CMR
INSTITUTE OF
TECHNOLOGY

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CMR

Internal Assesment Test - II

Sub:	ENGINEERING MATHEMATICS I							Code:	15MAT11
Date:	03/11/2016	Duration:	90 mins	Max Marks:	50	Sem:	I	Branch:	D,E,F,G,I,J,N
Answer All Questions									

		Marks	OBE CO	OBE RBT
1	Obtain the reduction formula of $\int Sin^n(x)dx$, Hence evaluate $\int_0^{\pi/2} Sin^n(x)dx$.	[8]	CO2	L3
2	If $u = x+y+z$, $uv = y+z$, $uvw = z$, then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.	[06]	CO2	L3
3(a)	If $u = Sin^{-1}\left(\frac{x+2y+3z}{x^8+y^8+z^8}\right)$ then find $xu_x + yu_y + zu_z$.	[03]	CO2	L3
(b)	Solve $\frac{dy}{dx} = xy^3 - xy$.	[03]	CO5	L3
4(a)	Find the largest Eigen value & corresponding Eigen vector of $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ using Rayleigh's power method. Perform 3 Iterations.	[03]	CO6	L3
(b)	If a substance cools from 370K to 330K in 10 Minutes. When the temperature of the surrounding air is 290K. Find the temperature of the substance after 40 Minutes.	[03]	CO5	L3
5(a)	Show the linear transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular hence find inverse transform.	[03]	CO6	L3
(b)	Solve the system of equations using Gauss Seidel method $3x+20y-z=-18$, $2x-3y+20z=25$, $20x+y-2z=17$. Perform 3 Iterations.	[03]	CO6	L3
6	Find the orthogonal trajectories of the family of cardioids $r = a\,(1+Cos(\theta))$, where 'a' is the parameter.	[06]	CO5	L3
7	Reduce the matrix $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$ to diagonal form.	[06]	CO6	L3
8(a)	Solve $(x^2+y^3+6x)dx + y^2x\,dy = 0$.	[03]	CO5	L3
(b)	Solve the system of equations using Gauss Elimination method $x+2y+z=3$, $2x+3y+3z=10$, $3x-y+3z=13$.	[03]	CO6	L3

Course Outcomes		PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1:	Apply nth derivate to obtain Taylor and Maclaurin's series of a given function. Evaluate the radius of curvature for cartesian, parametric polar and pedal equation.	2	3	-	-	-	-	-	1	1	1	-	-
CO2:	Apply partial derivatives to calculate rates of change of multivariate functions. Evaluate definite integrals.	2	3	-	-	-	-	-	1	1	1	-	-
CO3:	Analyze position, velocity, and acceleration in two or three dimensions using the calculus of vector valued functions.	2	3	-	-	-	-	-	1	1	1	-	-
CO4:	Evaluate curl and divergence of a vector valued functions which has various applications in electricity, magnetism and fluid flows.	2	3	-	-	-	-	-	1	1	1	-	-
CO5:	Solve first order ordinary Differential equations and model Newton's law of cooling.	2	3	-	-	-	-	-	1	1	1	-	-
CO6:	Evaluate matrices and determinants for solving systems of linear equations used in the different areas of Linear Algebra.	2	3	-	-	-	-	-	1	1	1	-	-

Cognitive level	KEYWORDS
L1	List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
L2	summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend
L3	Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover.
L4	Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer.
L5	Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize.

PO1 - *Engineering knowledge*; PO2 - *Problem analysis*; PO3 - *Design/development of solutions*; PO4 - *Conduct investigations of complex problems*; PO5 - *Modern tool usage*; PO6 - *The Engineer and society*; PO7- *Environment and sustainability*; PO8 – *Ethics*; PO9 - *Individual and team work*; PO10 - *Communication*; PO11 - *Project management and finance*; PO12 - *Life-long learning*

IAT-2 Solution

1) $I_n = \int \sin^n(x)\,dx$

$$= \int \underset{II}{\sin^{n-1}(x)\,dx}\ \underset{I}{\sin x\,dx} \qquad (1)$$

$$I_n = -\sin^{n-1}x\cos x + (n-1)\int \sin^{n-2}x\,(1-\sin^2 x)\,dx$$

$$I_n = \int \sin^n x\,dx = -\frac{\sin^{n-1}x\cos x}{n} + \frac{n-1}{n} I_{n-2} \qquad (3)$$

$$\int_0^{\frac{\pi}{2}} \sin^n x\,dx = -\frac{\sin^{n-1}x\cos x}{n}\Big]_0^{\frac{\pi}{2}} + \frac{n-1}{n} I_{n-2} \qquad (1)$$

$$I_{n-2} = \frac{n-3}{n-2} I_{n-4}$$

$$I_0 = \int_0^{\frac{\pi}{2}} \sin^0 x\,dx = \frac{\pi}{2} \qquad I_1 = \int_0^{\frac{\pi}{2}} \sin x\,dx = 1 \qquad (3)$$

$$\int_0^{\frac{\pi}{2}} \sin^n x\,dx = \frac{n-1}{n}\,\frac{n-2}{n-2}\cdots \begin{cases} 1 & n \text{ is odd} \\ \frac{\pi}{2} & n \text{ is even} \end{cases}$$

2) $$\frac{\partial(x\,y\,z)}{\partial(u\,v\,w)} = \begin{vmatrix} x_u & x_v & x_w \\ y_u & y_v & y_w \\ z_u & z_v & z_w \end{vmatrix} \qquad \begin{aligned} x &= u - uv \\ y &= uv - uvw \\ z &= uvw \end{aligned} \qquad (3)$$

$$J = \begin{vmatrix} 1-v & -u & 0 \\ v-vw & u-uw & -uv \\ vw & uw & uv \end{vmatrix} = u^2 v \qquad (3)$$

3 (a)

$$w = \sin u = \frac{x+2y+3z}{x^8+y^8+z^8} \qquad n=-7$$

w is homogeneous $\therefore$ euler theorem

$$x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z} = nw$$

$$x\frac{\partial(\sin u)}{\partial x} + y\frac{\partial}{\partial y}(\sin u) + z\frac{\partial(\sin u)}{\partial z} = -7\sin u$$ 2

$$x\left(+\cos u\frac{\partial u}{\partial x}\right) + y\left(+\cos u\frac{\partial u}{\partial y}\right) + z\left(+\cos u\frac{\partial u}{\partial y}\right) = -7\sin u$$

$$xu_x + yu_y + zu_z = \frac{-7\sin u}{+\cos u} = \cancel{7\cot u}\ -7\tan u$$ 1

(b)

$$\frac{dy}{dx} + xy = xy^3 \qquad \div y^3$$

$$\frac{1}{y^3}\frac{dy}{dx} + \frac{x}{y^2} = x \qquad \frac{1}{y^2} = t$$

$$\frac{dt}{dx} - 2xt = -2x \qquad If = e^{\int p\,dx} = e^{-x^2}$$

$$te^{-x^2} = \int e^{-x^2}(-2x)\,dx + c \qquad -x^2 = z \quad -2x\,dx = dz$$ 2

$$te^{-x^2} = \int e^z dz + c$$

$$te^{-x^2} = \cancel{ze^z - e^z + c} \qquad e^z + c$$

$$\frac{1}{y^2}e^{-x^2} = \cancel{-x^2e^{-x^2} - e^{-x^2} + c}$$ 1

$$\frac{1}{y^2}e^{-x^2} = e^{-x^2} + c$$

(4) (a)

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

let $x^{(0)} = [1, 0, 0]'$ be The inihal eigen vector.

$$Ax^{(0)} = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} = 2 \begin{bmatrix} 1 \\ 0 \\ 0.5 \end{bmatrix}$$

$$Ax^{(1)} = 2.5 \begin{bmatrix} 1 \\ 0 \\ 0.8 \end{bmatrix} \qquad Ax^{(2)} = 2.8 \begin{bmatrix} 1 \\ 0 \\ 0.93 \end{bmatrix}$$

$$Ax^{(3)} = 2.93 \begin{bmatrix} 1 \\ 0 \\ 0.98 \end{bmatrix}$$ - 3

(b)

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

when $t = 0$, $\theta = 370$ $\quad \therefore \underline{C = 80}$ 1

$\theta = 290 + 80e^{-kt}$

$$\underline{k = \frac{1}{10} \log 2}$$ - 1

at $t = 40$, $\underline{\underline{\theta = 295\,k}}$ - 1

(5)

(a) $$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & -2 \end{vmatrix} = -1 \neq 0$$ Regular — (1)

$$A^{-1} = \begin{pmatrix} 2 & -2 & -1 \\ -4 & 5 & 3 \\ 1 & -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2 & -2 & -1 \\ -4 & 5 & 3 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix}$$ — (2)

(b) $$x = \frac{1}{20}(17 - y + 2z)$$

$$y = \frac{1}{20}(-18 - 3x + z)$$

$$z = \frac{1}{20}(25 - 2x + 3y)$$

$x = 0,\ y = 0,\ z = 0$

$x^{(1)} = 0.85$, $y^{(1)} = -1.0275$ $z^{(1)} = 1.0109$ — (1)

$x^{(2)} = 1.0025$ $y^{(2)} = -0.9998$ $z^{(2)} = 0.9998$ — (1)

$x^{(3)} = 0.99997$ $y^{(3)} = -1.0000$ $z^{(3)} = 1.0000$ — (1)

$x = 1,\ y = -1,\ z = 1$

(6) (a) $r = a(1+\cos\theta)$

$$\log r = \log a + \log(1+\cos\theta)$$

$$\frac{1}{r}\frac{dr}{d\theta} = \frac{1}{1+\cos\theta} - \sin\theta \quad — (1)$$

$$\frac{1}{r} - r^2\frac{d\theta}{dr} = \frac{-\sin\theta}{1+\cos\theta}$$

$$\frac{-r\,d\theta}{dr} = \frac{-\sin\theta}{1+\cos\theta} \quad — (1)$$

$$\int \frac{(1+\cos\theta)\,d\theta}{\sin\theta} = \int \frac{dr}{r} + c$$

$$\int \frac{2\cos^2\theta/2}{2\sin\theta/2\,\cos\theta/2}\,d\theta = \int \frac{dr}{r} + c \quad — (1)$$

$$\int \cot\theta/2\,d\theta = \int \frac{dr}{r} + c \quad — (1)$$

$$2\log\sin\theta/2 = \log r + \log c$$

$$\log\sin^2\theta/2 - \log c = \log r \quad — (1)$$

$$\frac{\sin^2\theta/2}{c} = r$$

$$\boxed{r = \frac{1-\cos\theta}{c}} \quad — (1)$$

7. $A = \begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}$

Characteristic eqⁿ is $|A - \lambda I| = 0$

$\lambda^2 + 3\lambda - 10 = 0 \Rightarrow \lambda = 2, -5$ — (1)

Now consider $[A - \lambda I][X] = 0$

$(-19-\lambda)x + 7y = 0$

$-42x + (16-\lambda)y = 0$ — (1/2)

Case-(i) When $\lambda = 2$

$-21x + 7y = 0$

$-42x + 14y = 0$

both are same & gives $y = 3x$

Let $y = k$, $x = \frac{y}{3} = \frac{k}{3}$

Let $k = 3$

$\therefore X_1 = (1, 3)'$ — (1)

Case-(ii) When $\lambda = -5$

$-14x + 7y = 0$

$-42x + 21y = 0$

both are same & gives

$y = 2x$

Let $y = k_1$, $x = \frac{k_1}{2}$

take $k_1 = 2$

$\therefore X_2 = [1, 2]'$ — (1)

Modal matrix $P = [X_1\ X_2] = \begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

$|P| = -1$ — (1)

& $P^{-1} = -\begin{bmatrix} 2 & -1 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix}$ — (1)

$D = P^{-1}AP = \begin{bmatrix} -2 & 1 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} -19 & 7 \\ -42 & 16 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 2 \\ -15 & 5 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix}$ — (1)

8 (a) $$(x^2+y^3+6x)\,dx + y^2x\,dy = 0 \quad \text{—(1)}$$

$$M = x^2+y^3+6x \quad ; \quad N = y^2x$$

$$\frac{\partial M}{\partial y} = 3y^2 \quad ; \quad \frac{\partial N}{\partial x} = y^2$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x} = 2y^2 = \text{Near to } N \qquad \text{— (1)}$$

Now $$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = \frac{1}{y^2x} \times 2y^2 = \frac{2}{x} = f(x)$$

$$\therefore \quad I.F. = e^{\int f(x)\,dx} = e^{\int \frac{2}{x}\,dx} = x^2$$

Multiply I.F. in eqⁿ (1) — (1)

$$(x^4+y^3x^2+6x^3)\,dx + y^2x^3\,dy = 0$$

Now $$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} = 3x^2y^2$$

$\therefore$ The solution is

$$\int M\,dx + \int N(y)\,dy = c_1$$

$$\int (x^4+y^3x^2+6x^3)\,dx + 0 = c_1 \qquad \text{— (1)}$$

$$\frac{x^5}{5} + \frac{x^3y^3}{3} + \frac{3x^4}{2} = c_1 \Rightarrow 6x^5+10x^3y^3+45x^4 = 30c_1$$
$$= c$$

(b)
$$x+2y+z = 3$$
$$2x+3y+3z = 10$$
$$3x-y+3z = 13$$

The augmented matrix is $[A : B]$ — ①

$$= \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 2 & 3 & 3 & 10 \\ 3 & -1 & 3 & 13 \end{array}\right] \quad \begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 3R_1 \end{array}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & -7 & 0 & 4 \end{array}\right] \quad R_3 \rightarrow R_3 - 7R_2$$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 1 & 3 \\ 0 & -1 & 1 & 4 \\ 0 & 0 & -7 & -24 \end{array}\right]$$

$\therefore$ we have

$$x + 2y + z = 3$$
$$-y + z = 4 \quad \text{— ②}$$
$$-7z = -24$$

$$\Rightarrow z = \frac{24}{7}, \quad y = -\frac{4}{7}, \quad x = \frac{5}{7}$$