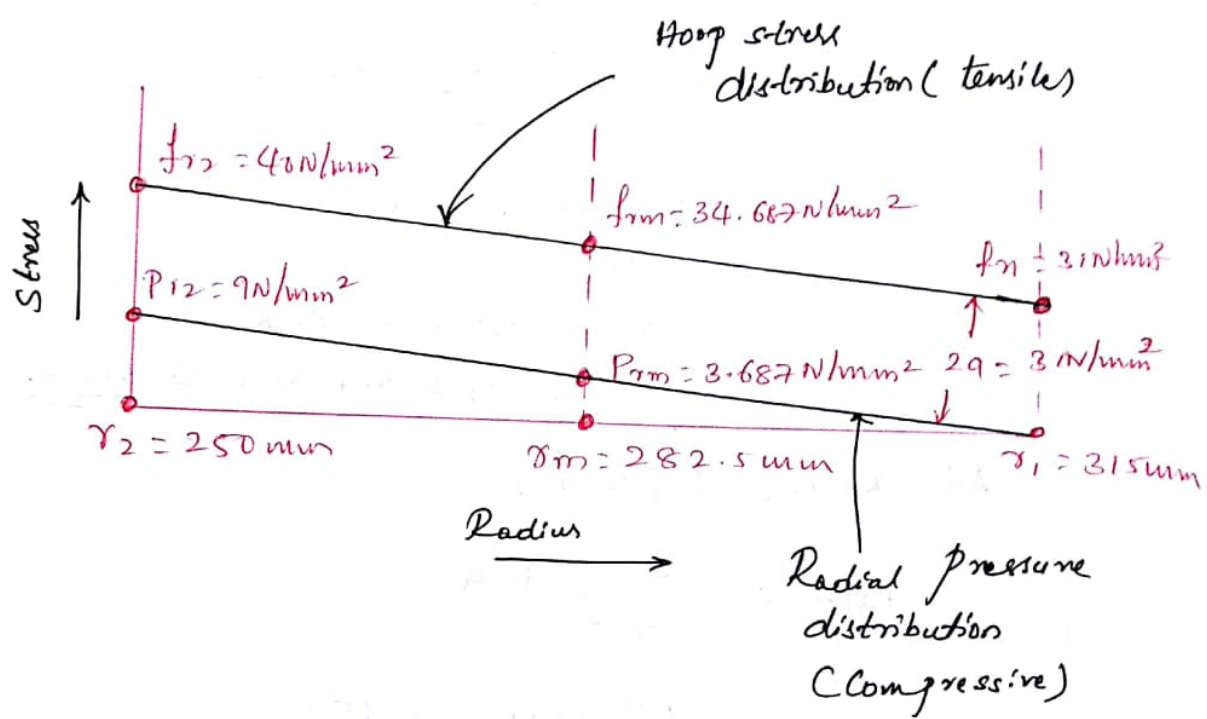
$$= \underline{\underline{3.687 \text{ N/mm}^2}}$$

$$\text{At } x = r_m; \quad f_x = f_{r_m} = f_{282.5}$$

$$f_{282.5} = \frac{1531250}{282.5^2} + 15.5$$

$$= \underline{\underline{34.687 \text{ N/mm}^2}}$$

$P_{r2} = 9 \text{ N/mm}^2$  ;  $P_{rm} = 3.687 \text{ N/mm}^2$  ;  $P_{r1} = 0$   
 $f_{r2} = 40 \text{ N/mm}^2$  ;  $f_{rm} = 34.687 \text{ N/mm}^2$  ;  $f_{r1} = 31 \text{ N/mm}^2$





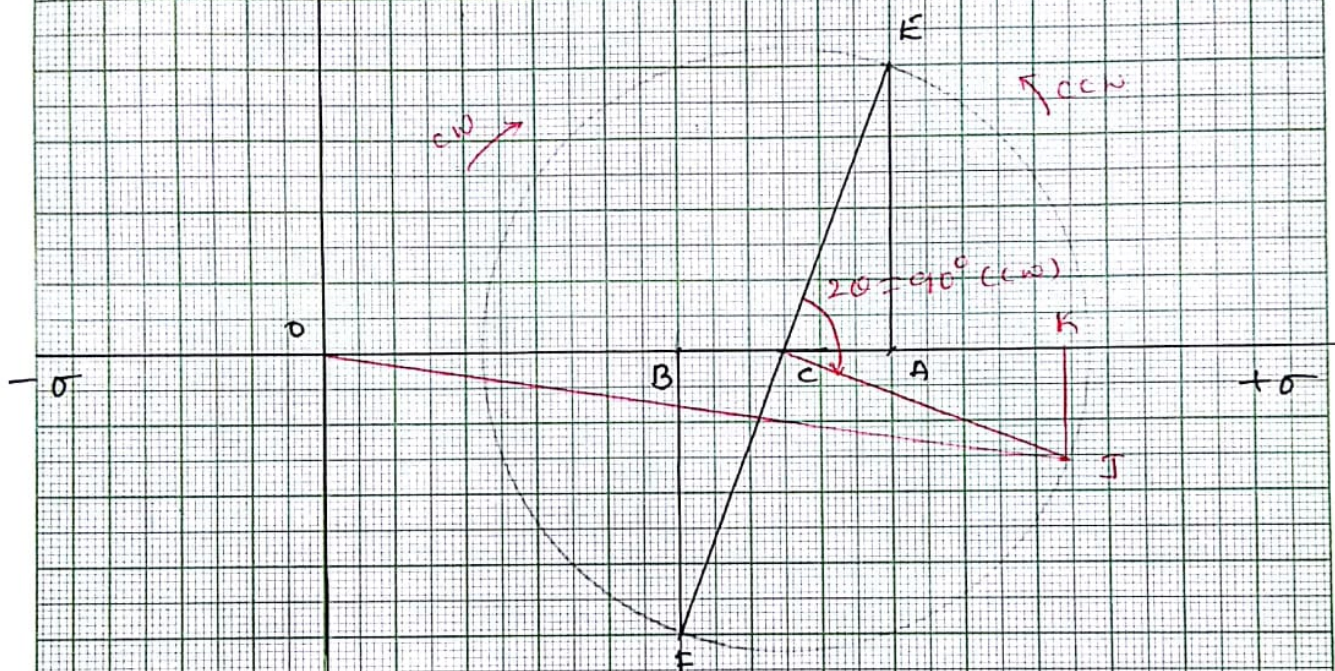
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$\tau_{xy}$

Scale:  $10 \text{ N/mm}^2 = 1 \text{ cm}$



Normal stress,  $\sigma_n = OK \times \text{Scale} = 10.4 \times 10 = 104 \text{ N/mm}^2$

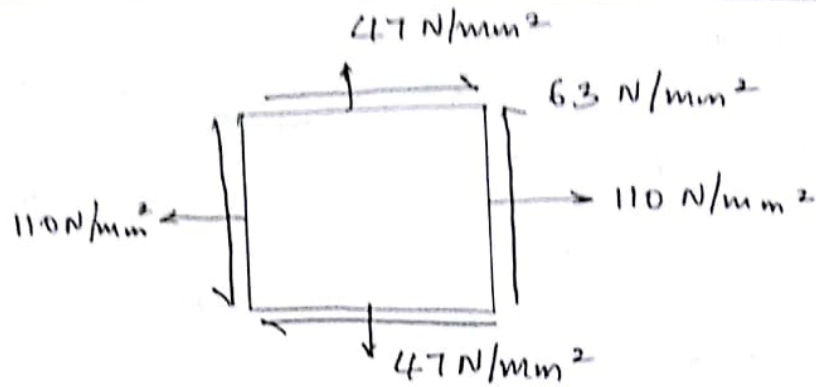
Tangential stress,  $\sigma_t = JK \times \text{Scale} = 1.6 \times 10 = 16 \text{ N/mm}^2$

$-\tau_{xy}$

Resultant stress,  $\sigma_R = OJ \times \text{Scale} = 10.5 \times 10 = 105 \text{ N/mm}^2$



8.



$$\sigma_x = +110 \text{ N/mm}^2, \quad \sigma_y = 47 \text{ N/mm}^2, \quad \tau_{xy} = +63 \text{ N/mm}^2$$

c) Direction and magnitude of principal stress

Direction of principal planes:

$$\tan 2\theta = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = \frac{2 \times 63}{110 - 47}$$

$$2\theta = \tan^{-1} \left( \frac{2 \times 63}{110 - 47} \right)$$

$$\therefore \theta = 31.71^\circ = \theta_1$$

$$\theta_2 = \theta_1 + 90^\circ = 121.71^\circ$$

Major principal stress

$$\begin{aligned} \sigma_1 &= \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{110 + 47}{2} + \sqrt{\left(\frac{110 - 47}{2}\right)^2 + 63^2} \end{aligned}$$

$$\therefore \sigma_1 = \underline{148.936 \text{ N/mm}^2} \approx \underline{149 \text{ N/mm}^2}$$

Minor principal stress

$$\begin{aligned} \sigma_2 &= \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \\ &= \frac{110 + 47}{2} - \sqrt{\left(\frac{110 - 47}{2}\right)^2 + 63^2} \end{aligned}$$

$$\therefore \sigma_2 = \underline{8.064 \text{ N/mm}^2}$$

(ii) Maximum Shear stress

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\left(\frac{110 - 47}{2}\right)^2 + 63^2}$$
$$= \underline{70.436 \text{ N/mm}^2}$$

or

$$\tau_{max} = \frac{\sigma_1 - \sigma_2}{2} = \frac{148.936 - 8.064}{2}$$

$$\therefore \tau_{max} = \underline{70.436 \text{ N/mm}^2}$$