CMR INSTITUTE OF TECHNOLOGY

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Internal Assesment Test - II

Sub:	Engineering Mathematics III							Code:	15MAT31
Date:	2/11/2016	Duration:	90 mins	Max Marks:	50	Sem:	III	Branch:	ECE- 3A, 3C, 3D

Note: Answer any THREE full questions choosing one full question from each module.

		Marks	OBE CO	OBE RBT
	Module-3			
1(a)	Fit a curve of the form $y = ae^{bx}$ for the data and hence estimate y when x = 8	[07]	CO4	L2
(b)	Use the formula $r = \dfrac{\sigma_x^2 + \sigma_y^2 - \sigma_{x-y}^2}{2\sigma_x\sigma_y}$ to find the correlation coefficient between industrial production and export for the given data	[07]	CO4	L2
(c)	Using Regula-Falsi method, find the real root of the equation $xe^x - \cos x = 0$ correct to 4 places of decimals.	[06]	CO6	L3
	OR			
2(a)	The following table gives the data on rainfall and discharge in a certain river. Obtain the lines of regression of y on x.	[07]	CO4	L2
(b)	Fit a parabola of second degree for the given data	[07]	CO4	L2
(c)	Using Newton-Raphson method find a real root of x + log10 x = 3.375 near 2.9, correct to 3-decimal places.	[06]	CO6	L3
	Module-4			
3(a)	From the following table, estimate the number of students who have obtained marks between 40 and 45:	[07]	CO6	L2

Table for 1(a):

x	1	2	3	4	5	6
y	2.98	4.26	5.21	6.1	6.8	7.5

Table for 1(b):

Production	55	56	58	59	60	60	62
Export	35	38	38	39	44	43	45

Table for 2(a):

Rainfall x in inches	1.53	1.78	2.60	2.95	3.42
Discharge y 1000 cc	33.5	36.3	40.0	45.8	53.5

Table for 2(b):

x	1	2	3	4	5	6	7
y	25	28	33	39	46	43	49

Table for 3(a):

Marks :	30–40	40–50	50–60	60–70	70–80
Numberofstudents :	31	42	51	35	31

(b) Using Lagrange's formula, find the interpolating polynomial that approximates to the function described by the following table : [07] CO6 L3

x	0	1	2	3	4
$f(x)$	3	6	11	18	27

Hence find f(0.5) and f(3.1).

(c) Evaluate $\int_0^1 \frac{x}{1+x^2}\,dx$ by using Simpson's $(3/8)^{th}$ rule, dividing the interval into 3 equal parts. Hence find an approximate value of $\log\sqrt{2}$. [06] CO6 L3

OR

4(a) Find y when x = 0.26 using the appropriate interpolation formula for the following data: [07] CO6 L3

x :	0.10	0.15	0.20	0.25	0.30
y	0.1003	0.1511	0.2027	0.2553	0.3093

(b) Construct an interpolating polynomial for the data given below using Newton's divided difference formula. [07] CO6 L3

x :	2	4	5	6	8	10
$f(x)$	10	96	196	350	868	1746

(c) Evaluate $\int_4^{5.2} \log_e x\,dx$, using Weddle's rule by taking 7 ordinates. Hence compare with exact value. [06] CO6 L3

Module-4

5(a) Find the geodesics on a surface given that the arc length on the surface is, $S = \int_{x_1}^{x_2} \sqrt{x(1+y'^2)}\,dx$ [05] CO3 L3

(b) Solve the variational problem: [05] CO3 L3

$\delta \int_0^1 (x + y + y'^2)\,dx = 0$

under the condition $y(0) = 1$ and $y(1) = 2$.

OR

6(a) Solve the variational problem: [05] CO3 L3

$\delta \int_0^{\pi/2} (y^2 - y'^2)\,dx$ under the conditions $y(0) = 0$, $y(\pi/2) = 2$.

(b) Prove that the shortest distance between two points in a plane is along the straight line joining them. [05] CO3 L3

Course Outcomes		PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1:	Evaluate the real form of the Fourier series for standard periodic and finite waveforms as half-range series which has its applications in finding out the sum of infinite series using Dirichlet's conditions.	3	0	0	0	0	0	0	0	1	0	0	0
CO2:	Apply integral expressions for the forward and inverse Fourier transform to a range of non-periodic waveforms such as rectangular, unit-step, sinusoidal and exponential decay functions and solve second order difference equations using Z transform and inverse Z transform.	3	0	0	0	0	0	0	0	1	0	0	0
CO3:	Solve Brachistochrone, shortest distance, minimal surface area and hanging chain problems and to find the geodesics of known surfaces using Euler-Lagrange method.	3	0	0	0	0	0	0	0	0	0	0	0
CO4:	Estimate the strength of the relationship between the variables using correlation coefficients and express the relationship in the form if an equation using regression analysis.	3	0	0	0	0	0	0	0	1	0	0	0
CO5:	Evaluate line and surface integrals using Green's, Stoke's and Gauss divergence theorems which have its application in computing the amount of work done, area and volume.	3	0	0	0	0	0	0	0	0	0	0	0
CO6:	Apply numerical techniques to perform various mathematical tasks such as solving equations, interpolation, integration and curve fitting	3	0	0	0	0	0	0	0	1	0	0	0

Cognitive level	KEYWORDS
L1	List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
L2	summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend
L3	Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover.
L4	Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer.
L5	Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize.

PO1 - *Engineering knowledge*; PO2 - *Problem analysis*; PO3 - *Design/development of solutions*; PO4 - *Conduct investigations of complex problems*; PO5 - *Modern tool usage*; PO6 - *The Engineer and society*; PO7- *Environment and sustainability*; PO8 – *Ethics*; PO9 - *Individual and team work*; PO10 - *Communication*; PO11 - *Project management and finance*; PO12 - *Life-long learning*

IAT-II

15MAT31 solution

Q1 a) Consider $y = ae^{bx}$

$\therefore \quad \log_e y = \log_e a + bx$

Let $\log_e y = Y, \quad \log_e a = A$

$\Rightarrow \quad Y = A + bx$, which is a straight line.

The associated normal equations are as follows:

$$\Sigma Y = nA + b\Sigma x$$

$$\Sigma xY = A\Sigma x + b\Sigma x^2 \quad (n = 6) \qquad \text{——— } 2$$

The relevant table is as follows:

x	y	$Y = \log_e y$	xY	x^2
1	2.98	1.09192	1.09192	1
2	4.26	1.449269	2.898538	4
3	5.21	1.650579	4.951737	9
4	6.1	1.808288	7.233152	16
5	6.8	1.916923	9.584615	25
6	7.5	2.014903	12.089418	36
21		9.931882	37.84938	91

——— 2

The normal equations become

$$6A + 21b = 9.931882$$

$$21A + 91b = 37.84938$$

On solving we have $A = 1.03775$, $b = 0.176445$

$\log_e a = A \Rightarrow \quad a = e^A = 2.82286$

Thus the required curve of fit is

$$y = (2.82286)\, e^{0.17645 x} \qquad 2$$

when $x = 8$ $\quad y = 11.8649 \qquad 1$

~~b Here $n = 7$~~

~~x $\quad y$ $\quad Z = x - y$ $\quad X = x - \bar{x}$ $\quad Y = y - \bar{y}$ $\quad Z^2$ $\quad X^2$ $\quad Y^2$~~

~~55~~

~~56~~

b Here $n = 7$

x	y	$Z = x - y$	x^2	y^2	z^2
55	35	20	3025	1225	400
56	38	18	3136	1444	324
58	38	20	3364	1444	400
59	39	20	3481	1521	400
60	44	16	3600	1936	256
60	43	17	3600	1849	289
62	45	17	3844	2025	289
410	282	128	24050	11444	2358

2

$\bar{x} = \frac{410}{7} = 58.57, \quad \bar{y} = \frac{282}{7} = 40.28, \quad \bar{z} = \frac{128}{7} = 18.28$

$\sigma_x^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2 = 5.2694$

$$\sigma_y^2 = \frac{\Sigma y^2}{n} - (\bar{y})^2 = 12.3787$$

$$\sigma_z^2 = \frac{\Sigma z^2}{n} - (\bar{z})^2 = 2.6987 \qquad — \quad 3$$

Then $\quad r = \dfrac{5.2694 + 12.3787 - 2.6987}{2 \times 2.2955 \times 3.5183}$

$$= 0.92 \qquad — \quad 2$$

$\underline{\underline{c}}$ The root lies in $(0.5, 0.6)$ — 2

$\underline{I^{st}\ \text{iteration}}$ Let $x_0 = 0.5, \quad x_1 = 0.6$

$$x_2 = x_1 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0)$$

$$= 0.5166 \qquad — \quad 1$$

$\underline{II^{nd}\ \text{iteration}}$: $f(0.5166) < 0$

$\therefore$ the root lies in $(0.5166, 0.6)$

$x_0 = 0.5166, \quad x_1 = 0.6$

$$x_3 = 0.5177 \qquad — \quad 1$$

$\underline{III^{rd}\ \text{iteration}}$: $f(0.5177) < 0$

$\therefore$ the root lies in $(0.5177, 0.6)$

$x_0 = 0.5177, \quad x_1 = 0.6$

$$x_4 = 0.5178. \qquad — \quad 1$$

Thus the required root is 0.5178. — 1

Q2 a) Here $n = 5$

x	y	X	Y	X^2	Y^2	XY
1.53	33.5	-0.92	-8.32	0.84	69.22	7.65
1.78	36.3	-0.67	-5.52	0.44	30.47	3.69
2.60	40	0.15	-1.82	0.02	3.31	-0.27
2.95	45.8	0.5	3.98	0.25	15.84	1.99
3.42	53.5	0.97	11.68	0.94	136.42	11.32
12.28	209.1	0.03	0	2.45	255.26	24.38

— 3

$\bar{x} = 2.45$, $\bar{y} = 41.82$

$$r = \frac{\Sigma XY}{\sqrt{\Sigma X^2 \, \Sigma Y^2}} = 0.9748$$ — 2

Regression line of y on x

$$y - \bar{y} = \frac{r\sigma_y}{\sigma_x}(x - \bar{x})$$

$$y = 9.9429x - 17.4598.$$ — 2

b) Let $y = ax^2 + bx + c$ be the equation for the second degree parabola. Then the normal eqns are.

$$\Sigma y = a\Sigma x^2 + b\Sigma x + cn$$

$$\Sigma xy = a\Sigma x^3 + b\Sigma x^2 + c\Sigma x$$

$$\Sigma x^2 y = a\Sigma x^4 + b\Sigma x^3 + c\Sigma x^2$$ — 2

Here $n = 7$

x	y	xy	x^2	x^2y	x^3	x^4
1	25	25	1	25	1	1
2	28	56	4	112	8	16
3	33	99	9	297	27	81
4	39	156	16	624	64	256
5	46	230	25	1150	125	625
6	43	258	36	1548	216	1296
7	49	343	49	2401	343	2401
28	263	1167	140	6157	784	4676

—— 3

Then the normal eqns. beomes

$$140a + 28b + 7c = 263$$

$$784a + 140b + 28c = 1167$$

$$4676a + 784b + 140c = 6157$$

On solving we get,

$a = -0.273$, $b = 6.297$, $c = 17.85$

$\therefore y = -0.273x^2 + 6.297x + 17.85$ —— 2

c) Here $f(x) = x + \log_{10} x - 3.375$

$\therefore f'(x) = 1 + \frac{1}{x}\log_{10} e$ —— 2

Also, $x_0 = 2.9$

$$\therefore\ x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.91096$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.91096$$

Thus, the required root is 2.91096. — 4

Q 3 a) First we prepare cumulative frequency table as follows:

Marks below (x):	40	50	60	70	80
No. of stud (y):	31	73	124	159	190

Difference table is — 1

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
40	$\boxed{31}$				
		$\boxed{42}$			
50	73		$\boxed{9}$		
		51		$\boxed{-25}$	
60	124		-16		$\boxed{37}$
		35		12	
70	159		-4		
		31			
80	190				

— 3

Now, $x_0 = 40$, $h = 10$, $p = \dfrac{x - x_0}{h} = \dfrac{45 - 40}{h} = 0.5$

forward intapolation formulae is

$$y = f(x) = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \cdots$$ — 1

$$\therefore\ f(45) = 47.86 \approx 48$$

Number of students obtained marks between 40 and 45

$= f(45) - f(40) = 48 - 31 = 17.$ — 2

b Here $\omega(x) = x(x-1)(x-2)(x-3)(x-4)$

$$\omega'(0) = 24$$

$$\omega'(x_1) = (x_1 - x_0)(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)$$
$$= -1 \times -2 \times -3$$
$$= -6$$

$$\omega'(x_2) = 2 \times 1 \times -1 \times -2 = 4$$

$$\omega'(x_3) = 3 \cdot 2 \cdot 1 \cdot -1 = -6$$

$$\omega'(x_4) = 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

$$y = f(x) = \omega(x)\left[\frac{y_0}{(x-x_0)\,\omega'(x_0)} + \cdots + \frac{y_4}{(x-x_4)\,\omega'(x_4)}\right]$$

$$= \frac{(x-1)(x-2)(x-3)(x-4)}{8} + \frac{6}{6}x(x-2)(x-3)(x-4)$$
$$+ \frac{11}{4}x(x-1)(x-3)(x-4) - 3x(x-1)(x-2)(x-4)$$
$$+ \frac{\cancel{27}9}{\cancel{24}8}x(x-1)(x-2)(x-3)$$

$$= \frac{(x-1)(x-2)(x-3)}{8}(x-4+9x) - x(x-2)(x-4)\left[x-3+3x-6\right]$$
$$+ \frac{22}{8}x(x-1)(x-3)(x-4)$$

$$= \frac{(x-1)(x-3)}{8}\left[(x-2)(10x-4) + 22x(x-4)\right] - x(x-2)(x-4)(4x-9)$$

$$f(0.5) = 4.25$$

— 3 — 4

c) Here $n = 3$, $h = \frac{1-0}{n} = \frac{1}{3}$

x	0	$1/3$	$2/3$	1
$y = \frac{x}{1+x^2}$	0	0.3	0.46154	0.5

$$I = \int_a^b y\,dx = \frac{3h}{8}\left[y_0 + y_3 + 3(y_1 + y_2)\right]$$

c) Here $n = 4$, $h = 0.25$

x	0	0.25	0.5	0.75	1
$y = \frac{x}{1+x^2}$	0	0.23529	0.4	0.48	0.5
	y_0	y_1	y_2	y_3	y_4

$I =$

$1+x^2 = t$

$2x\,d$

$\frac{1}{2}$

c) Here $n = 3$, $h = 1/3$

x	0	1/3	2/3	1
$y = \frac{x}{1+x^2}$	0	0.3	0.46154	0.5
	y_0	y_1	y_2	y_3

— 2

$$I = \int_a^b y\,dx = \frac{3h}{8}\left[y_0 + y_3 + 3(y_1 + y_2)\right]$$

$$= \frac{1}{8}\left[0.5 + 3(0.76154)\right]$$

$$= 0.3480.$$ — 2

Now $I = \int_0^1 \frac{x}{1+x^2}\,dx = \frac{1}{2}\log(1+x^2)\Big|_0^1$

$$= \frac{1}{2}\log 2 = 0.3484$$

$$\Rightarrow \log_e 2 = 0.69612$$ — 2

Q4 a) Here the value $x = 0.26$ is near the end ~~of~~ value 0.30

<u>Difference table:</u>

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
0.10	0.1003				
		0.0508			
0.15	0.1511		0.008		
		0.0516		0.0002	
0.20	0.2027		0.001		0.0002
		0.0526		0.0004	
0.25	0.2553		0.0014		
		0.0540			
0.30	0.3093				

— 3

Newton Backward interpolation formula is

$$y = y_n + p \nabla y_n + \frac{p(p+1)}{2!} \nabla^2 y_n + \cdots$$ — 2

$$p = \frac{x - x_n}{h} = -0.8$$

$f(0.26) = \underline{0.26602}$. — 2

b) <u>Divided difference table</u>:

x	$f(x)$	1st diff	2nd diff	3rd diff
2	10	43		
			19	
				2
4	96	100		
			27	
5	196	154		2
			35	
6	350	259		
				2
		439	45	
8	868			
10	1746			

— 3

The Newton divided difference formula is

$$y = f(x) = f(x_0) + (x - x_0) f(x_0, x_1) + \cdots$$

$$= 2x^3 - 3x^2 + 5x - 4.$$ — 4

c) Here $n = 6$

x	4	4.2	4.4	4.6	4.8	5	5.2
$y = \log_e x$	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487
	y_0	y_1	y_2	y_3	y_4	y_5	y_6

— 4

$$I = \int_a^b y\,dx = \frac{3h}{10}\left[y_0 + 5y_1 + y_2 + 6y_3 + y_4 + 5y_5 + y_6\right]$$

$= 1.8279.$ — 2

Q 5 a) Here $f(x, y, y') = \sqrt{x(1+y'^2)}$

f is independent of y

$\Rightarrow$ Euler-Lagrange eqn: is

$$\frac{\partial f}{\partial y'} = K \quad \text{— 2}$$

$$\frac{1}{2}\sqrt{x}\,\frac{2y'}{\sqrt{1+y'^2}} = K$$

$$x y'^2 = K^2(1+y'^2)$$

$\Rightarrow y'^2(x-K^2) = K^2 \Rightarrow y' = \frac{K}{\sqrt{x-K^2}}$ — 2

$\Rightarrow dy = \frac{K\,dx}{\sqrt{x-K^2}}$; let $x - K^2 = t$
$dx = dt$

$dy = \frac{K\,dt}{\sqrt{t}}$, Integrating we get

$y = 2K(x-K^2)^{1/2} + C$

or, $(y-C)^2 = 4K^2(x-K^2) = 4a(x-a)$, $a = K^2$

$(y-c)^2 = 4a(x-a)$, is the required geodesic 1

b) $f(x, y, y') = x + y + y'^2$

Euler - Lagrange eqns is

$$\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0 \quad — 2$$

$$1 - \frac{d}{dx}(2y') = 0$$

$$1 - 2y'' = 0 \Rightarrow y'' = \frac{1}{2}$$

Integrating we get

$$y' = \frac{1}{2}x + a$$

again integrating

$$y = \frac{x^2}{4} + ax + b \quad — 2$$

Now $y(0) = 1 \Rightarrow b = 1$

$y(1) = 2 \Rightarrow 2 = \frac{1}{4} + a + 1 \Rightarrow a = 3/4$

$$\therefore y = \frac{x^2 + 3x}{4} + 1 \quad — 1$$

Q 6 a $\quad f(x, y, y') = y^2 - y'^2$, independent of x

$$f - y'\frac{\partial f}{\partial y'} = K$$

$— 2$

$$y^2 - y'^2 - y'(-2y') = K$$

$$y^2 + y'^2 = K \Rightarrow y' = \sqrt{K - y^2}$$

$\frac{dy}{\sqrt{K-y^2}} = dx$, let $y = \sqrt{K}\sin\theta$

$dy = \sqrt{K}\cos\theta \, d\theta$

$$\Rightarrow \frac{\cancel{\sqrt{K}\cos\theta}\, d\theta}{\cancel{\sqrt{K}\cos\theta}} = dx$$

$$\theta = x + c$$

$$\Rightarrow \quad \sin^{-1}\frac{y}{\sqrt{k}} = x + c$$

Or, $\quad y = \sqrt{k}\sin(x+c)$ — 2

Now, $\quad y(0) = 0 \Rightarrow \sin c = 0 \Rightarrow c = 0$

$$y = \sqrt{k}\sin x$$

$$y(\pi/2) = 2 \Rightarrow 2 = \sqrt{k}\,1 \Rightarrow \sqrt{k} = 2$$

$$\therefore \boxed{y = 2\sin x}$$ — 1

b) Let $A(x_1, y_1)$, $B(x_2, y_2)$ be two points in a plane and $y(x)$ is a curve joining A and B.

$$I[y] = \int_{x_1}^{x_2} \sqrt{1+y'^2}\,dx$$ — 1

$$f(x, y, y') = \sqrt{1+y'^2} \quad \text{ind. of } y$$

$$\frac{\partial f}{\partial y'} = k \qquad \text{E-L eqn}$$ — 2

$$y' = k\sqrt{1+y'^2} \Rightarrow y'^2 = k^2(1+y'^2)$$

$$\Rightarrow \quad y'^2(1-k^2) = k^2 \Rightarrow y' = \frac{k}{\sqrt{1+k^2}} = m$$

$$\Rightarrow \quad dy = m\,dx$$ — 2

$\Rightarrow \quad y = mx + c$, which is a st. line.