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CMR INSTITUTE OF TECHNOLOGY

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Improvement Test

CMR

Sub:	Engineering Maths-III						Code:	15MAT31
Date:	18 / 11 /2016	Duration:	90 mins	Max Marks:	50	Sem: 3	Branch:	ME-B,EC-E,CS-D

NOTE: First question is compulsory. Answer any six questions from the rest.

	Marks	OBE CO	OBE RBT
1 . Evaluate $\int_0^{\frac{\pi}{2}} \cos x \, dx$ by Simpson's one third rule by taking eleven ordinates .Compare the value with the theoretical value.	[8]	CO3	L3
2 .Apply Lagrange's formula inversely to find a root of the equation $f(x)=0$ given that $f(30)=-30, f(34)=-13, f(38)=3, f(42)=18$.	[7]	CO3	L3
3 .Find $y(1.4)$ by Newton's forward interpolation formula from the given data	[7]	CO3	L3

x	1	2	3	4	5
y	10	26	58	112	194

4 .Find the correlation coefficient and the equation of lines of regression from the given data [7] CO6 L3

x	1	2	3	4	5
y	2	5	3	8	7

5 .If x and y are random variables with standard deviations σ_x and σ_y ,it was found that random variables $(x+y),(x-y),(2x+y)$ respectively have variance 15,11,29.Compute standard deviations of x,y and also the coefficient of correlation. [7] CO6 L3

6 . Fit a straight line of the form $y = ax + b$ for the following data [7] CO3 L3

x	5	10	15	20	25
y	16	19	23	26	30

7 .Find the negative root of the equation $x^3 - 4x + 9 = 0$ by Regula –Falsi method .Carry out two iterations. [7] CO3 L3

8 . Obtain the constant term and the coefficients of first cosine and sine terms in the Fourier expansion of y from the table [7] CO1 L3

x	0	1	2	3	4	5
y	9	18	24	28	26	20

	Course Outcomes	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1:	Evaluate the real form of the Fourier series for standard periodic and a periodic waveforms which has its applications in finding out the sum of infinite series using Dirichlet's conditions.	3	0	0	0	0	0	0	0	0	0	0	0
CO2:	Apply integral expressions for the forward and inverse Fourier transform to a range of non-periodic waveforms such as rectangular, unit-step, sinusoidal and exponential decay functions and to solve second order difference equations using Z transform and inverse Z transform	3	0	0	0	0	0	0	0	1	0	0	0
CO3:	Apply numerical techniques to perform various mathematical task such as solving equations, interpolation, integration and curve fitting	3	0	0	0	0	0	0	0	1	0	0	0
CO4:	Solve Brachistochrone, shortest distance, minimal surface area and hanging chain problems and to find the geodesics of known surfaces using Euler-Lagrange method.	3	0	0	0	0	0	0	0	0	0	0	0
CO5:	Evaluate line and surface integrals using Green's, Stoke's and Gauss divergence theorems which have its application in computing the amount of work done, area and volume.	3	0	0	0	0	0	0	0	0	0	0	0
CO6:	Estimate the strength of the relationship between the variables using correlation coefficients and to express the relationship in the form if an equation using regression.	3	0	0	0	0	0	0	0	1	0	0	0

Cognitive level	KEYWORDS
L1	List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
L2	summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend
L3	Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover.
L4	Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer.
L5	Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize.

PO1 - *Engineering knowledge*; PO2 - *Problem analysis*; PO3 - *Design/development of solutions*; PO4 - *Conduct investigations of complex problems*; PO5 - *Modern tool usage*; PO6 - *The Engineer and society*; PO7- *Environment and sustainability*; PO8 – *Ethics*; PO9 - *Individual and team work*; PO10 - *Communication*; PO11 - *Project management and finance*; PO12 - *Life-long learning*

Improvement test.

ME - B.
(Regular)
C_4 dip
EC-E, CS-D

Maths - 3.

1. $$I = \int_0^{\pi/2} \cos x \, dx$$

$h = \pi/20 = 9^\circ$, $n = 10$ — 1m

x°	0°	9	18	27	36	45	54	63	72	81	90
$y = \cos x$	1	0.9877	0.9511	0.891	0.8090	0.7071	0.5878	0.454	0.3090	0.1564	0
	y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	y_9	y_{10}

3m

By Simpson's $\frac{1}{3}$rd rule.

$$I = \frac{h}{3}\left[(y_0 + y_{10}) + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8)\right]$$ 1m

$$= \frac{1}{3} \cdot \frac{\pi}{20}\left[(1+0) + 4(0.9877 + 0.8910 + 0.7071 + 0.454 + 0.1564) + 2(0.9511 + 0.8090 + 0.5878 + 0.3090)\right]$$

$I = 1.00000358$ $\pi = \frac{22}{7}$ 2m

Theoritical value.

$$\int_0^{\pi/2} \cos x \, dx = [\sin x]_0^{\pi/2} = \sin \pi/2 - 0$$

$$= 1$$ 1m

5. By data $\sigma_{x+y}^2 = 15$, $\sigma_{x-y}^2 = 11$

$\sigma_{2x+y}^2 = 29$ — (1) 1m

$\sigma_{ax+by}^2 = a^2\sigma_x^2 + b^2\sigma_y^2 + 2rab\,\sigma_x\sigma_y$

$\sigma_{x+y}^2 = \sigma_x^2 + \sigma_y^2 + 2r\sigma_x\sigma_y$
$\sigma_{x-y}^2 = \sigma_x^2 + \sigma_y^2 - 2r\sigma_x\sigma_y$
$\sigma_{2x+y}^2 = 4\sigma_x^2 + \sigma_y^2 + 4r\sigma_x\sigma_y$ 2m

using (1) in these we have

$\sigma_x^2 + \sigma_y^2 + 2r\sigma_x\sigma_y = 15$ — (2), $\sigma_x^2 + \sigma_y^2 - 2r\sigma_x\sigma_y = 11$ — (3)

$4\sigma_x^2 + \sigma_y^2 + 4r\sigma_x\sigma_y = 29$ — (4)

(2) + (3) $2\sigma_x^2 + 2\sigma_y^2 = 26$

$\sigma_x^2 + \sigma_y^2 = 13$ — (5)

$2\times(5) + (4) \Rightarrow 6\sigma_x^2 + 3\sigma_y^2 = 51$

$2\sigma_x^2 + \sigma_y^2 = 17$ — (6) 2m

Solving (5) & (6)

$\sigma_x^2 = 4$, $\sigma_y^2 = 9$

$\sigma_x = 2$, $\sigma_y = 3$ 1m

using σ_x, σ_y in (2)

$4 + 9 + 12r = 15$, $r = 1/6 = 0.17$ 1m

2) $f(30) = -30$ $f(34) = -13$, $f(38) = 3$, $f(42) = 18$

$x_0 = 30$, $x_1 = 34$, $x_2 = 38$, $x_3 = 42$, $y_0 = -30$, $y_1 = -13$, $y_2 = 3$, $y_3 = 18$

$$x = \frac{(y-y_1)(y-y_2)(y-y_3)x_0}{(y_0-y_1)(y_0-y_2)(y_0-y_3)} + \frac{(y-y_0)(y-y_2)(y-y_3)x_1}{(y_1-y_0)(y_1-y_2)(y_1-y_3)}$$ 1m

$$+ \frac{(y-y_0)(y-y_1)(y-y_3)x_2}{(y_2-y_0)(y_2-y_1)(y_2-y_3)} + \frac{(y-y_0)(y-y_1)(y-y_2)x_3}{(y_3-y_0)(y_3-y_1)(y_3-y_2)}$$

2m

$$x(0) = \frac{(13)(-3)(-18)30}{(-17)(-33)(-48)} + \frac{30(-3)(-18)(34)}{(17)(-16)(-31)} + \frac{30(13)(-18)(38)}{(33)(16)(-15)}$$

$$+ \frac{(30)(13)(-3)42}{(48)(31)(15)}$$ 2m

$$= -0.7821 + 6.5322 + 33.6818 - 2.2016$$ m

$$x(0) = 37.2303$$ 1m

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3) Find $y(1.4)$

x	1	2	3	4	5
y	10	26	58	112	194

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1	10				
		16			
2	26		16		
		32		6	
3	58		22		0
		54		6	
4	112		28		
		82			
5	194				

2m

$$y_r = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \cdots$$ 1m

$$r = \frac{x - x_0}{h} = \frac{1.4-1}{1} = 0.4$$ 1m

$\Delta y_0 = 16$, $\Delta^2 y_0 = 16$, $\Delta^3 y_0 = 6$, $\Delta^4 y_0 = 0$

$$y(1.4) = f(1.4) = 10 + (0.4)16 + \frac{(0.4)(0.4-1)}{2}(16) + \frac{(0.4)(0.4-1)(0.4-2)}{6}(6)$$ 2m

$y(1.4) = 14.864$ 1m

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4) Correlation coefficient.

x	y	$z = x-y$	x^2	y^2	z^2
1	2	-1	1	4	1
2	5	-3	4	25	9
3	3	0	9	9	0
4	8	-4	16	64	16
5	7	-2	25	49	4
$\Sigma x = 15$	$\Sigma y = 25$	$\Sigma z = -10$	$\Sigma x^2 = 55$	$\Sigma y^2 = 151$	$30 = \Sigma z^2$

2m

$$\sigma_x^2 = \frac{\Sigma x^2}{n} - (\bar{x})^2 = \frac{55}{5} - (3)^2 = 2$$

$$\sigma_y^2 = \frac{\Sigma y^2}{n} - (\bar{y})^2 = \frac{151}{5} - (5)^2 = 5.2.$$ 1 m

$$\sigma_z^2 = \frac{\Sigma z^2}{n} - (\bar{z})^2 = \frac{30}{5} - (-2)^2 = 2.$$

$$r = \frac{\sigma_x^2 + \sigma_y^2 - \sigma_z^2}{2\sigma_x \sigma_y} = \frac{2 + 5.2 - 2}{2\sqrt{2}\sqrt{5.2}} \sim 0.81$$ 1m

$$(y - \bar{y}) = r \frac{\sigma_y}{\sigma_x} (x - \bar{x})$$

$$y - 5 = (0.81) \frac{\sqrt{5.2}}{\sqrt{2}} (x - 3)$$

$$y = 1.306x + 1.082$$ 2m

$$(x - \bar{x}) = r \frac{\sigma_x}{\sigma_y} (y - \bar{y})$$

$$x - 3 = (0.81) \frac{\sqrt{2}}{\sqrt{5.2}} (y - 5)$$

$$x = 0.502y + 0.49$$ 1m

6) Find the equation of best fitting st line.

x	y	xy	x^2
5	16	80	25
10	19	190	100
15	23	345	225
20	26	520	400
25	30	750	625
$\Sigma x = 75$	$\Sigma y = 114$	$\Sigma xy = 1885$	$\Sigma x^2 = 1375$

2m

1m

The normal eqs

$\Sigma y = a \Sigma x + nb$

$\Sigma xy = a \Sigma x^2 + b \Sigma x$.

1m

$75a + 5b = 114$

$1375a + 75b = 1885$ 1m

$a = 0.7, \quad b = 12.3$ 1m

$y = 0.7x + 12.3$ is the equation 1m

~~when~~

7) $x^3 - 4x + 9$, $f(-2) = 9 > 0$, $f(-3) = -6 < 0$ 1m

$f(x) = x^3 - 4x + 9$

$\therefore$ Negative ~~root~~ lies between $(-3, -2)$ 1m

neighbourhood of -3 $\quad f(-2.8) = -1.752, \; f(-2.7) = 0.117$

$\therefore$ I iteration

$a = -2.8$

$b = -2.7$

$$x_1 = \frac{a f(b) - b f(a)}{f(b) - f(a)}$$

$= -2.7063$ 2m

II iteration *

$f(-2.7063) = 0.0041 > 0$

$a = -2.8, \quad b = -2.7063$

$x_2 = -2.7065$ 2m

$\therefore x = -2.7065$ is the root correct to 3 decimal places 1m

8.

x	y	θ°	$\cos\theta$	$y\cos\theta$	$\sin\theta$	$y\sin\theta$
0	9	0	1	9	0	0
1	18	60	0.5	9	0.866	15.588
2	24	120	-0.5	-12	0.866	20.784
3	28	180	-1	-28	0	0
4	26	240	-0.5	-13	-0.866	-25.516
5	20	300	0.5	10	-0.866	-17.32

2m

$y = f(x) = \frac{a_0}{2} + a_1 \cos\theta + b_1 \sin\theta$ 1m

$\Sigma y = 125$ $\quad a_0 = \frac{2}{N} \Sigma y = \frac{1}{3}(125) = 41.67$

$\Sigma y\cos\theta = -25$ $\quad a_1 = \frac{2}{N} \Sigma y\cos\theta = \frac{1}{3}(-25) = -8.333$

1m

$\Sigma y\sin\theta = -3.464$ $\quad b_1 = \frac{2}{N} \Sigma y \sin\theta = \frac{1}{3}(-3.464) = -1.155$

2m

Constant term $= \frac{a_0}{2} = 20.835$

Coeff of first cosine term $= a_1 = -8.333$

coeff of first sine term $= b_1 = -1.155$

1m

8. By Green's theorem

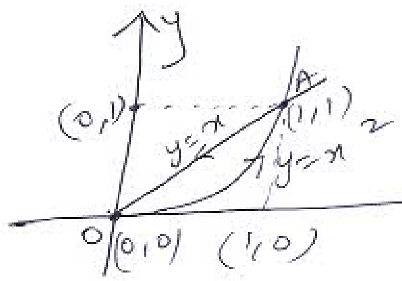
$$\int_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy \quad 1m$$

The pts of intersection are (0,0) (1,1)

$y = x,\ y = x^2$

$$\int_C (xy + y^2)\,dx + x^2\,dy = \int_{OA} (xy+y^2)\,dx + x^2\,dy + \int_{AO} (xy+y^2)\,dx + x^2\,dy$$

$$I_1 + I_2$$



1m

$$I_1 = \int_{x=0}^{1} (x \cdot x^2 + x^4)\,dx + 2x^3\,dx$$

$I_1 \Rightarrow y = x^2$

$dy = 2x\,dx$

$x \to 0$ to 1

$$= \int_0^1 (3x^3 + x^4)\,dx = 3\left[\frac{x^4}{4}\right]_0^1 + \left[\frac{x^5}{5}\right]_0^1 = \frac{19}{20}$$

Along I_2 i.e. $y = x$, $dy = dx$ $x \to 1$ to 0

$$I_2 = \int_{x=0}^{1} (x \cdot x + x^2)\,dx = \int_1^0 3x^2\,dx = -1$$

$$\therefore \int_C (xy + y^2)\,dx + x^2\,dy = \frac{19}{20} - 1 = \frac{-1}{20} \quad 2m$$

$M = xy + y^2,\ N = x^2$

$$\iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) dx\,dy$$

$$\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} = x - 2y \quad 1m$$

$$= \int_{x=0}^{1}\int_{y=x^2}^{x} (x - 2y)\,dy\,dx$$

$\therefore$ Theorem is verified $$= \int_{x=0}^{1} \left[xy - y^2\right]_{y=x^2}^{x} dx = \int_{x=0}^{1} (x^4 - x^3)\,dx = \frac{-1}{20} \quad 2m$$