SOLUTION AND SCHEME OF EVALUATION

IAT-II

Elements of civil Engineering and Engineering Mechanics

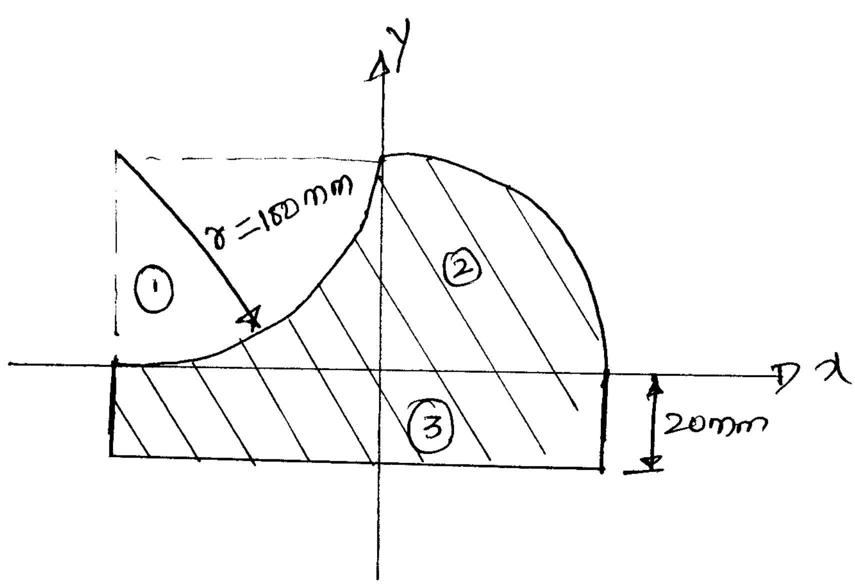
Sub code : 15CIV13

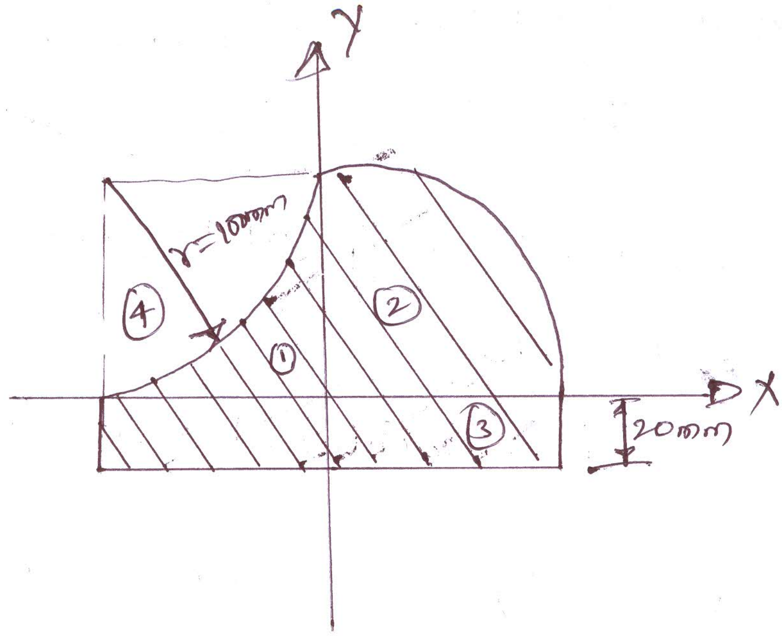
1 (a) Differentiate between centroid and centre of gravity

01×02 = 02

centre of gravity	centroid.
(i) Applicable to bodies with weight	(i) Applies to plane areas
(ii) Point through which the resultant weight of body acts	(ii) Point moment of area is zero about the axis.

1 (b) Locate the position of centroid of the shaded area shown in fig.

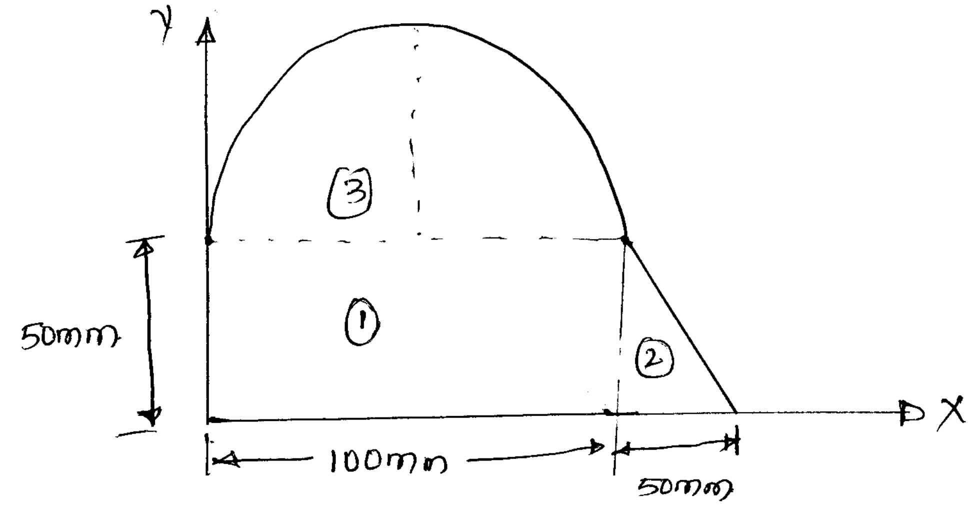




component	Area (mm^2)	$\bar{x}$ mm	$\bar{y}$ mm	$a\bar{x}$ mm^3	$a\bar{y}$ mm^3
① square	100×100 $= 10000$	-50	$\frac{100}{2}$ $= 50$	-500000	$+500000$
② ~~semicircle~~ quarter	$\frac{\pi \times 100^2}{4}$ $= 7853.98$	$\frac{4 \times 100}{3\pi}$ $= 42.44$	$\frac{4 \times 100}{3\pi}$ $= 42.44$	333322.98	333322.98
③ Rectangle	20×200 $= 4000$	0	$-\frac{20}{2}$ $= -10$	0	-400000
④ quarter circle	-7853.98	$-\left[100 - \frac{4r}{3\pi}\right]$ $= -57.56$	$+57.56$	452075.088	-452075.088
02	14000		02	285398.068 ~~162156.932~~	341247.892 ~~18752.108~~ 02

$\bar{X} = 20.38$ mm 02 $\bar{Y} = 24.37$ mm

2a) Compute the moment of inertia of the shaded area shown in fig about its horizontal centroidal axis and also find the radius of gyration about the same axis



component	area mm^2	$\bar{y}$ (mm)	$a\bar{y}$ (mm^3)	I_{xc} (mm^4)	d_x (mm)
① Rectangle	50×100 $= 5000$	$\frac{50}{2}$ $= 25$	125000	$\frac{100 \times 50^3}{12}$	$41.81 - 25$ $= 16.81$
② Triangle	$\frac{1}{2} \times 50 \times 50$ $= 1250$	$\frac{1}{3} \times 50$ $= 16.67$	20833.34	$\frac{50 \times 50^3}{36}$	$41.81 - 16.67$ $= 25.14$
③ Semi-circle	$\frac{\pi \times 50^2}{2}$ $= 3927$	$50 + \left(\frac{4 \times 50}{3\pi}\right)$ $= 71.22$	279683.53	0.11×50^4	$41.81 - 71.22$ $= -29.41$
	10177		425516.87		

05

$$\bar{y} = 41.81 mm$$

$$I_{xx} = \left[\frac{100 \times 50^3}{12} + (5000)(16.81)^2\right] + \left[\frac{50 \times 50^3}{36} + (1250)(25.14)^2\right]$$

$$+ \left[0.11 \times 50^4 + (3927)(-29.41)^2\right]$$

05

$$I_{xx} = 75 \times 10^5 mm^4$$

3a laws of friction :-

$01 \times 04 = 04$

(1) The frictional force is always tangential to the contact surface and acts opposite to the direction of impending motion.

(2) The value of frictional force (F) increases as the applied disturbing force increases till it reaches the limiting value Fmax. At this limiting stage the body is on the verge of motion.

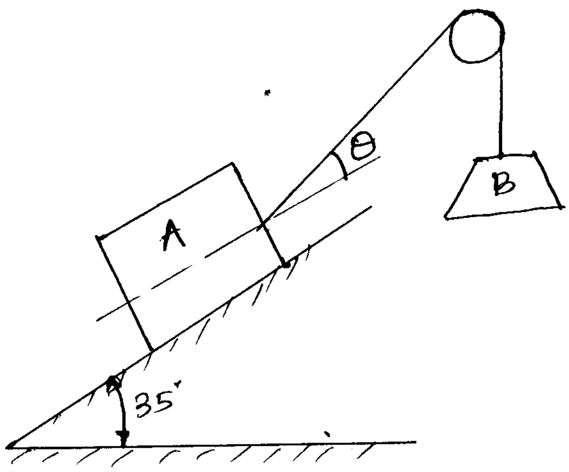
(3) The frictional force F generated between the two rubbing surfaces is independent of the area of contact.

(4) The ratio of limiting frictional force Fmax and the normal reaction N is constant and is reffered as coefficient of static friction (μs)

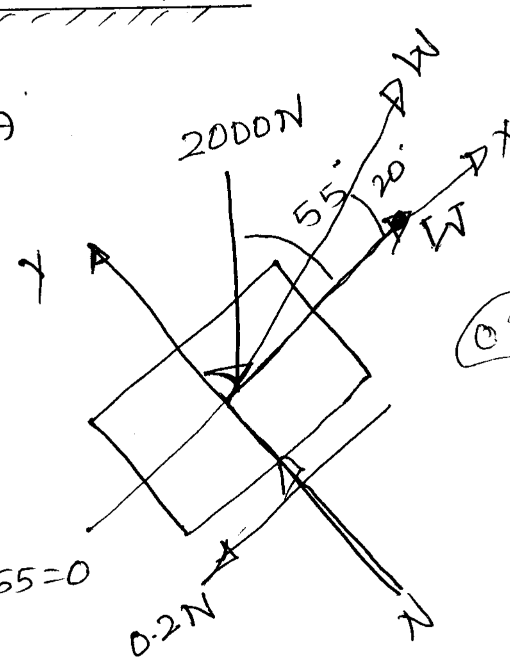
(5) For bodies in motion, frictional force developed (Fk) is less than the limiting frictional force (Fmax). The ratio of Fk and Normal reaction N is a constant and is reffered as coefficient of kinetic friction (μk)

3(b) Block 'A' of weight 2000N is kept on a plane inclined at 35°. It is connected to weight 'B' by an inextensible string passing over a smooth pulley. Determine weight of 'B' just moves down. Take $\theta = 20°$, $\mu = 0.2$.

Soln:



FBD of block 'A'



(02)

$\Sigma F_x = 0$

$-0.2N + W\cos 20 - 2000\cos 55 = 0$

$0.2N - 0.94W = -1147.152 \longrightarrow ①$ (02)

$\Sigma F_y = 0$

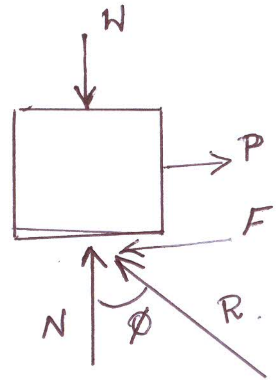
$N + 0.342W = 1638.30 \longrightarrow ②$

$W = \cancel{9240.21}\ 1462.52\ N$ (02)

4 a) **Angle of Friction**

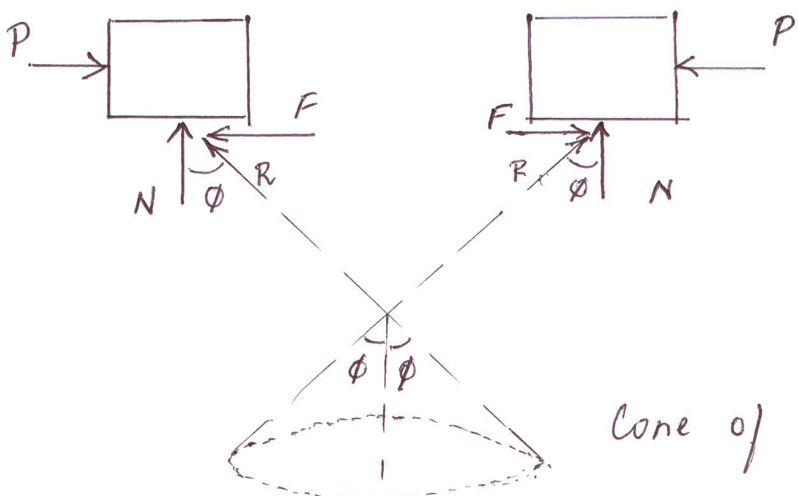
The angle of friction for two contacting surfaces is the angle between the resultant 'R' (of friction force F and the normal reaction 'N') and the normal reaction N. It is denoted by 'ϕ'.

1.5



Cone of Friction

The combination of resultant 'R' of frictional force F and normal reaction N obtained by applying forces in opposite directions successively form a right circular cone of angle 2ϕ, known as the cone of friction, as shown in figure.



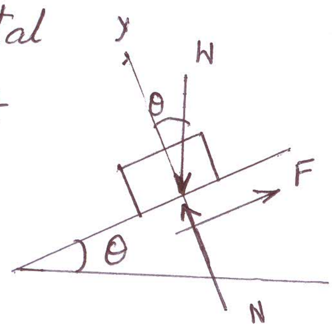
1.5

Cone of friction.

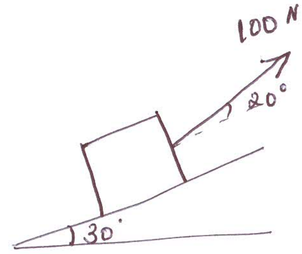
Angle of Repose (θ) :

01

When a plane is inclined to the horizontal by a certain angle, the body placed on it will remain at rest up to a certain angle of inclination, beyond which the body just begins to move. This maximum angle made by the inclined plane with the horizontal, when the body placed on that ~~placed~~ plane is just at the point of sliding down the plane, is known as the angle of repose.

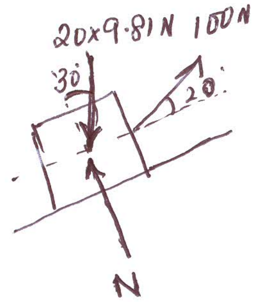


4b). Determine the state of 20kg block shown in ~~fig~~ Fig 4b, i.e., whether it is in equilibrium or not ? Find the magnitude and direction of the frictional force. Take $\mu_s = 0.2$ and $\mu_k = 0.15$.



$\Sigma F_x = 100 \cos 20^\circ - 20 \times 9.81 \sin 30^\circ$

$\Sigma F_x = -4.13 = 4.13 (\leftarrow) N$ 02



$\Sigma F_y = -20 \times 9.81 \cos 30^\circ + N + 100 \sin 20^\circ = 0$

$N = 135.71 N$

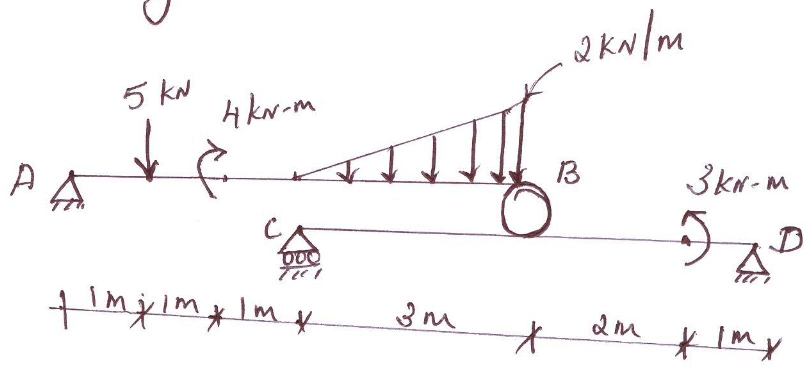
$F_{r\,max} = \mu_s \times 135.71 = 0.2 \times 135.71$ 02

$F_{r\,max} = 27.14 N (\rightarrow)$

$\Sigma F_x < F_{r\,max}$ $\therefore$ It is in equilibrium.

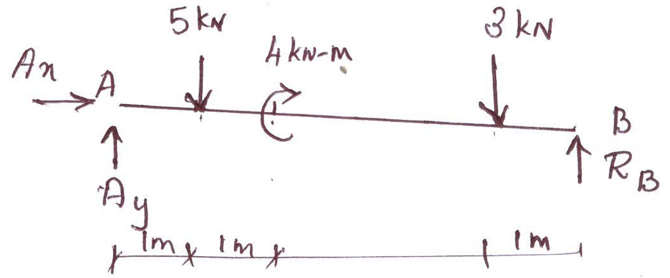
$\therefore F_r = F_{r\,max} (\rightarrow)$ 02

5(a) Determine the reactions at the beam loaded as shown in Fig 5.a.



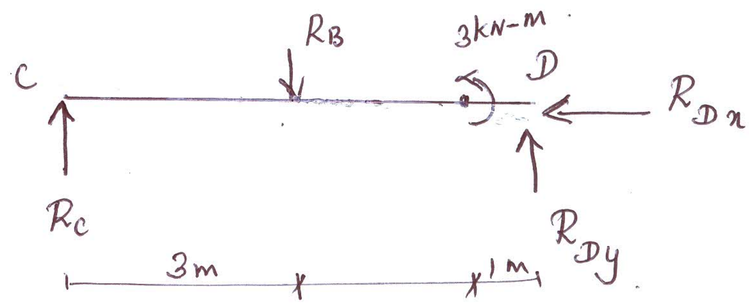
Soln

FBD of AB



02

FBD of CD.



For AB, $\Sigma F_x = 0$

$A_x = 0$

$\Sigma M_A = 0$: $-5\times1 - 4 - 3\times5 + R_B\times6 = 0$ 02

$\underline{R_B = 4\text{ kN}}$

$\Sigma F_y = 0$: $A_y - 5 - 3 + R_B = 0$ 02

$\underline{A_y = 4\text{ kN}}$

For CD, $\Sigma F_x = 0$ $\underline{R_{Dx} = 0}$

$\Sigma M_C = 0$:

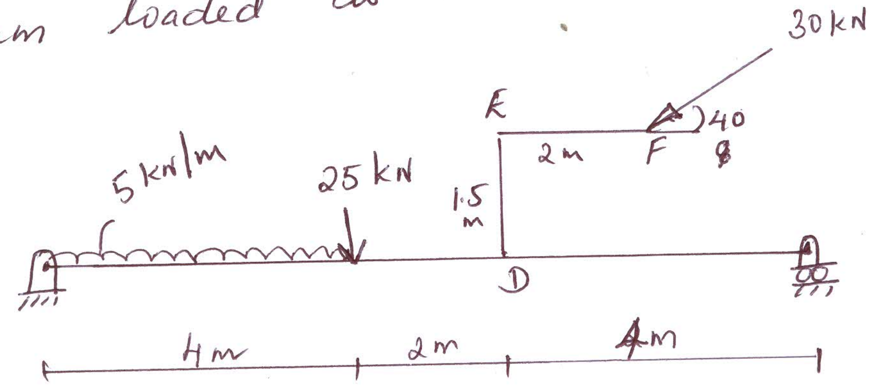
$$-R_B \times 3 + 3 - R_{Dy} \times 6 = 0$$ 02

$$\underline{R_{Dy} = 1.5 \text{ kN}}$$

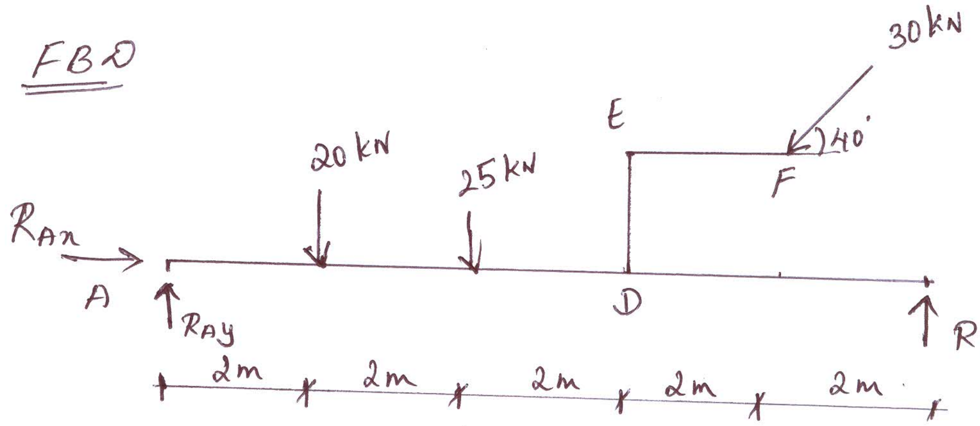
$\Sigma F_y = 0$: $\quad R_C - R_B + R_{Dy} = 0$

$$\underline{R_C = 2.5 \text{ kN}}$$ 02

6 a) Determine the reactions at the supports for the beam loaded as shown in Fig 6a.



soln FBD



$\Sigma H = 0$

$$R_{Ax} - 30 \cos 40 = 0 \qquad \underline{R_{Ax} = 22.98 \text{ kN}}$$ 02

$\Sigma M_A = 0$

$$20 \times 2 + 25 \times 4 + 30 \sin 40 \times 8 - 30 \cos 40 \times 1.5 - R_{By} \times 10 = 0$$

$$\underline{R_{By} = 25.98 \text{ kN}}$$ 05

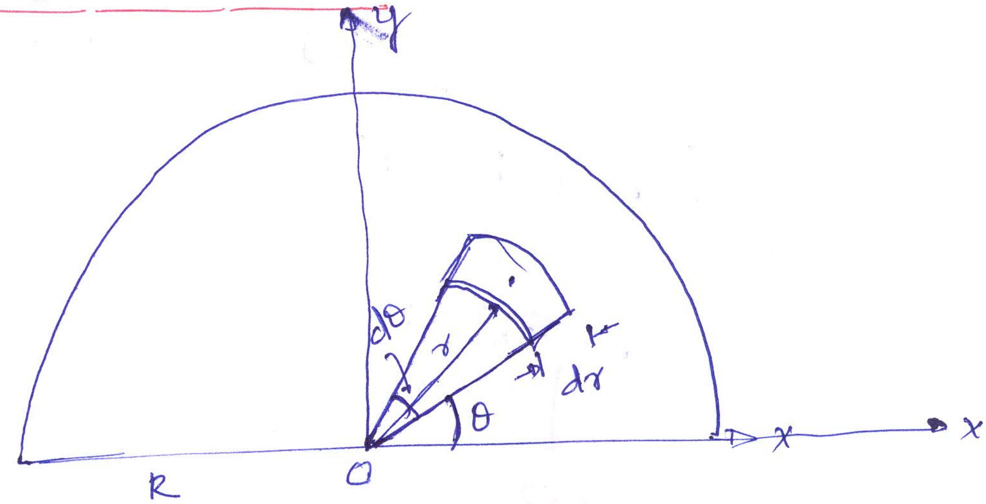
$$\Sigma V = 0$$

$$R_{Ay} - 20 - 25 - 30 \sin 40 + R_{By} = 0$$

$$R_{Ay} = 38.30 \text{ kN}$$

03

(19) Centroid of a semicircle :-



Consider the semicircle of radius 'R' as shown in figure. Due to symmetry centroid lies on y-axis. Let its distance from diametric axis be $\bar{y}$.

To find $\bar{y}$ consider an element at a distance 'r' from the centre 'O' of the semicircle, radial width being 'dr' and bound by radii at θ and 'θ + dθ'

The elemental area may be treated as a rectangle of sides $r\,d\theta$ and dr.

$\therefore$ Area of element $= r\,d\theta \cdot dr$

Its moment about x-axis is given by

$$r\,d\theta \cdot dr \cdot r\sin\theta$$

$\therefore$ Total moment of area about diametetral axis

$$= \int_0^{\pi}\int_0^{R} r^2 \sin\theta \, dr \cdot d\theta$$

$$= \int_0^{\pi} \left[\frac{r^3}{3}\right]_0^R \sin\theta \cdot d\theta$$

$$= \frac{R^3}{3} \left[-\cos\theta \right]_0^{\pi}$$

$-[-1-1]$

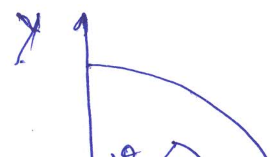
$$= \frac{R^3}{3} [1+1]$$

$$= \frac{2R^3}{3}.$$

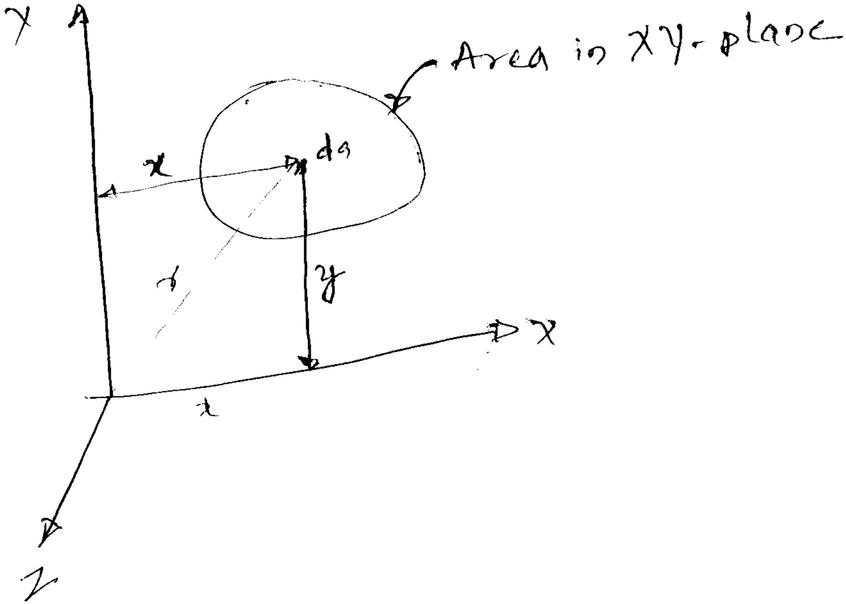
Area of semi circle $= \frac{\pi R^2}{2}$

$$\bar{y} = \frac{\text{moment of area}}{\text{Total area}} = \frac{2}{3} R^3 \cdot \frac{2}{\pi R^2}$$

$$\boxed{\bar{y} = \frac{4R}{3\pi}}$$



Polar moment of Inertia:

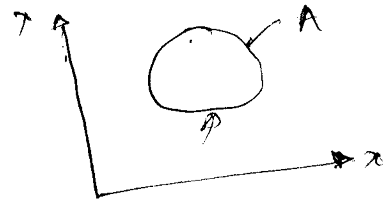


If a plane lamina lies in the xy plane as shown in figure. Then the axis perpendicular to xy plane is Z-axis. The moment of inertia of an area about an axis perpendicular to its plane is known as Polar moment of inertia.

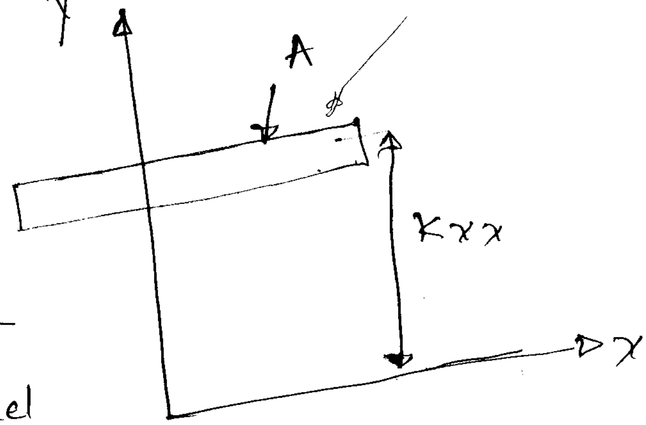
$$I_{zz} = \Sigma da \cdot r^2$$

$$= \Sigma da\ (x^2 + y^2)$$

$$I_{zz} = I_{xx} + I_{yy}$$



Radius of gyration :-



If the area shown in fig is concentrated into a strip of the same area and is placed parallel to x-axis at a distance of Kxx as shown, such that this strip has the same moment of inertia about x-axis as that of area A, then Kxx is known as the radius of gyration about x-axis.

$$I_{xx} = K_{xx}^2 A$$

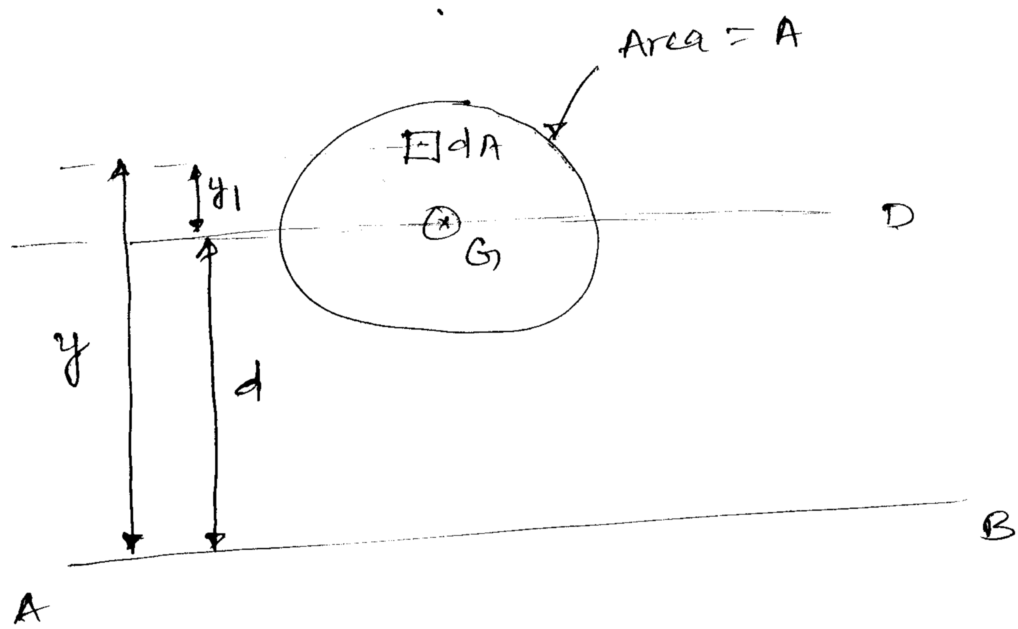
$$K_{xx} = \sqrt{\frac{I_{xx}}{A}}$$

IIIly $K_{yy} = \sqrt{\frac{I_{yy}}{A}}$

$$K_{zz} = \sqrt{\frac{I_{zz}}{A}}$$

Parallel axes theorem:

The moment of inertia of any area about an axis in its plane is the sum of moment of inertia about a parallel axis passing through the centroid of the area (known as centroidal axis) and the product of area and square of the distance between the two parallel axes.



In the above figure

$$I_{AB} = \int y^2 . dA$$

$$y = d + y_1$$

$$I_{AB} = \int (y_1 + d)^2 \, dA$$

$$= \int (y_1^2 + d^2 + 2 y_1 d) \, dA$$

$$I_{AB} = \int y_1^2 \, dA + \int d^2 \, dA + 2d \int y_1 \, d$$

The moment of inertia of ~~the~~ area A about centroidal axis CD is

$$I_G = \int y_1^2 \, dA$$

$\int y_1 \, dA$ is the moment of area A about its centroidal axis. As area is always symmetrical about its centroidal axis.

$$\int y_1 \, dA = 0 \qquad \text{also } \int dA = A$$

$$I_{AB} = I_G + 0 + d^2 A$$

$$I_{AB} = I_G + A d^2$$

This proves the parallel axes theorem

$$I_{AB} = K_{AB}^2 . A \quad \& \quad I_G = K_G^2 A$$

$$K_{AB}^2 A = K_G^2 A + A d^2$$

$$K_{AB}^2 = K_G^2 + d^2$$