CMR
INSTITUTE OF
TECHNOLOGY

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Improvement Test

Sub:	Engineering Mathematics III							Code:	15MAT31
Date:	18/11/2016	Duration:	90 mins	Max Marks:	50	Sem:	III	Branch:	ECE- 3A, 3C, 3D

Note: First question is *compulsory*. Answer ANY SIX questions from the rest.

		Marks	OBE CO	OBE RBT
1.	Using Z-transform, solve $y_{n+2} - 4y_n = 0$, given that $y_0 = 0,\ y_1 = 2$.	[08]	CO2	L3
2.	Find the Fourier sine tranform of $e^{-\lvert x\rvert}$. Hence show that $\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m},\ m > 0$.	[07]	CO2	L3
3.	Obtain the Fourier series of $f(x) = \lvert x \rvert$ valid in the interval $(-l, l)$.	[07]	CO1	L3
4.	Calculate the Karl Pearson's coefficient of correlation for 10 students who have obtained the following percentage of marks in Mathematics and Electronics:	[07]	CO6	L2

Roll No.	1	2	3	4	5	6	7	8	9	10
Marks in Mathematics	78	36	98	25	75	82	90	62	65	39
Marks in Electronics	84	51	91	60	68	62	86	58	53	47

		Marks	CO	RBT
5.	Find the real root of the equation near correct four decimal places, $x \sin x + \cos x = 0$ near $x = \pi$ correct four decimal places, using Newton- Raphson method. Carryout three iterations.	[07]	CO3	L2
6.	From the following table, which gives the distance y (in nautical miles) of the visible horizon for the given heights x (in feet) above the earth' surface, find the value of y at x= 410:	[07]	CO3	L3

x	100	150	200	250	300	350	400
y	10.63	13.03	15.04	16.81	18.42	19.90	21.27

		Marks	CO	RBT
7.	Derive Euler's equation in the standard form.	[07]	CO4	L3
8.	Using Stoke's theorem, evaluate $\iint_S (\nabla \times \vec{F}) \bullet \hat{n}\, dS$ where $\vec{F} = 3y\,\hat{i} - xz\,\hat{j} + yz^2\,\hat{k}$ and S is the surface of the paraboloid $2z = x^2 + y^2$ bounded by $z = 2$.	[07]	CO5	L3

Course Outcomes		PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1:	Evaluate the real form of the Fourier series for standard periodic and a periodic waveforms which has its applications in finding out the sum of infinite series using Dirichlet's conditions.	3	0	0	0	0	0	0	0	0	0	0	0
CO2:	Apply integral expressions for the forward and inverse Fourier transform to a range of non-periodic waveforms such as rectangular, unit-step, sinusoidal and exponential decay functions and to solve second order difference equations using Z transform and inverse Z transform	3	0	0	0	0	0	0	0	1	0	0	0
CO3:	Apply numerical techniques to perform various mathematical task such as solving equations, interpolation, integration and curve fitting	3	0	0	0	0	0	0	0	1	0	0	0
CO4:	Solve Brachistochrone, shortest distance, minimal surface area and hanging chain problems and to find the geodesics of known surfaces using Euler-Lagrange method.	3	0	0	0	0	0	0	0	0	0	0	0
CO5:	Evaluate line and surface integrals using Green's, Stoke's and Gauss divergence theorems which have its application in computing the amount of work done, area and volume.	3	0	0	0	0	0	0	0	0	0	0	0
CO6:	Estimate the strength of the relationship between the variables using correlation coefficients and to express the relationship in the form if an equation using regression.	3	0	0	0	0	0	0	0	1	0	0	0

Cognitive level	KEYWORDS
L1	List, define, tell, describe, identify, show, label, collect, examine, tabulate, quote, name, who, when, where, etc.
L2	summarize, describe, interpret, contrast, predict, associate, distinguish, estimate, differentiate, discuss, extend
L3	Apply, demonstrate, calculate, complete, illustrate, show, solve, examine, modify, relate, change, classify, experiment, discover.
L4	Analyze, separate, order, explain, connect, classify, arrange, divide, compare, select, explain, infer.
L5	Assess, decide, rank, grade, test, measure, recommend, convince, select, judge, explain, discriminate, support, conclude, compare, summarize.

PO1 - *Engineering knowledge*; PO2 - *Problem analysis*; PO3 - *Design/development of solutions*; PO4 - *Conduct investigations of complex problems*; PO5 - *Modern tool usage*; PO6 - *The Engineer and society*; PO7- *Environment and sustainability*; PO8 – *Ethics*; PO9 - *Individual and team work*; PO10 - *Communication*; PO11 - *Project management and finance*; PO12 - *Life-long learning*

II Improvement test

Eng. Math III

Solution

Q1

$$y_{n+2} - 4y_n = 0$$

Taking Z-transform on both sides

$$Z_T(y_{n+2}) - 4 Z_T(y_n) = 0$$

$$\Rightarrow \quad z^2\left[Z_T(y_n) - y_0 - y_1 z^{-1}\right] - 4 Z_T(y_n) = 0$$

$$\Rightarrow \quad z^2\left(\bar{y}(z) - 2z^{-1}\right) - 4\bar{y}(z) = 0$$

$Z_T(y_n) = \bar{y}(z)$
$y_0 = 0, \; y_1 = 2$

$$\Rightarrow \quad (z^2-4)\,\bar{y}(z) = 2z$$

or,

$$\bar{y}(z) = \frac{2z}{(z+2)(z-2)} = A\frac{z}{z+2} + B\frac{z}{z-2}$$

or,

$$2z = Az(z-2) + Bz(z+2)$$

or,

$$2 = A(z-2) + B(z+2)$$

Put $z = 2 \Rightarrow B = 1/2$

$z = -2 \Rightarrow A = -1/2$

$$\therefore \quad \bar{y}(z) = \frac{1}{2}\frac{z}{z-2} - \frac{1}{2}\frac{z}{z+2}$$

$$= \frac{1}{2}2^n - \frac{1}{2}(-2)^n$$

$$= 2^{n-1} + (-2)^{n-1}$$ is the required solution.

Q2 $f(x) = e^{-|x|}$

Fourier sine transform

$$F_s(u) = \int_0^\infty e^{-|x|} \sin ux \, dx = \int_0^\infty e^{-x} \sin ux \, dx$$

$$= \left| \frac{e^{-x}(-\sin ux - u\cos ux)}{1+u^2} \right|_0^\infty$$

$$= \frac{u}{1+u^2}$$

By taking inverse fourier sine transform we have

$$f(x) = \frac{2}{\pi} \int_0^\infty F_s(u) \sin ux \, du$$

$$\Rightarrow \quad e^{-|x|} = \frac{2}{\pi} \int_0^\infty \frac{u}{1+u^2} \sin ux \, du$$

putting $x = m$ where $m > 0$ we have $e^{-|m|} = e^{-m}$

and

$$\int_0^\infty \frac{x \sin mx}{1+x^2} dx = \frac{\pi}{2} e^{-m}$$

Q3 Here $f(x) = |x|$ in $(-l, l)$

The period is $l + l = 2l$ and fourier series of period $2l$ is given by

$$f(x) \approx \frac{a_0}{2} + \sum_{n=1}^\infty a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l}$$

$f(-x) = |-x| = |x| = f(x)$

$\Rightarrow f(x)$ is an even fn! in $(-l, l)$

$\Rightarrow \quad b_n = 0 \quad \forall n$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx = \frac{2}{l}\int_0^l x\,dx = \frac{2}{l}\left|\frac{x^2}{2}\right|_0^l$$

$$= \frac{2}{l}\,\frac{l^2}{2} = l$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\frac{n\pi x}{l}\,dx$$

$$= \frac{2}{l}\int_0^l \underset{u}{x}\,\underset{v}{\cos\frac{n\pi x}{l}}\,dx$$

$$= \frac{2}{l}\left|x\sin\frac{n\pi x}{l}\times\frac{l}{n\pi} + 1\cdot\frac{l^2}{n^2\pi^2}\cos\frac{n\pi x}{l}\right|_0^l$$

$$= \frac{2}{l}\,\frac{l^2}{n^2\pi^2}\left(\cos n\pi - 1\right)$$

$$= \frac{2l}{n^2\pi^2}\left(\cos n\pi - 1\right)$$

$$\therefore\ f(x) \approx \frac{l}{2} + \sum_{n=1}^{\infty}\frac{2l}{n^2\pi^2}\left\{(-1)^n - 1\right\}\cos\frac{n\pi x}{l}.$$

Q4 We have $\bar{x} = \dfrac{\Sigma x}{n} = \dfrac{650}{10} = 65$

$\bar{y} = \dfrac{\Sigma y}{n} = \dfrac{660}{10} = 66$

i.e. x	y	$X = x - \bar{x}$	$Y = y - \bar{y}$	X^2	Y^2	XY
78	84	13	18	169	325	234
36	51	-29	-15	841	225	435
98	91	33	25	1089	625	825
25	60	-40	-6	1600	36	240
75	68	10	2	100	4	20
82	62	17	-4	289	16	-68
90	86	25	20	625	400	500
62	58	-3	-8	9	64	24
65	53	0	-13	0	169	0
39	47	-26	-19	676	361	494
				$\Sigma X^2 = 5398$	$\Sigma Y^2 = 2225$	$\Sigma XY = 2704$

$$r = \frac{\Sigma xy}{\sqrt{\Sigma x^2 \Sigma y^2}} = 0.7802.$$

Q.5 Let $f(x) = x \sin x + \cos x$

$\therefore f'(x) = x \cos x$

Also $x_0 = \pi$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 2.8233$$

Now $$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.7986$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.7984$$

Required real root is 2.7984 after three iterations.

Q6 Finite difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
100	10.63						
		2.4					
150	13.03		−0.39				
		2.01		0.15			
200	15.04		−0.24		−0.07		
		1.77		.08		0.02	
250	16.81		−0.16		−0.05		$\boxed{0.02}$
		1.61		.03		$\boxed{0.04}$	
300	18.42		−0.13		$\boxed{-0.01}$		
		1.48		$\boxed{.02}$			
350	19.90		$\boxed{-0.11}$				
		$\boxed{1.37}$					
400	$\boxed{21.27}$						

To find $y(410)$ we will use Newton's backward interpolation

$x = 410$, $\quad p = \frac{x - x_n}{h} = \frac{410-400}{50} = 0.2$

$$y = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n + \cdots$$

$\Rightarrow y(410) = 21.27 + 0.2 \times 1.37 + \frac{0.2(1.2)}{2!}(-0.11) + \frac{0.2(1.2)(2.2)}{6} 0.02 + \frac{0.2(1.2)(2.2)(3.2)}{4!}(-0.01) + \frac{0.2(1.2)(2.2)(3.2)(4.2)}{5!} 0.04 + \frac{.2 \; 1.2 \; 2.2 \; 3.2 \; 4.2 \; 5.2}{6!} 0.02$

$= 21.53524647$

Q7 Let $y(x)$ be such a function which minimises $I[y]$.

Let $\bar{y}(x)$ is any other admissible function

Then $I[y] \leq I[\bar{y}]$

Let $\bar{y}(x) = y(x) + \varepsilon\, n(x)$

Since $y(x_1) = \bar{y}(x_1)$ and

$y(x_2) = \bar{y}(x_2)$

$\therefore n(x_1) = n(x_2) = 0$

and $\bar{y}(x) = y(x)$ if $\varepsilon = 0$

$$I[\bar{y}] = \int_{x_1}^{x_2} f(x, \bar{y}, \bar{y}')\, dx$$

$$= \int_{x_1}^{x_2} f(x, y + \varepsilon n, y' + \varepsilon n')\, dx = \phi(\varepsilon)$$

as $\phi(\varepsilon)$ is a function of parameter ε. Thus the problem of determining $y(x)$ reduces to finding the extremum of $\phi(\varepsilon)$ at $\varepsilon = 0$

$\Rightarrow \phi'(\varepsilon) = 0$ at $\varepsilon = 0$

$$\Rightarrow \frac{d}{d\varepsilon} \int_{x_1}^{x_2} f(x, y + \varepsilon n, y' + \varepsilon n')\, dx = 0 \quad \text{at } \varepsilon = 0$$

Using Leibnitz rule of differentiation under integration

$$\int_{x_1}^{x_2} \left(n(x) \frac{\partial f}{\partial \bar{y}} + n'(x) \frac{\partial f}{\partial \bar{y}'} \right) dx = 0 \quad \text{at } \varepsilon = 0$$

$$\Rightarrow \int_{x_1}^{x_2} \left[\frac{\partial f}{\partial y} n + \frac{\partial f}{\partial y'} n' \right] dx = 0 \quad \text{at } \varepsilon = 0 \; y = \bar{y}$$

Let Consider $\displaystyle\int_{x_1}^{x_2} \underset{u}{\frac{\partial f}{\partial y'}} \underset{v}{n'} \, dx$

$$= \left| \frac{\partial f}{\partial y'} n \right|_{x_1}^{x_2} - \int_{x_1}^{x_2} \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) n \, dx$$

$$= - \int_{x_1}^{x_2} \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) n(x) \, dx \qquad \because n(x_2) = n(x_1) = 0$$

$$\therefore \int_{x_1}^{x_2} \left[\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) \right] n(x) \, dx = 0$$

Since $n(x)$ is arbitrary

$$\Rightarrow \frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) = 0 .$$

Q8 $\displaystyle\iint_S \bar{\nabla} \times \bar{F} \cdot \hat{n} \, ds = \oint_C \bar{F} \cdot d\bar{r}$

$$= \oint_C 3y\,dx - xz\,dy + yz^2\,dz \qquad z = 2 \Rightarrow dz = 0$$

$$= \oint_C 3y\,dx - 2x\,dy$$

$x^2 + y^2 = 2z = 4$ Let $x = 2\cos\theta$, $y = 2\sin\theta$

$\Rightarrow dx = -2\sin\theta\, d\theta, \qquad dy = 2\cos\theta\, d\theta$

$$\oint_C \left[6\sin\theta(-2\sin\theta) - 4\cos\theta(2\cos\theta)\right] d\theta$$

$$= \int_{\theta=0}^{2\pi} \left(-12\sin^2\theta - 8\cos^2\theta\right) d\theta$$

$$= \int_0^{2\pi} \left(-8 - 4\sin^2\theta\right) d\theta$$

$$= -8 \times 2\pi - 2\int_0^{4\pi} (1 - \cos 2\theta)\, d\theta$$

$$= -16\pi - 4\pi + 0 = \underline{\underline{-20\pi}}.$$